Formal Methods for Software Development

Reasoning about Programs with Dynamic Logic

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Dynamic Logic

(JAVA) Dynamic Logic

Typed FOL

- + (JAVA) programs p
- + modalities $\langle p \rangle \phi$, $[p]\phi$ (p program, ϕ DL formula)
- + ... (later)

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Remark on Hoare Logic and DL

In Hoare logic $\{\mathrm{Pre}\}\ \mathrm{p}\ \{\mathrm{Post}\}$ (Pre, Post must be FOL)

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Remark on Hoare Logic and DL

In Hoare logic $\{\mathrm{Pre}\}\ \mathrm{p}\ \{\mathrm{Post}\}$ (Pre, Post must be FOL)

In DL $\mathrm{Pre} \rightarrow [\mathrm{p}]\mathrm{Post}$ (Pre, Post any DL formula)

Proving DL Formulas

An Example

$$
\begin{array}{l}
\forall\ \texttt{int}\ x;\\
\quad (x = \texttt{n} \land x >= 0 \rightarrow\\
\qquad [\ \texttt{i} = 0; \texttt{r} = 0;\\
\qquad\ \ \texttt{while}(\texttt{i} < \texttt{n})\{\texttt{i} = \texttt{i} + 1; \texttt{r} = \texttt{r} + \texttt{i};\}\\
\qquad\ \ \texttt{r} = \texttt{r} + \texttt{r} - \texttt{n};\\
\qquad]\texttt{r} = x * x)
\end{array}
$$

How can we prove that the above formula is valid
(i.e. satisfied in all states)?

Semantics of DL Sequents

$\Gamma = \{\phi_1, \ldots, \phi_n\}$ and $\Delta = \{\psi_1, \ldots, \psi_m\}$ sets of DL formulas
where all logical variables occur bound.

Recall: $\mathcal{S} \models (\Gamma \Longrightarrow \Delta)$ iff $\mathcal{S} \models (\phi_1 \wedge \cdots \wedge \phi_n) \rightarrow (\psi_1 \vee \cdots \vee \psi_m)$

Define semantics of DL sequents identical to semantics of FOL sequents

Definition (Validity of Sequents over DL Formulas)

A sequent $\Gamma \Longrightarrow \Delta$ over DL formulas is valid iff

$$\mathcal{S} \models (\Gamma \Longrightarrow \Delta) \text{ in all states } \mathcal{S}$$

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Consequence for program variables

Initial value of program variables implicitly "universally quantified"

Symbolic Execution of Programs

Sequent calculus decomposes top-level operator in formula.
What is "top-level" in a sequential program `p; q; r;` ?

Symbolic Execution

- Follow the natural control flow when analysing a program
- Values of some variables unknown: symbolic state representation

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Example

Compute the final state after termination of

```
x=x+y; y=x-y; x=x-y;
```

Symbolic Execution of Programs Cont'd

Typical form of DL formulas in symbolic execution

$$\langle \texttt{stmt; rest} \rangle \phi \qquad [\texttt{stmt; rest}]\phi$$

- Rules symbolically execute *first* statement ("active statement")
- Repeated application of such rules corresponds to symbolic program execution

Symbolic Execution of Programs Cont'd

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Example (`symbolicExecution/simpleIf.key`, Demo, active statement only)

```
\programVariables {
 int x; int y; boolean b;
}
\problem {
 \<{ if (b) { x = 1; } else { x = 2; } y = 3; }\> y > x
}
```

Symbolic Execution of Programs Cont'd

Symbolic execution of conditional

$$\text{if}\ \frac{\Gamma, \mathtt{b} = \mathsf{TRUE} \Longrightarrow \langle \mathtt{p;\ rest} \rangle \phi, \Delta \qquad \Gamma, \mathtt{b} = \mathsf{FALSE} \Longrightarrow \langle \mathtt{q;\ rest} \rangle \phi, \Delta}{\Gamma \Longrightarrow \langle \mathbf{if}\ \mathtt{(b)\ \{\ p\ \}}\ \mathbf{else}\ \mathtt{\{\ q\ \}\ ;\ rest} \rangle \phi, \Delta}$$

Symbolic execution must consider all possible execution branches

Symbolic Execution of Programs Cont'd

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Symbolic execution of loops: unwind

$$\text{unwindLoop}\ \frac{\Gamma \Longrightarrow \langle \mathbf{if}\ \mathtt{(b)\ \{\ p;}\ \mathbf{while}\ \mathtt{(b)\ p\ \};\ rest} \rangle \phi, \Delta}{\Gamma \Longrightarrow \langle \mathbf{while}\ \mathtt{(b)\ \{p\};\ rest} \rangle \phi, \Delta}$$

Updates for KeY-Style Symbolic Execution

Needed: a Notation for Symbolic State Changes

- Symbolic execution should “walk” through program in natural forward direction
- Need succint representation of state changes, effected by each symbolic execution step
- Want to simplify effects of program execution early
- Want to apply state changes late (to branching conditions and post condition)

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We use dedicated notation for state changes: updates

Explicit State Updates

Definition (Syntax of Updates, Updated Terms/Formulas)

If $\mathtt{v}$ is program variable, t FOL term type-conformant to $\mathtt{v}$,
t' any FOL term, and ϕ any DL formula, then

- $\{\mathtt{v} := t\}$ is an update
- $\{\mathtt{v} := t\}t'$ is DL term
- $\{\mathtt{v} := t\}\phi$ is DL formula

Definition (Semantics of Updates)

State $\mathcal{S}$ interprets program variables v with $\mathcal{I}_{\mathcal{S}}(v)$
β variable assignment for logical variables in t, define semantics ρ as:

$\rho_\beta(\{\mathtt{v} := t\})(\mathcal{S}) = \mathcal{S}'$ where $\mathcal{S}'$ identical to $\mathcal{S}$ except $\mathcal{I}_{\mathcal{S}'}(\mathtt{v}) = val_{\mathcal{S},\beta}(t)$

Explicit State Updates Cont'd

Facts about updates $\{\mathtt{v} := t\}$

- Update semantics similar to that of assignment
- Value of update also depends on $\mathcal{S}$ and logical variables in t, i.e., β
- Updates are not assignments: right-hand side is FOL term

  $\{\mathtt{x} := n\}\phi$ cannot be turned into assignment (n logical variable)

  $\langle\mathtt{x=i++;}\rangle\phi$ cannot (immediately) be turned into update
- Updates are not equations: they change value of $\mathtt{v}$

Computing Effect of Updates (Automated)

Rewrite rules for update followed by ...

program variable
$$\begin{cases} \{\mathrm{x} := t\}\mathrm{x} & \leadsto & t \\ \{\mathrm{x} := t\}\mathrm{y} & \leadsto & \mathrm{y} \end{cases}$$

logical variable $\{\mathrm{x} := t\}w \leadsto w$

complex term $\{\mathrm{x} := t\}f(t_1, \ldots, t_n) \leadsto f(\{\mathrm{x} := t\}t_1, \ldots, \{\mathrm{x} := t\}t_n)$

(because f is rigid)

atomic formula $\{\mathrm{x} := t\}p(t_1, \ldots, t_n) \leadsto p(\{\mathrm{x} := t\}t_1, \ldots, \{\mathrm{x} := t\}t_n)$

FOL formula
$$\begin{cases} \{\mathrm{x} := t\}(\phi \ \& \ \psi) \leadsto \{\mathrm{x} := t\}\phi \ \& \ \{\mathrm{x} := t\}\psi \\ \ldots \\ \{\mathrm{x} := t\}(\forall \tau \ y;\ \phi) \leadsto \forall \tau \ y;\ (\{\mathrm{x} := t\}\phi) \end{cases}$$

program formula No rewrite rule for $\{\mathrm{x} := t\}(\langle \mathrm{p} \rangle \phi)$ unchanged!

Update rewriting delayed until p symbolically executed

Assignment Rule Using Updates

Symbolic execution of assignment using updates

$$\text{assign}\ \frac{\Gamma \Longrightarrow \{\mathtt{x} := t\}\langle\mathtt{rest}\rangle\phi, \Delta}{\Gamma \Longrightarrow \langle\mathtt{x\ =\ t;\ rest}\rangle\phi, \Delta}$$

- Simple! No variable renaming, etc.
- Works as long as t is 'simple' (has no side effects)

Demo

`updates/assignmentToUpdate.key`

Parallel Updates

How to apply updates on updates?

Example

Symbolic execution of

```
t=x; x=y; y=t;
```

yields:

```
{t := x}{x := y}{y := t}
```

Need to compose three sequential state changes into a single one:

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Need to compose three sequential state changes into a single one:
parallel updates

Parallel Updates Cont'd

Definition (Parallel Update)

A parallel update has the form $\{v_1 := r_1 || \cdots || v_n := r_n\}$, where each $\{v_i := r_i\}$ is simple update

- All r_i computed in old state before update is applied
- Updates of all program variables v_i executed simultaneously
- Upon conflict $v_i = v_j,\ r_i \neq r_j$ later update ($\max\{i, j\}$) wins

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Definition (Parallelising Updates, Conflict Resolution)

$$\{v_1 := r_1\}\{v_2 := r_2\} \ = \ \{v_1 := r_1 || v_2 := \{v_1 := r_1\}r_2\}$$

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$$\{v_1 := r_1\}\{v_2 := r_2\} \;=\; \{v_1 := r_1 || v_2 := \{v_1 := r_1\} r_2\}$$

$$\{v_1 := r_1 || \cdots || v_n := r_n\}\mathtt{x} \;=\; \begin{cases} \mathtt{x} & \text{if } \mathtt{x} \notin \{v_1, \ldots, v_n\} \\ r_k & \text{if } \mathtt{x} = v_k,\ \mathtt{x} \notin \{v_{k+1}, \ldots, v_n\} \end{cases}$$

Symbolic Execution with Updates (by Example)

$$\Longrightarrow \texttt{x} < \texttt{y} \; \texttt{->} \; \langle \texttt{t=x; x=y; y=t;} \rangle \; \texttt{y} < \texttt{x}$$

Symbolic Execution with Updates (by Example)

$$\mathtt{x} < \mathtt{y} \implies \{\mathtt{t:=x}\}\langle\mathtt{x=y;\ y=t;}\rangle\, \mathtt{y} < \mathtt{x}$$

$$\vdots$$

$$\implies \mathtt{x} < \mathtt{y} \ \mathtt{\text{->}}\ \langle\mathtt{t=x;\ x=y;\ y=t;}\rangle\, \mathtt{y} < \mathtt{x}$$

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Parallel Updates Cont'd

Demo

`updates/swap1.key`

Parallel Updates Cont'd

Example

symbolic execution of `x=x+y; y=x-y; x=x-y;` gives

`({x := x+y}{y := x-y}){x := x-y}`

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symbolic execution of `x=x+y; y=x-y; x=x-y;` gives

```
({x := x+y}{y := x-y}){x := x-y}
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```

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KeY automatically deletes overwritten (unnecessary) updates

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Parallel updates store intermediate state of symbolic computation

Another use of Updates

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Not allowed: $\forall \tau\ \mathtt{i}; \langle \mathtt{...i...} \rangle \phi$
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Not allowed: $\forall\,\tau\ \mathtt{i};\ \langle\ldots\mathtt{i}\ldots\rangle\phi$
(program variables $\cap$ logical variables $= \emptyset$)

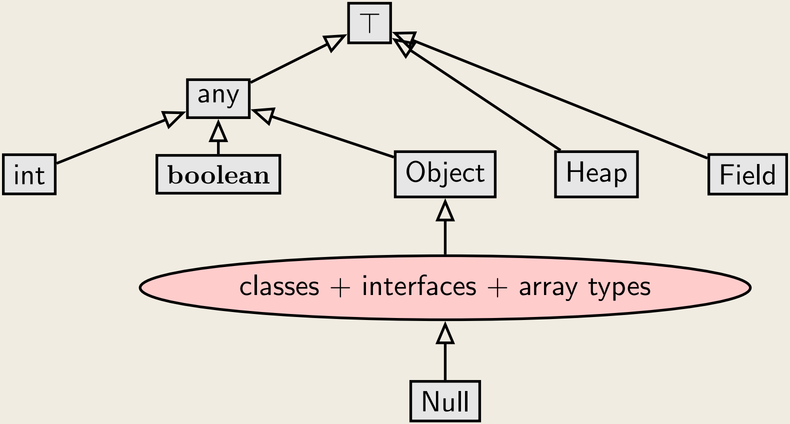
Instead

Quantify over value, and assign it to program variable:

$\forall\,\tau\ x;\ \{\mathtt{i} := x\}\langle\ldots\mathtt{i}\ldots\rangle\phi$

Modelling Java in FOL: Fixing a Type Hierarchy

Signature based on Java's type hierarchy



Each interface and class in API and in target program becomes type with appropriate subtype relation

Modelling the Heap in FOL

The Java Heap

Objects are stored on (i.e., in) the heap.

- Status of heap changes during execution
- Each heap associates values to object/field pairs

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Each element of data type Heap represents a certain heap status.

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- in F_{Σ}: Heap `store`(Heap, Object, Field, any);
  `store`(h, o, f, v) returns heap like h, but with v associated to (o, f)

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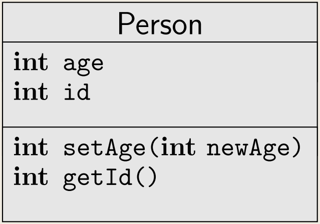
- in F_Σ: any `select`(Heap, Object, Field)`;`
  `select`(h, o, f) returns value associated to (o, f) in h

Modelling the Heap in FOL

Modelling instance fields



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Modelling the Heap in FOL

Modelling instance fields

Person
`int age` `int id`
`int setAge(int newAge)` `int getId()`

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Reading Field `id` of Person `p`

FOL notation $\mathtt{select}(h, \mathtt{p}, \mathtt{id})$

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KeY notation $\mathtt{p.id@}h$ (abbreviating $\mathtt{select}(h, \mathtt{p}, \mathtt{id})$)

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KeY notation $\mathtt{p.id@}h$ (abbreviating $\mathtt{select}(h, \mathtt{p}, \mathtt{id})$)

$\mathtt{p.id}$ (abbreviating $\mathtt{select}(\mathtt{heap}, \mathtt{p}, \mathtt{id})$)[a]

[a] `heap` is special program variable for "current" heap; mostly implicit in *o.f*

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Modelling instance fields

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Writing to Field `id` of Person `p`

FOL notation $\mathtt{store}(h, \mathtt{p}, \mathtt{id}, 6238)$

Modelling the Heap in FOL

Modelling instance fields

Person
int age int id
int setAge(int newAge) int getId()

- for each JAVA reference type C there is a type $C \in T_\Sigma$, for example, Person
- for each field `f` there is a unique constant `f` of type Field, for example, `id`
- domain of all Person objects: D^{Person}
- a heap relates objects and fields to values

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FOL notation $\mathtt{store}(h, \mathtt{p}, \mathtt{id}, 6238)$

KeY notation $h[\mathtt{p.id} := 6238]$ (notation for `store`, not update)

The Algebra of Heaps

We do *not* formalise the *structure* (implementation) of heaps.
We formalise the *behaviour*, with an algebra of heap operations:

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Example

$\mathtt{select}(\mathtt{store}(h, \mathtt{o}, \mathtt{f}, 15), \mathtt{o}, \mathtt{f}) \rightsquigarrow 15$
$\mathtt{select}(\mathtt{store}(h, \mathtt{o}, \mathtt{f}, 15), \mathtt{o}, \mathtt{g}) \rightsquigarrow \mathtt{select}(h, \mathtt{o}, \mathtt{g})$
$\mathtt{select}(\mathtt{store}(h, \mathtt{o}, \mathtt{f}, 15), \mathtt{u}, \mathtt{f}) \rightsquigarrow$
$\quad \text{if } (\mathtt{o} = \mathtt{u}) \text{ then } (15) \text{ else } (\mathtt{select}(h, \mathtt{u}, \mathtt{f}))$

Pretty Printing

Shorthand Notations for Heap Operations

$\mathtt{o.f@h}$	is	$\mathtt{select}(\mathtt{h}, \mathtt{o}, \mathtt{f})$
$\mathtt{h}[\mathtt{o.f} := \mathtt{v}]$	is	$\mathtt{store}(\mathtt{h}, \mathtt{o}, \mathtt{f}, \mathtt{v})$

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Very-Shorthand Notations for Current Heap

Current heap always in special variable $\mathtt{heap}$.

$\mathtt{o.f}$	is	$\mathtt{select}(\mathtt{heap}, \mathtt{o}, \mathtt{f})$
$\{\mathtt{o.f} := \mathtt{v}\}$	is update	$\{\mathtt{heap} := \mathtt{heap}[\mathtt{o.f} := \mathtt{x}]\}$

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$\mathtt{int::select}(h, \mathrm{p}, \mathsf{Person::\$id}) >= 0$ is type-safe

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General

For each `T` typed field `f` of class `C`, F_Σ contains

- a constant declared as Field `C::$f`
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Everything blue is a function name

Modelling the Heap in FOL—The Full Story

Writing to Fields

Declaration: Heap `store`(Heap, Object, Field, any)`;`

Usage: `store`(h, `p`, Person::`$id`, `42`)

Field Update Assignment Rule

Changing the value of fields

How to translate assignment to field, for example, `p.age=18;` ?

$$\text{assign}\ \frac{\Gamma \Longrightarrow \{\mathtt{o.f} := t\}\langle\mathtt{rest}\rangle\phi, \Delta}{\Gamma \Longrightarrow \langle\mathtt{o.f\ =\ t;\ rest}\rangle\phi, \Delta}$$

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Dynamic Logic: KeY input file

```
\javaSource "path to source code referenced in problem";

\programVariables { Person p; }

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      \<{   p.age = 18;  }\> p.age = 18
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Demo

`updates/firstAttributeExample.key`

Refined Semantics of Program Modalities

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- $[\mathrm{p}]\phi$: If p terminates normally then formula ϕ holds in final state (partial correctness)

Abrupt termination on top-level counts as non-termination!

Example Reconsidered: Exception Handling

```
\javaSource "path to source code";

\programVariables {
  ...
}

\problem {
       p != null -> \<{   p.age = 18;  }\> p.age = 18
}
```

Only provable when no top-level exception thrown

Demo

`updates/secondAttributeExample.key`

The Self Reference

Modeling reference this to the receiving object

Special name for the object whose JAVA code is currently executed:

in JML: `Object` **this**;

in Java: `Object` **this**;

in KeY: `Object self;`

Default assumption in JML-KeY translation: $\mathtt{self} \mathbin{!=} \mathbf{null}$

Which Objects do Exist?

How to model object creation with **new** ?

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Constant Domain Assumption

Assume that domain $\mathcal{D}$ is the same in all states $(\mathcal{D}, \delta, \mathcal{I}) \in \mathit{States}$

Consequence:
Quantifiers and modalities commute:

$$\models (\forall\ T\ x;\ [\mathrm{p}]\phi) \leftrightarrow [\mathrm{p}](\forall\ T\ x;\ \phi)$$

Object Creation (background; no need to remember this)

Realizing Constant Domain Assumption

- Implicitly declared field **boolean** `<created>` in class `Object`
- `<created>` has value **true** iff argument object has been created
- Object creation modeled as {heap := create(heap, ob)} for not (yet) created `ob` (essentially sets `<created>` field of `ob` to **true**)

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`ob` is a fresh program variable

Alternatives exisit in the literature. E.g.:
[Ahrendt, de Boer, Grabe, *Abstract Object Creation in Dynamic Logic – To Be or Not To Be Created*, Springer, LNCS 5850]

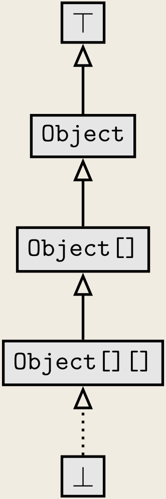
Dynamic Logic to (almost) full Java

KeY supports full sequential Java, with some limitations:

- Limited concurrency
- No generics
- No I/O
- Only preliminary support for floats
- No dynamic class loading or reflexion
- API method calls: need either JML contract or implementation

Java Features in Dynamic Logic: Arrays

Arrays



- JAVA type hierarchy includes array types
- Types ordered according to JAVA subtyping rules
- Function $\mathtt{arr} : \mathrm{int} \to \mathrm{Field}$ turns integer index into type Field (required in `store`).
- Store array elements on heap
- Value of `a[i]` on the heap $\mathtt{store}(\mathtt{heap}, \mathtt{a}, \mathtt{arr}(\mathtt{i}), 17)$ is 17
- Arrays `a` and `b` can refer to same object (aliasing)

Java Features in Dynamic Logic: Complex Expressions

Complex expressions with side effects

- JAVA expressions may have side effects, due to method calls, increment/decrement operators, nested assignments
- FOL terms have no side effect on the state

Example (Complex expression with side effects in Java)

`int i = 0; if ((i=2)>= 2) i++;`   value of `i` ?

Complex Expressions Cont'd

Decomposition of complex terms by symbolic execution

Follow the rules laid down in JAVA Language Specification

Local code transformations

$$\text{evalOrderIteratedAssgnmt}\ \frac{\Gamma \Longrightarrow \langle \texttt{y = t; x = y; } \omega\rangle\phi, \Delta}{\Gamma \Longrightarrow \langle \texttt{x = y = t; } \omega\rangle\phi, \Delta}\quad \texttt{t}\ \text{simple}$$

Temporary variables store result of evaluating subexpression

$$\text{ifEval}\ \frac{\Gamma \Longrightarrow \langle \textbf{boolean}\ \texttt{v0; v0 = b; } \textbf{if}\ \texttt{(v0) p; } \omega\rangle\phi, \Delta}{\Gamma \Longrightarrow \langle \textbf{if}\ \texttt{(b) p; } \omega\rangle\phi, \Delta}\quad \texttt{b}\ \text{complex}$$

Java Features in Dynamic Logic: Abrupt Termination

Abrupt Termination: Exceptions and Jumps

Redirection of control flow via **return**, **break**, **continue**, exceptions

$$\langle \mathbf{try}\ \{\mathtt{p}\}\ \mathbf{catch}(\mathtt{T\ e})\ \{\mathtt{q}\}\ \mathbf{finally}\ \{\mathtt{r}\}\ \omega\rangle\phi$$

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Rule tryThrow matches try–catch in pre-/postfix and active throw

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Demo

`exceptions/try-catch.key`

Java Features in Dynamic Logic: Aliasing

Demo

`aliasing/attributeAlias1.key`

Java Features in Dynamic Logic: Aliasing

Demo

`aliasing/attributeAlias1.key`

Reference Aliasing

Alias resolution causes proof split

A Round Tour of Java Features in DL Cont'd

Null pointer exceptions

There are no "exceptions" in FOL: $\mathcal{I}$ total on FSym

Need to model possibility that $\texttt{o} = \mathbf{null}$ in $\texttt{o.a}$

- KeY branches over $\texttt{o} \mathrel{!}= \mathbf{null}$ upon each field access

Summary

- Most JAVA features covered in KeY
- Several of remaining features available in experimental version
  - Simplified multi-threaded JMM
  - Floats
- Degree of automation for loop-free programs is very high
- Proving loops requires user to provide invariant
  - Automatic invariant generation sometimes possible
- Symbolic execution paradigm lets you use KeY w/o understanding details of logic

Literature for this Lecture

KeYbook *W. Ahrendt*, B. Beckert, R. Bubel, R. Hähnle, P. Schmitt, M. Ulbrich, editors.
Deductive Software Verification - The KeY Book
Vol 10001 of *LNCS*, Springer, 2016
(E-book at `link.springer.com`)

- B. Beckert, V. Klebanov, B. Weiß, Dynamic Logic for Java
  Chapter 3 in [KeYbook]
  on the surface only: Sections 3.1, 3.2, 3.4, 3.5.5, 3.5.6, 3.5.7, 3.6
- *W. Ahrendt*, S. Grebing, Using the KeY Prover
  Chapter 15 in [KeYbook]