Advanced Algorithms. Assignment 2

Exercise 3.
We are given k finite sets $S_1, \dots S_k$. (Note that the S_i are arbitrary, and in general they are not disjoint.)

(a) We wish to select exactly one element s_i from each set S_i such that the total number of distinct elements s_i is minimized.

(b) We wish to select exactly one element s_i from each set S_i such that the total number of distinct elements s_i is maximized.

Although these problems look so similar, they have different complexities:

Show that problem (a) is NP-complete, by a polynomial-time reduction from a known NP-complete problem.

Show that problem (b) can be solved in polynomial time, by reducing it to Maximum Flow.

Exercise 4.
The floor of a storage hall is partitioned like a chess board into unit squares, say of 1×1 meters. But many of the squares are occupied by heavy objects that cannot be easily moved. That is, only a certain set F of the squares are free. Nothing special is assumed about the shape of F.

Now a number of boxes of size 1×2 meters shall be stored. Every box must be placed exactly on two neighbored free squares, but the boxes may be rotated (stand in N-S or W-E direction). The problem is to place as many boxes as possible on the given space F.

Solve this problem by reducing it to Maximum Flow.

Here one can easily get stuck first in approaches that do not succeed, therefore we give a few hints: Color the squares black and white like a chess board(!), represent the squares by nodes and the boxes by edges, connect all white/black squares with a source/sink ... now you should be able to finish.

Remarks: In 3(b) and 4, give the full description of your network construction (directed graph with source, sink, and capacities), but also explain how you obtain the solution of the given problem from the flow, and do not forget to prove that your construction guarantees the optimal solution.