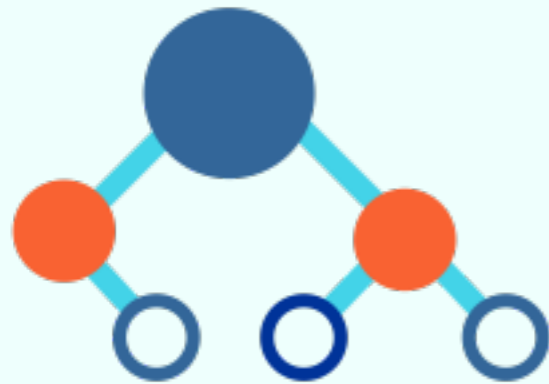


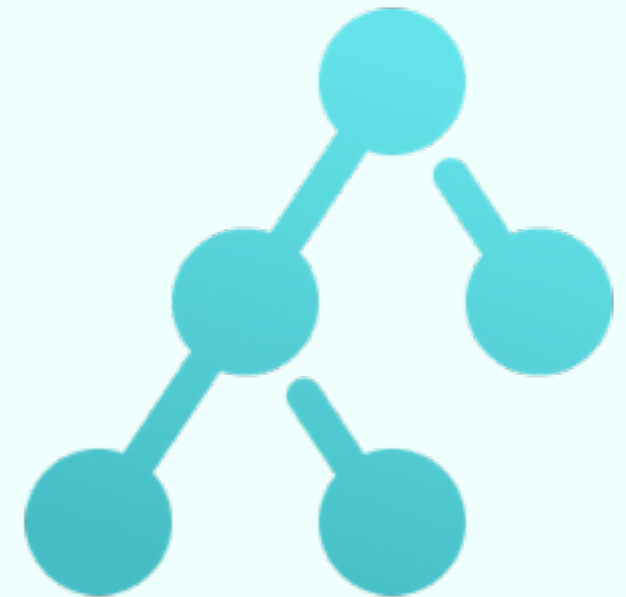


Data Structures

Exercise Session



Marco Vassena



Exercise 1 from 11/12

Analyze the time complexity in terms of N

```
// A : Empty Dynamic Array
// PQ : Priority Queue, |PQ| =  $N^3$ 
for(int i = 0; i < N; i++)
    A.add(0 , PQ.deleteMin())
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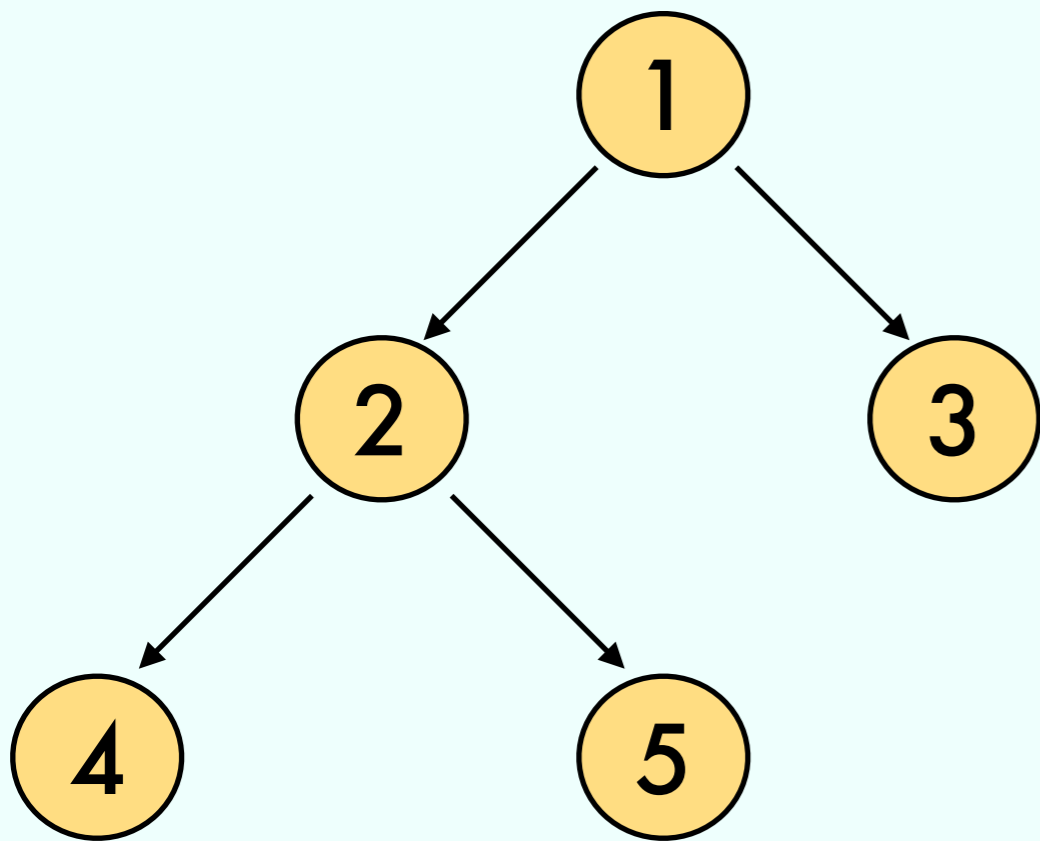
$O(\log(|PQ| - i))$

Exercise 2 from 11/12

Add to each node a reference to the parent

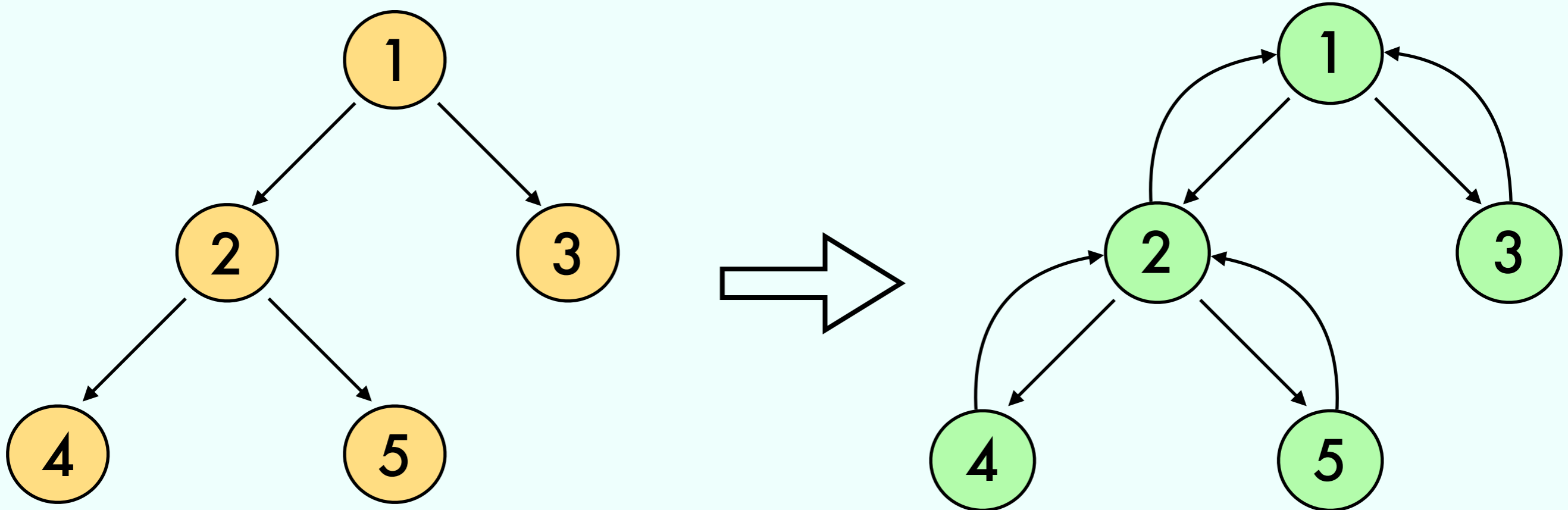
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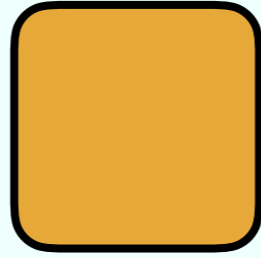
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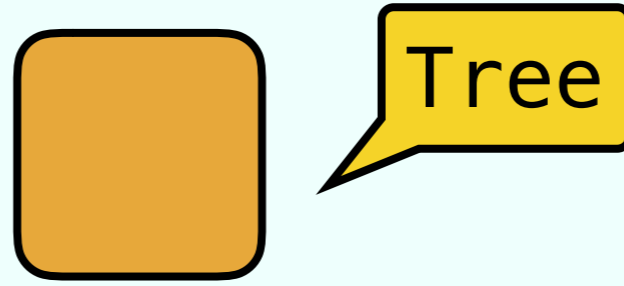


Tree

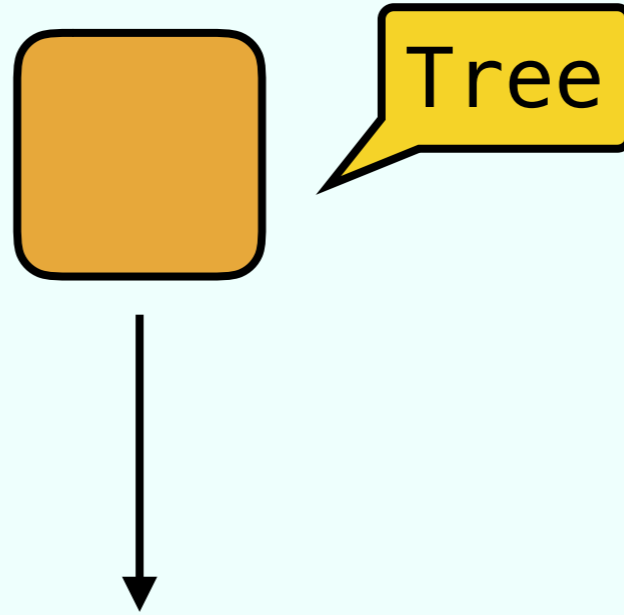
Tree



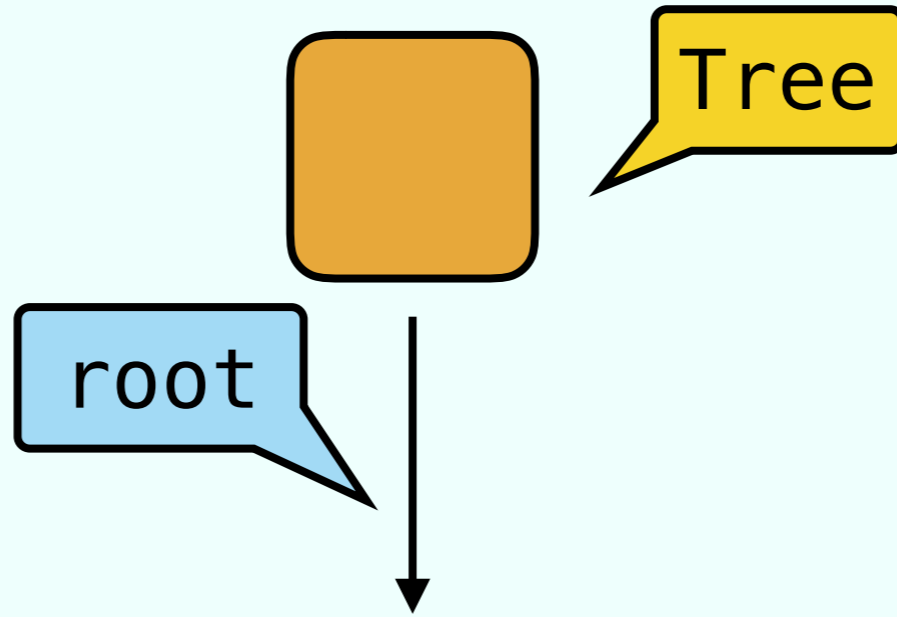
Tree



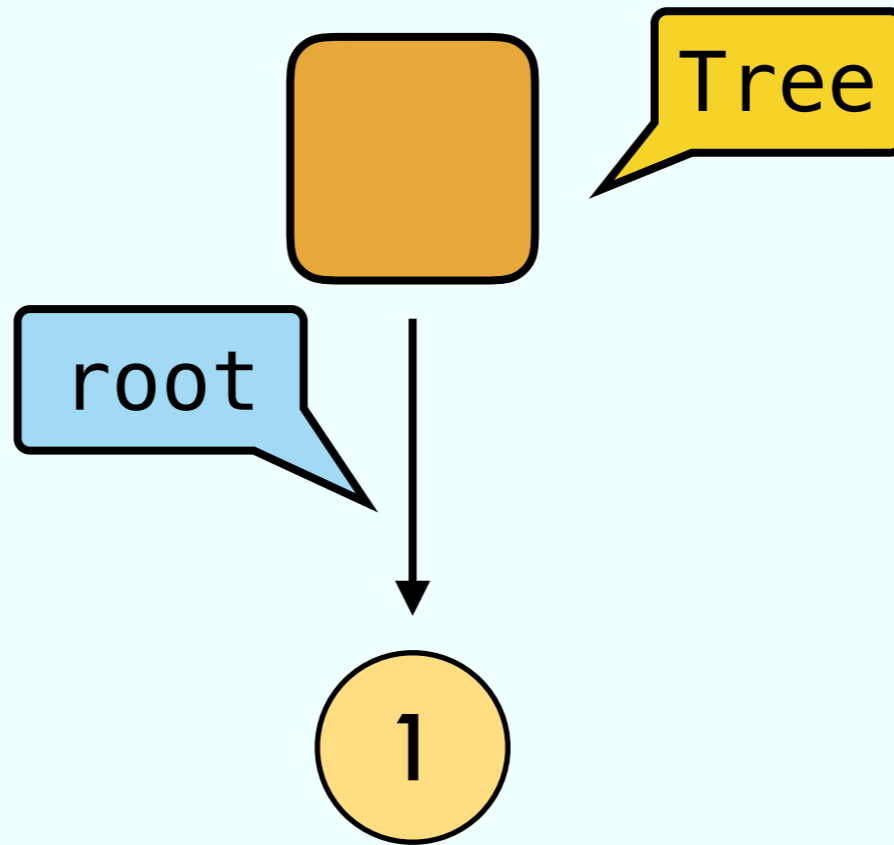
Tree



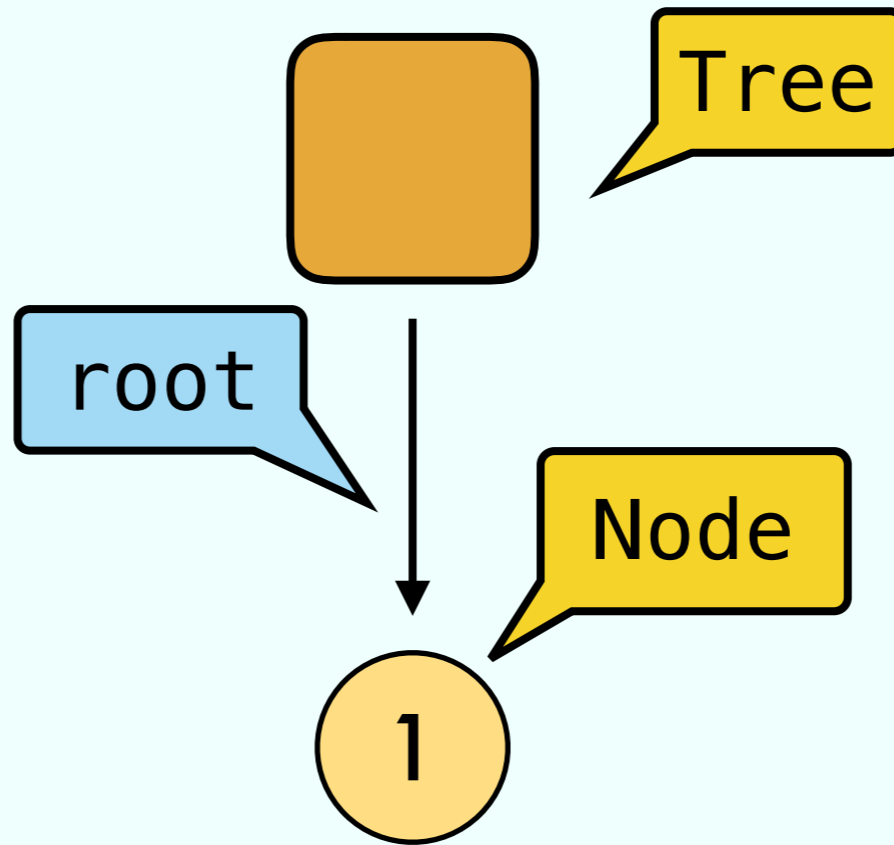
Tree



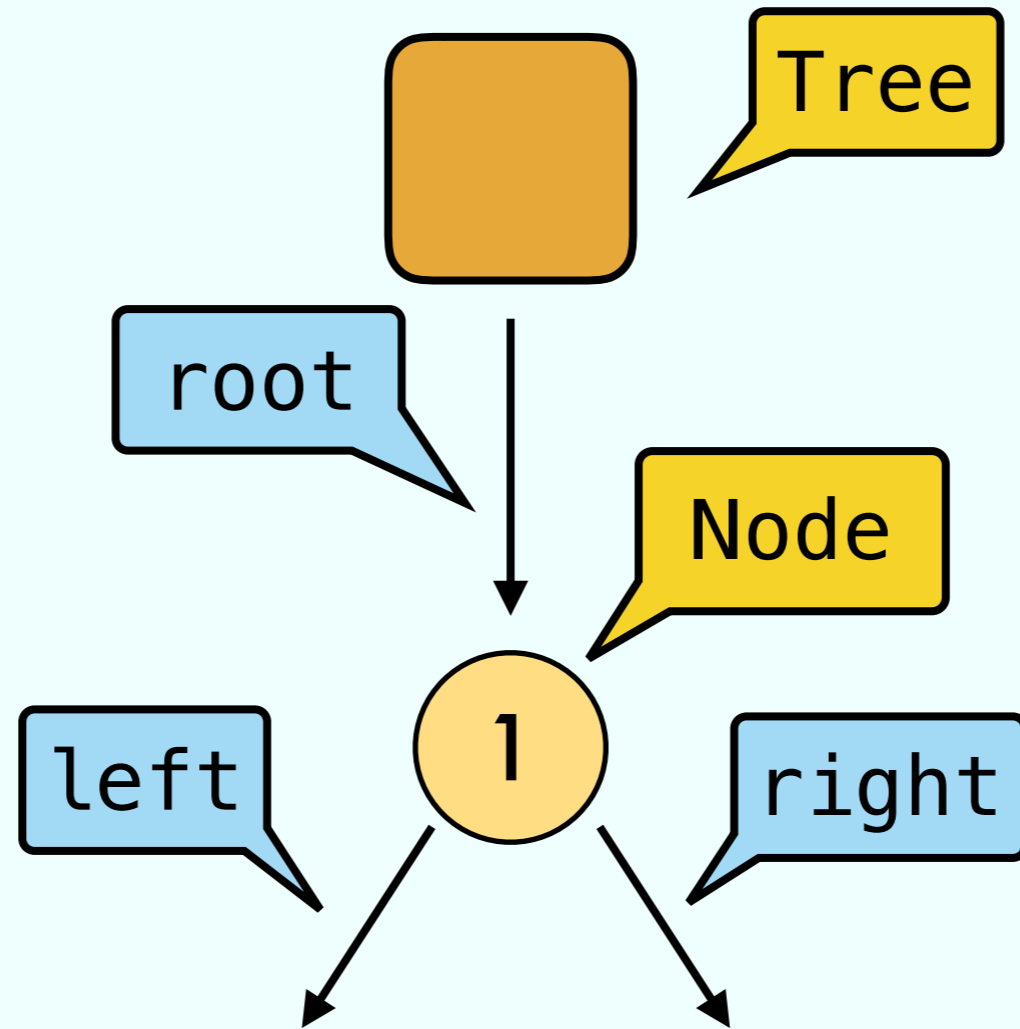
Tree



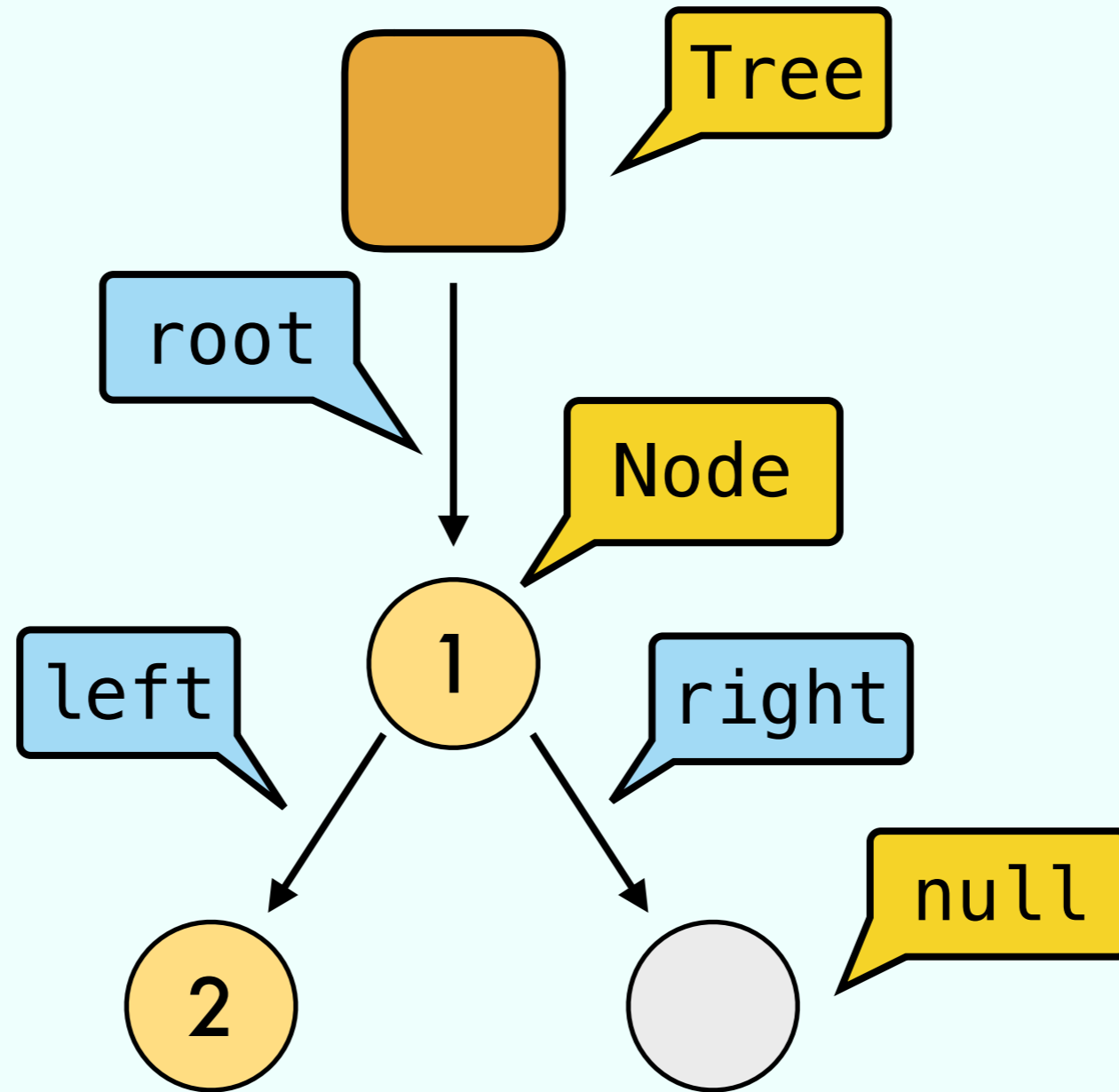
Tree



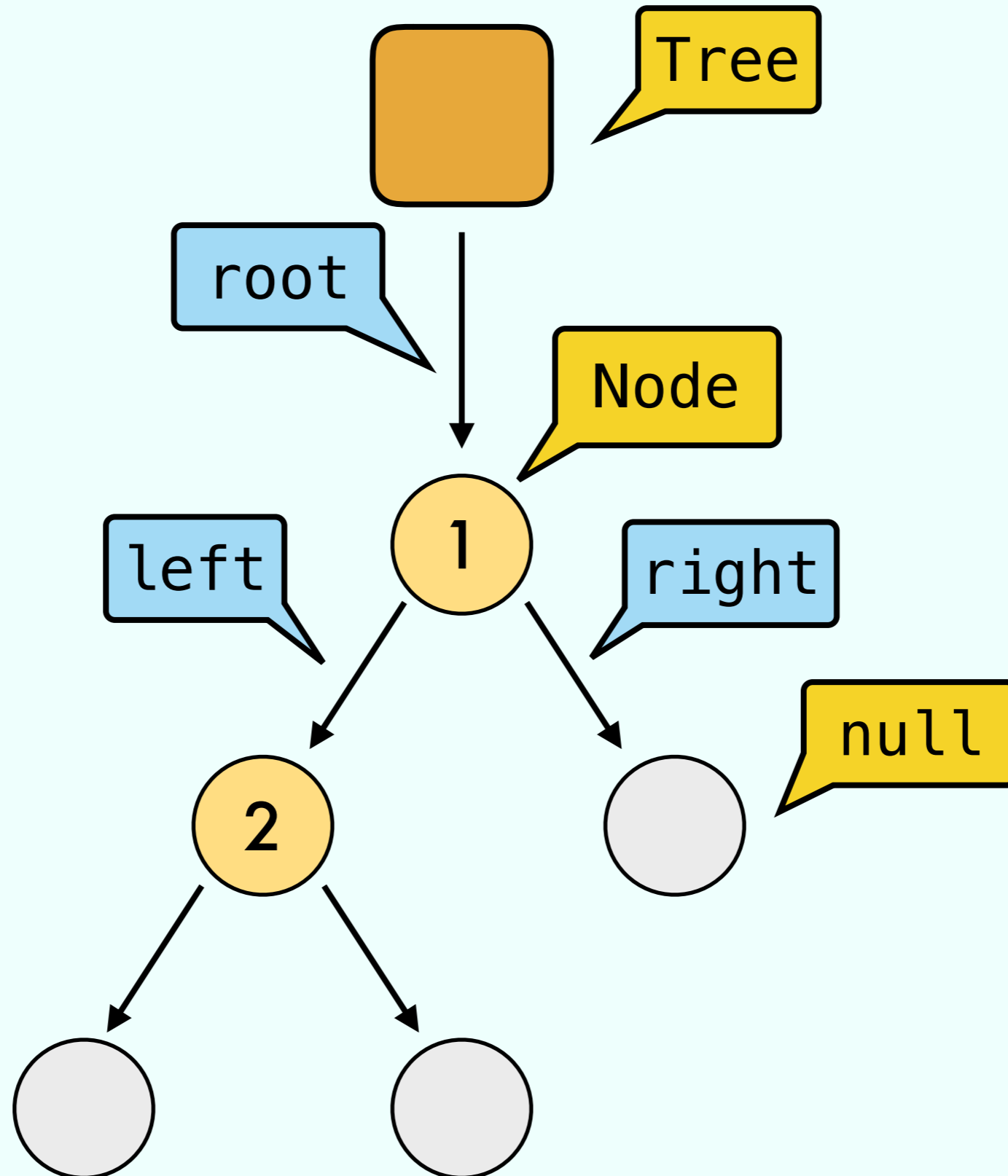
Tree



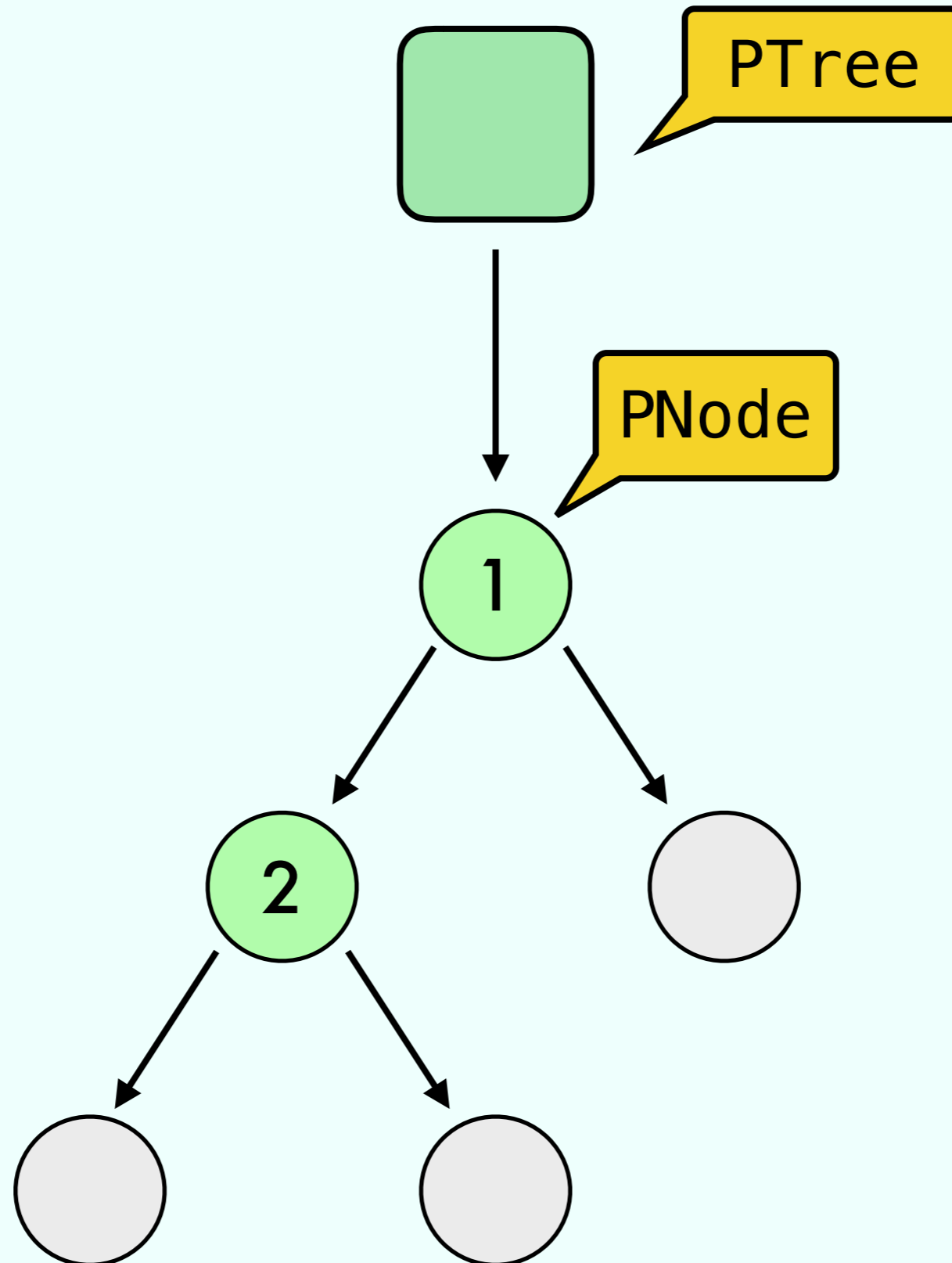
Tree



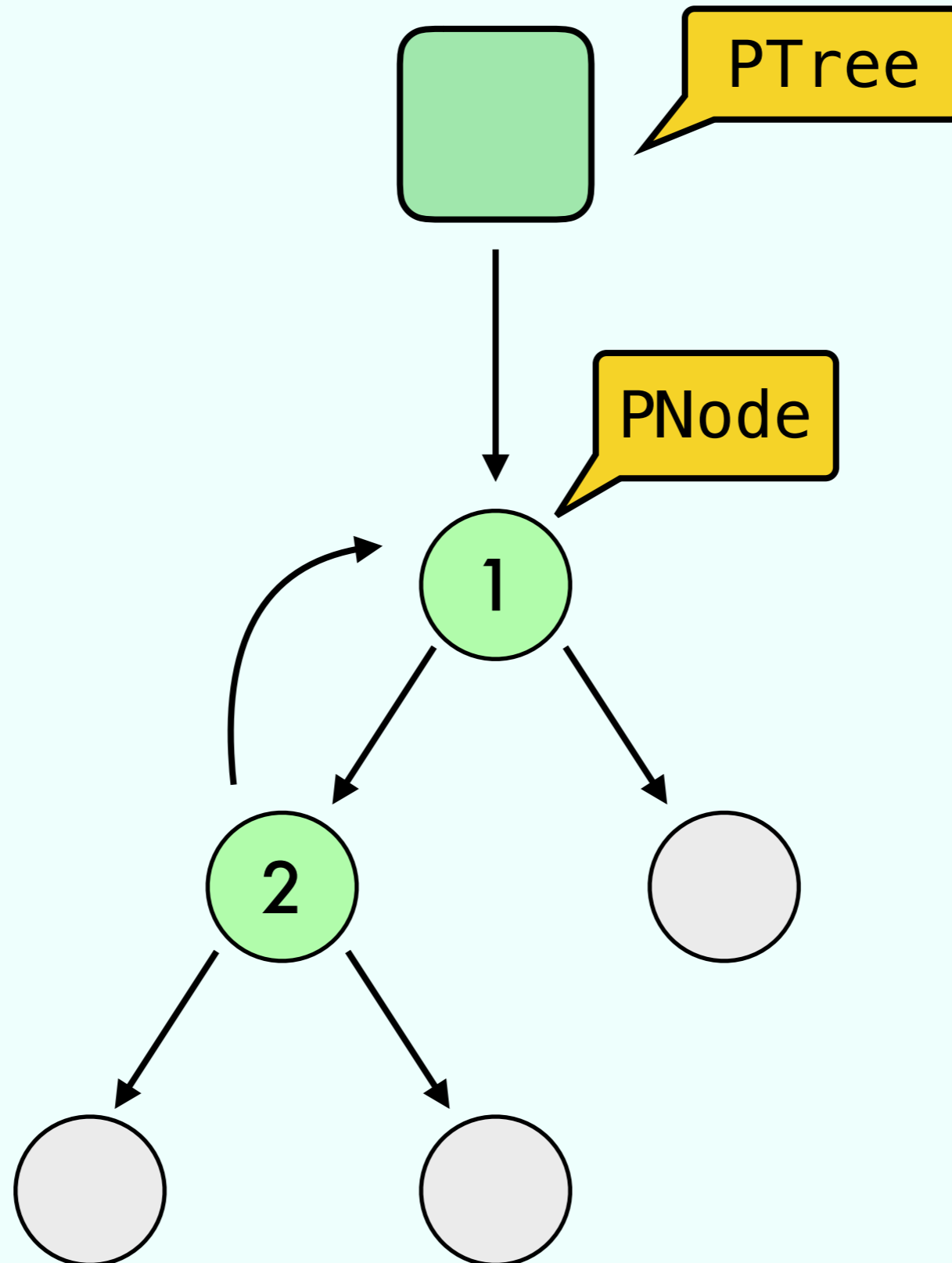
Tree



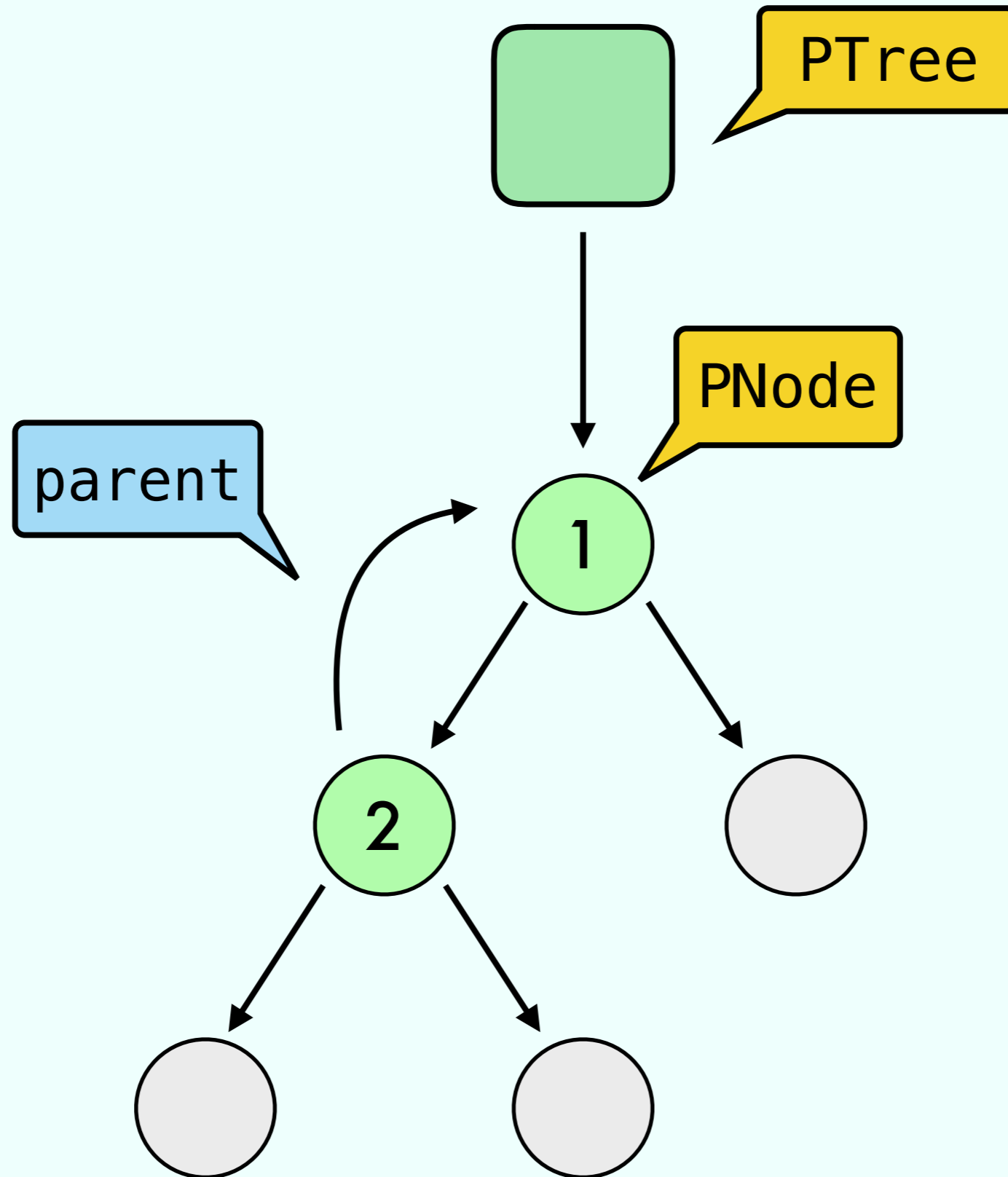
Parent Tree



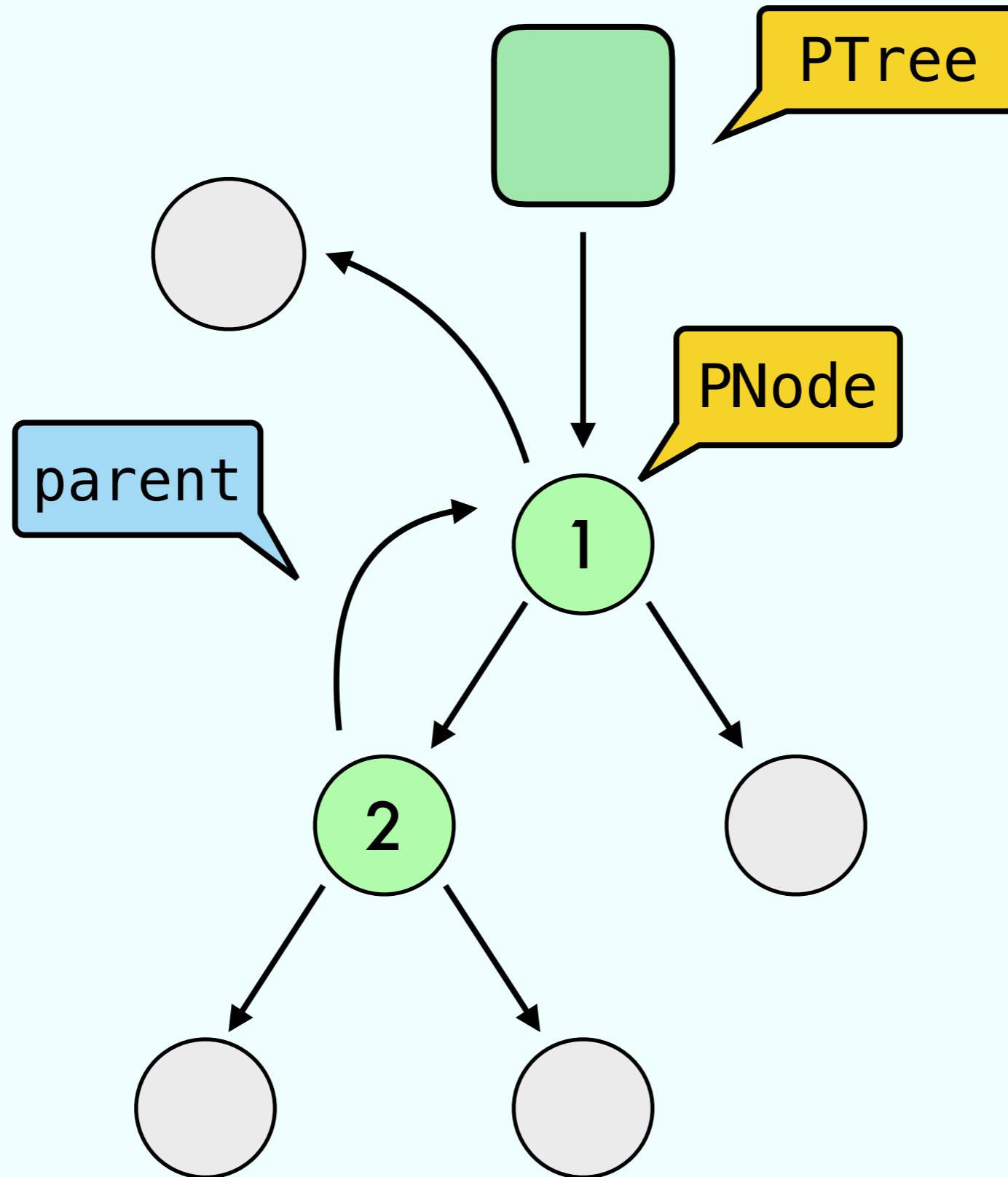
Parent Tree



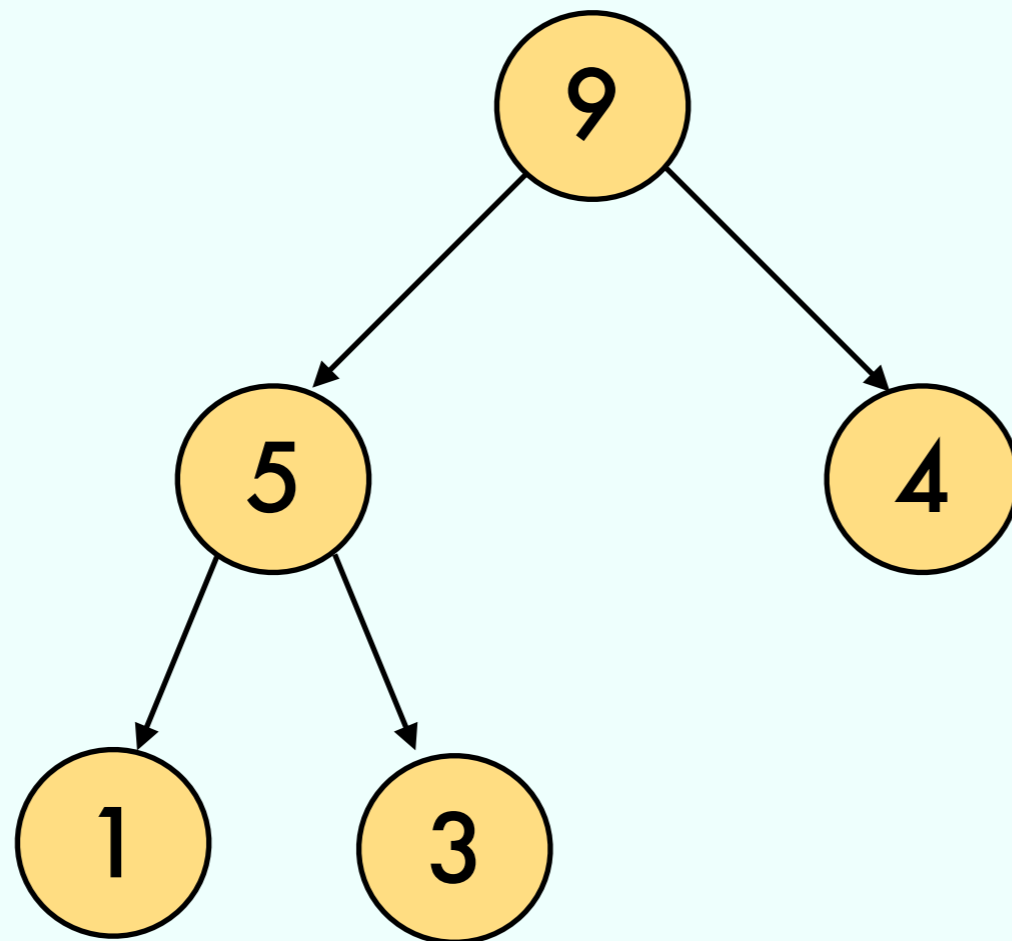
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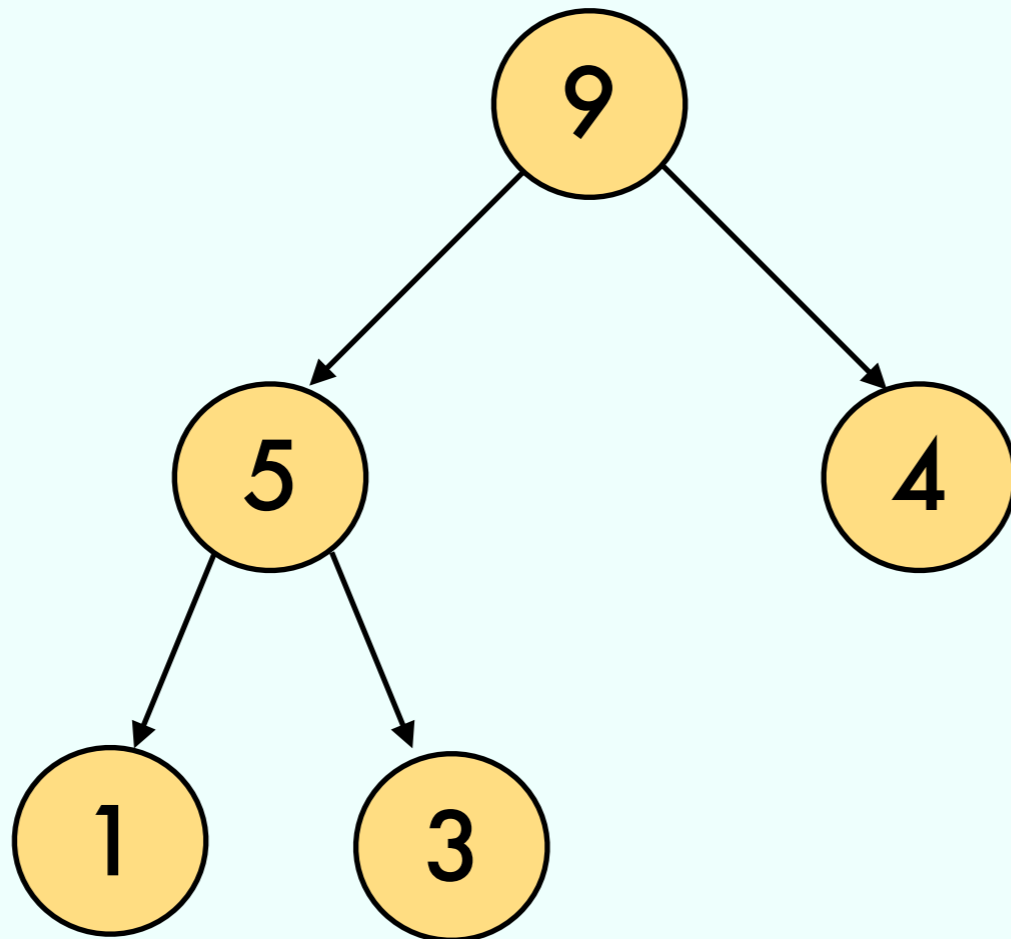


Binary Heap Recap



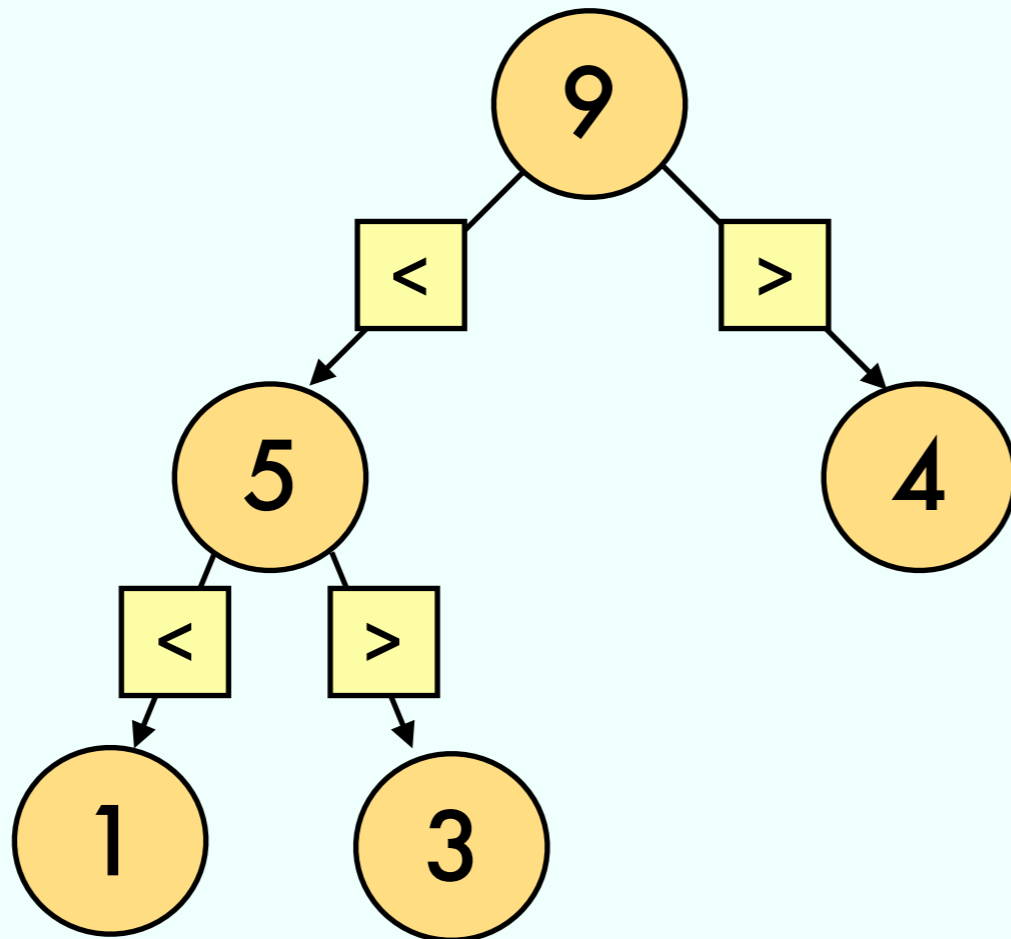
Binary Heap Recap

Order Property



Binary Heap Recap

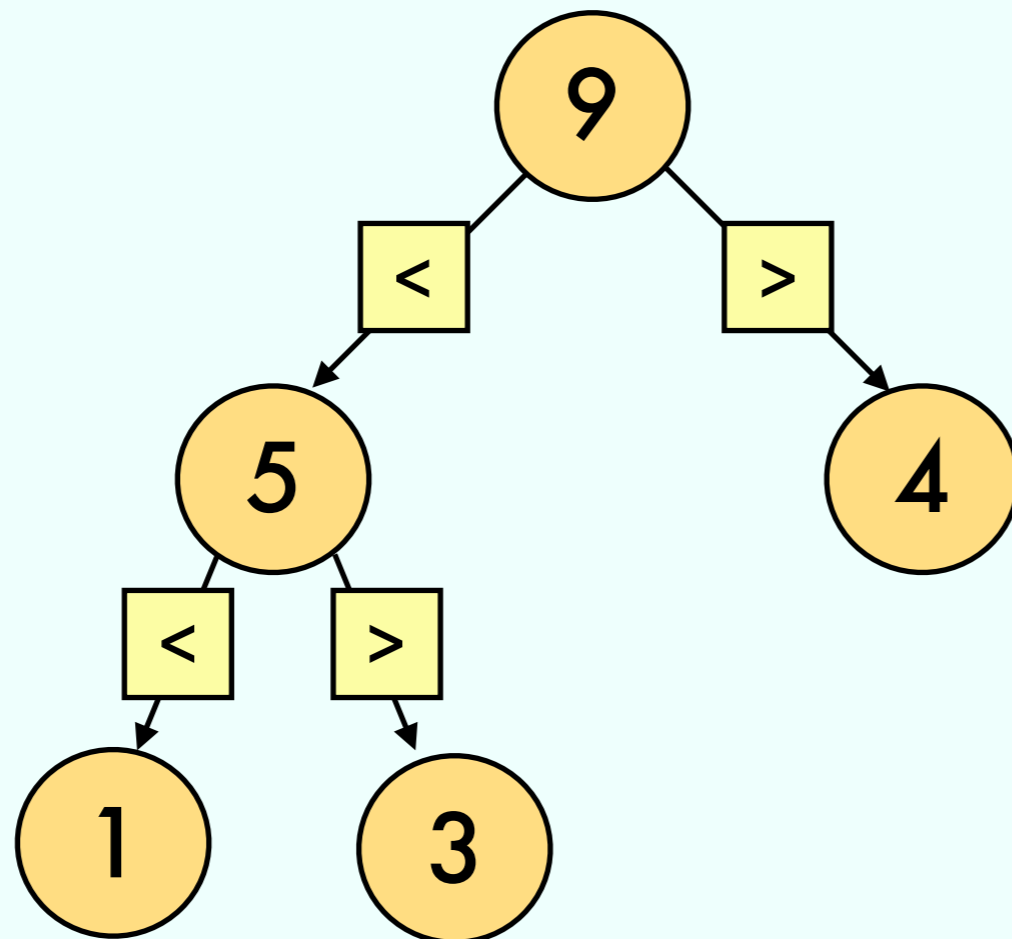
Order Property



Binary Heap Recap

Order Property

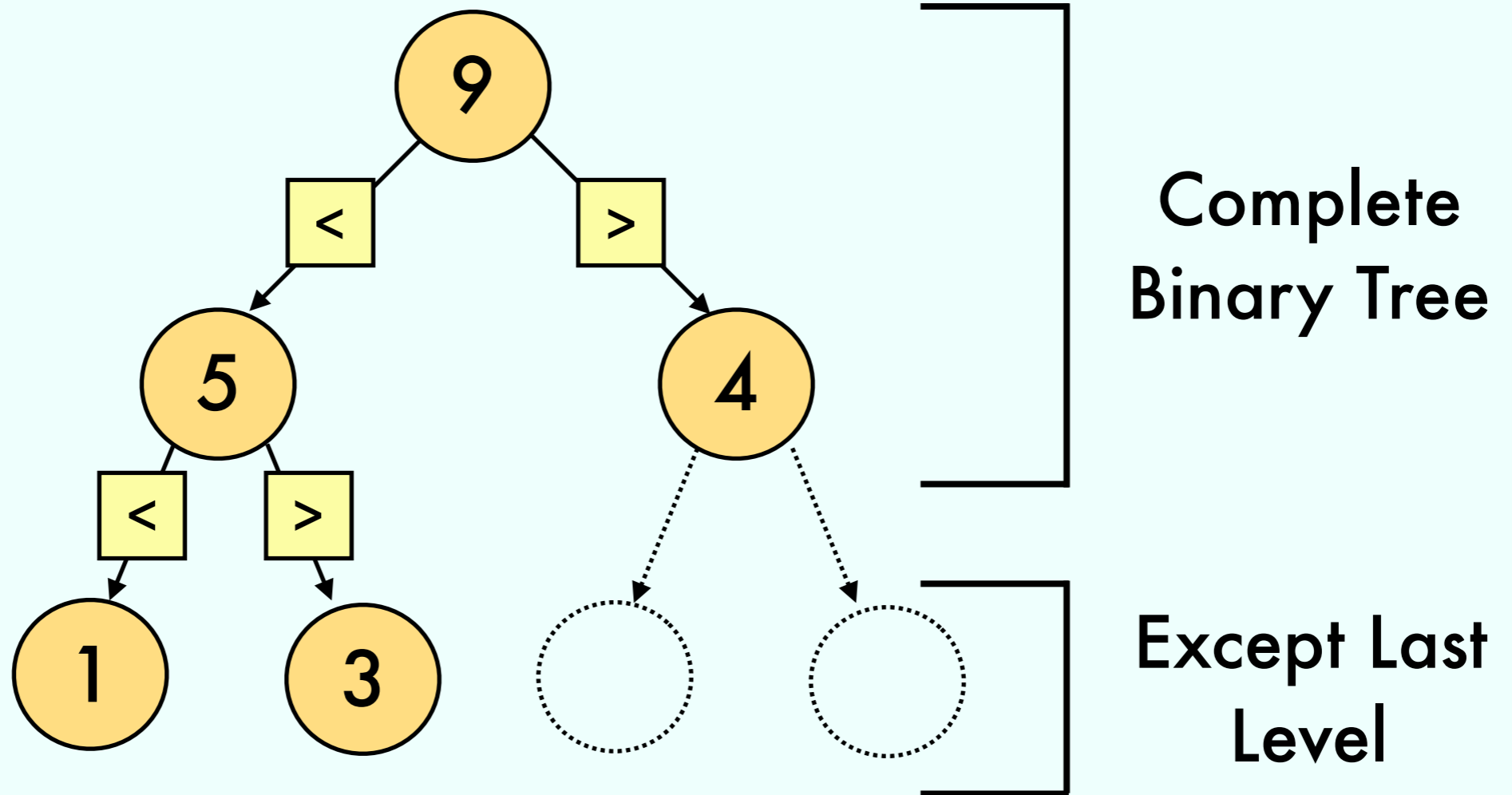
Structure Property



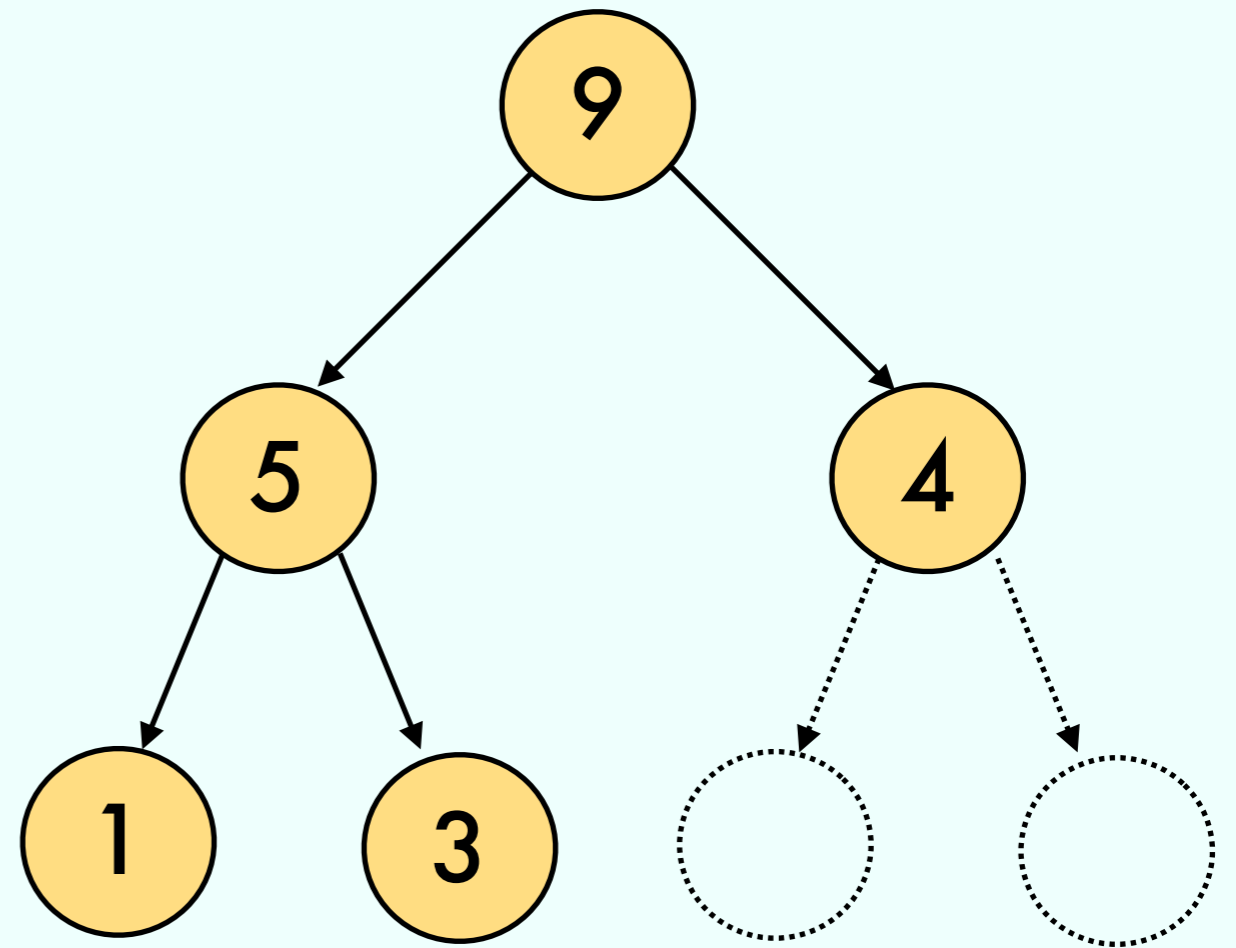
Binary Heap Recap

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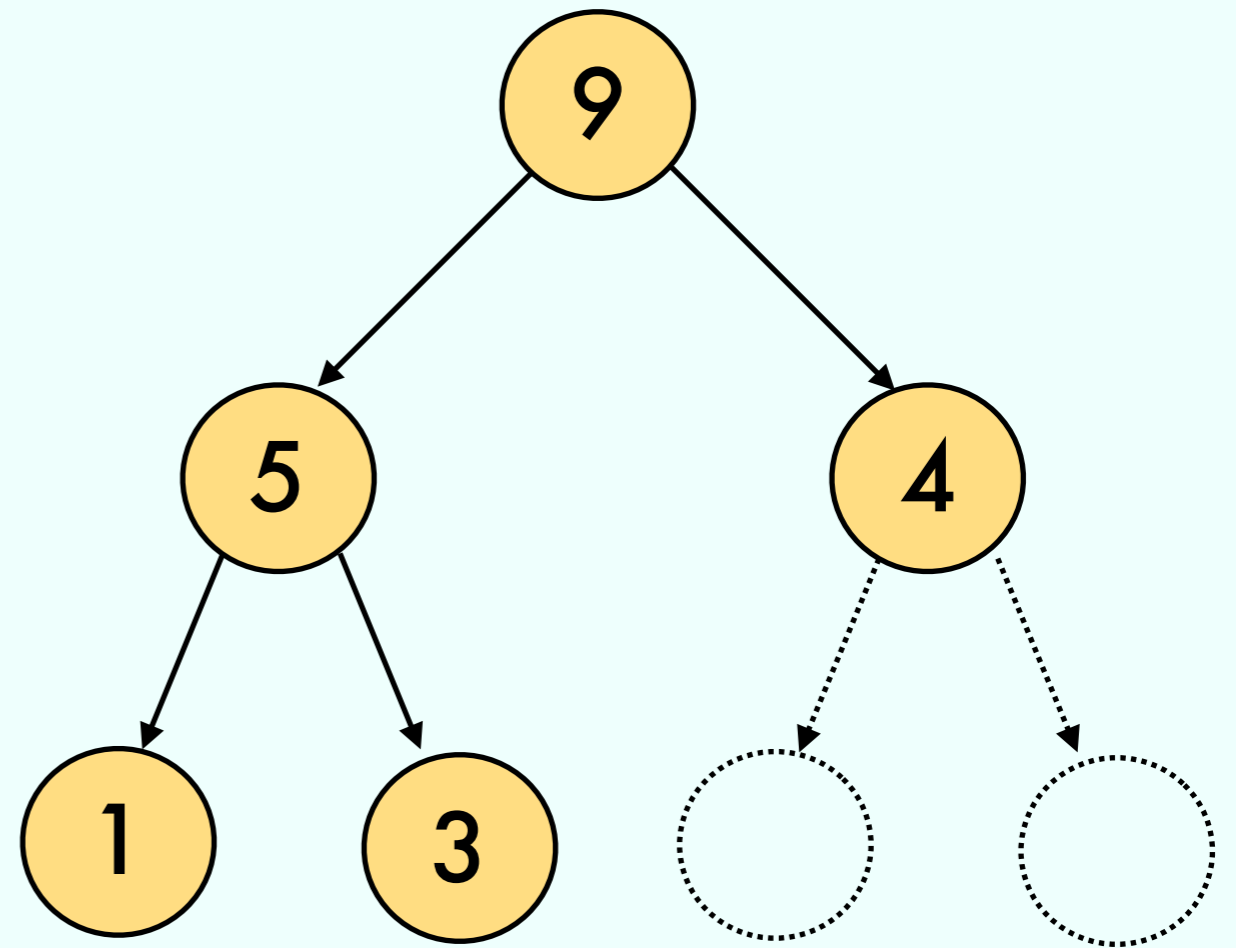
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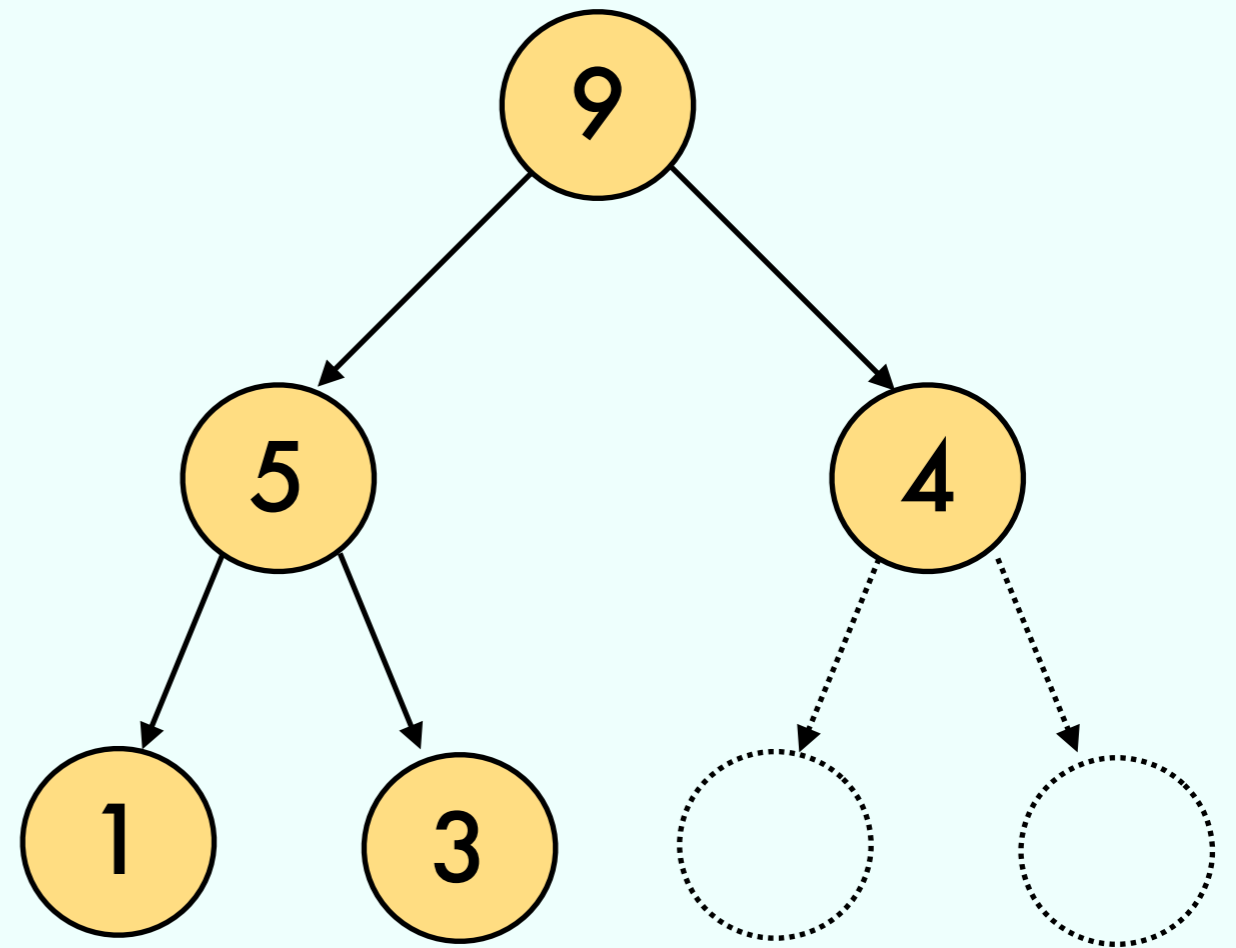
Array Representation



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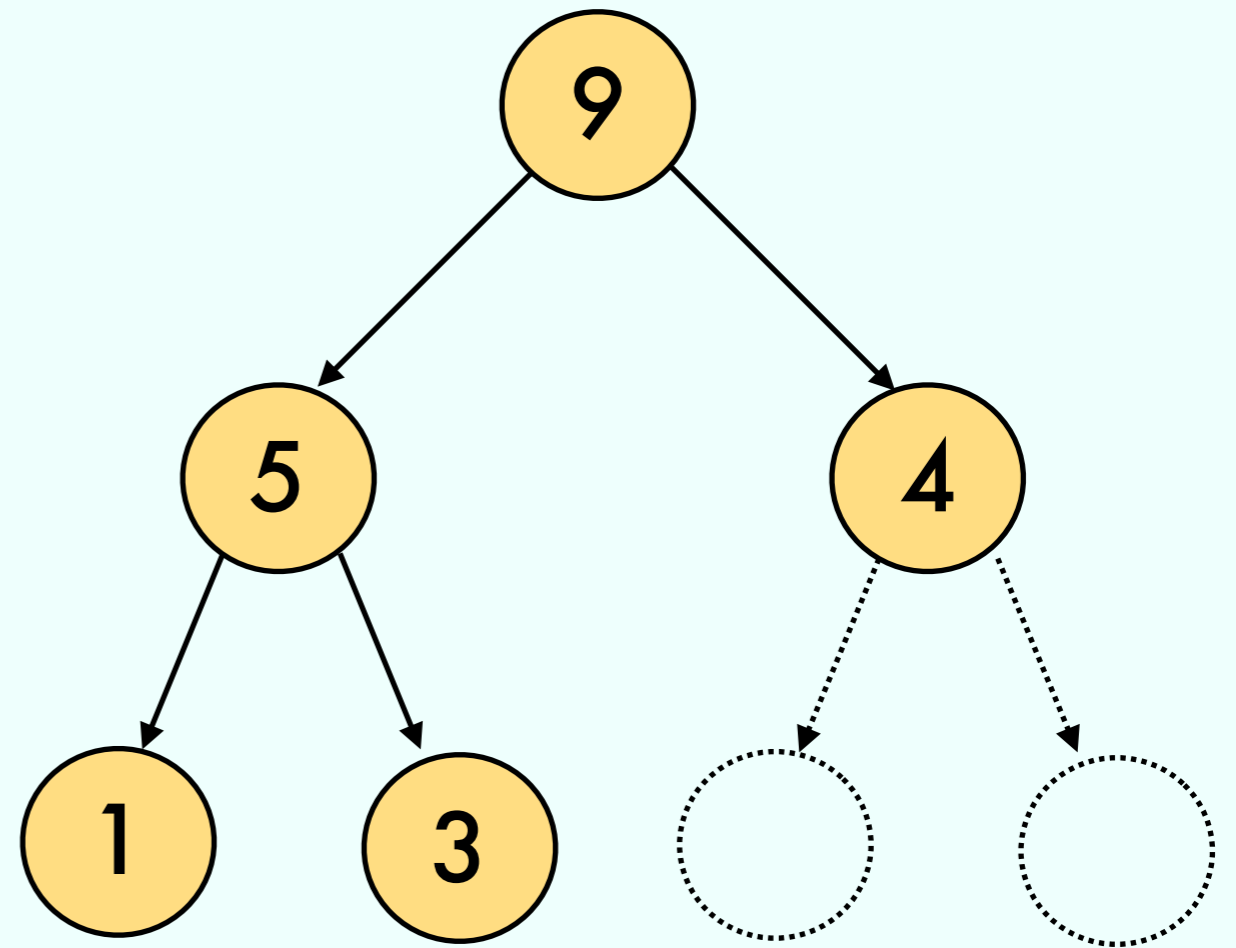
Array Representation



root = A[1]



Array Representation



A[0] unused

root = A[1]



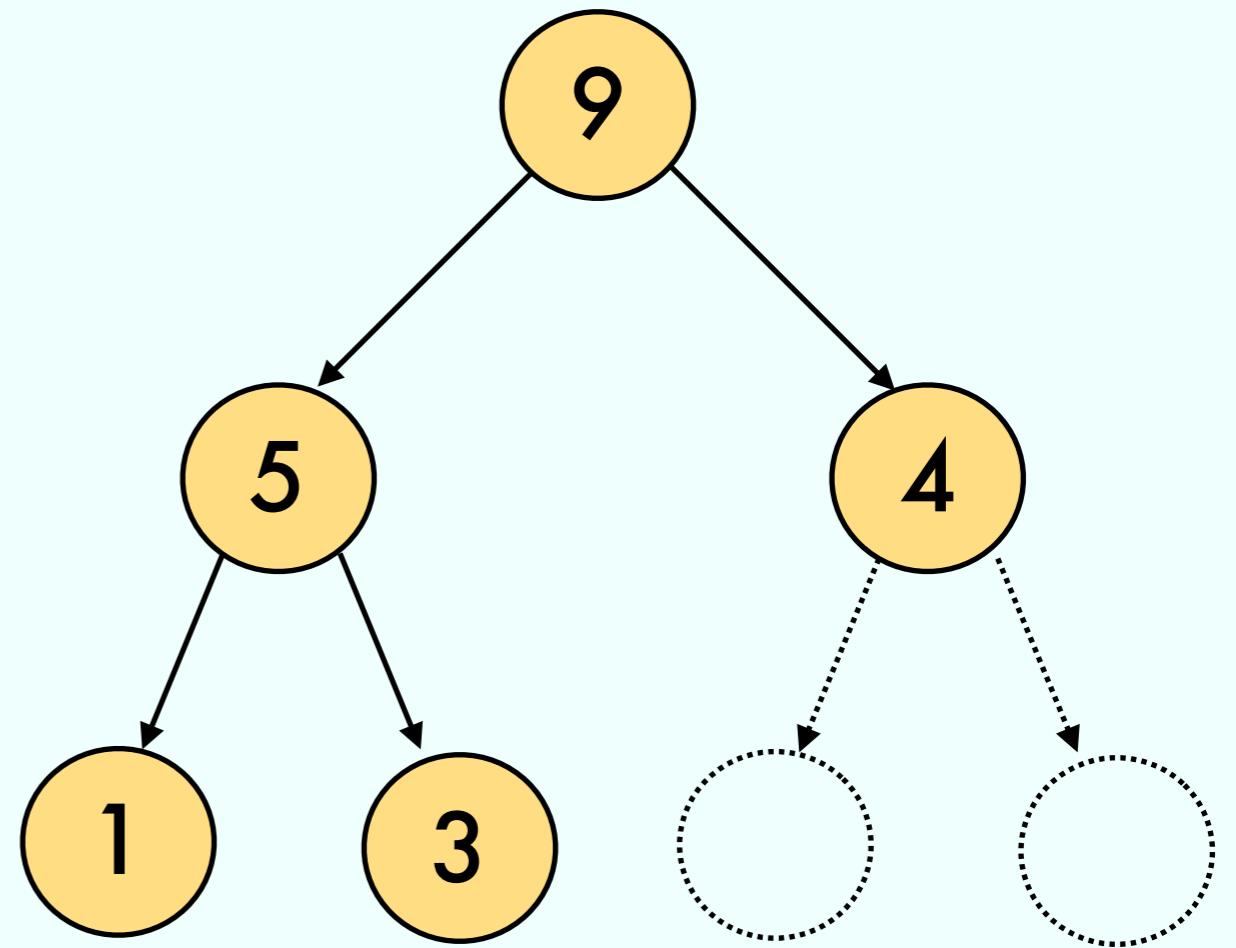
Array Representation

Index Operations

$$\text{left}(i) = i * 2$$

$$\text{right}(i) = i * 2 + 1$$

$$\text{parent}(i) = \text{floor}(i/2)$$

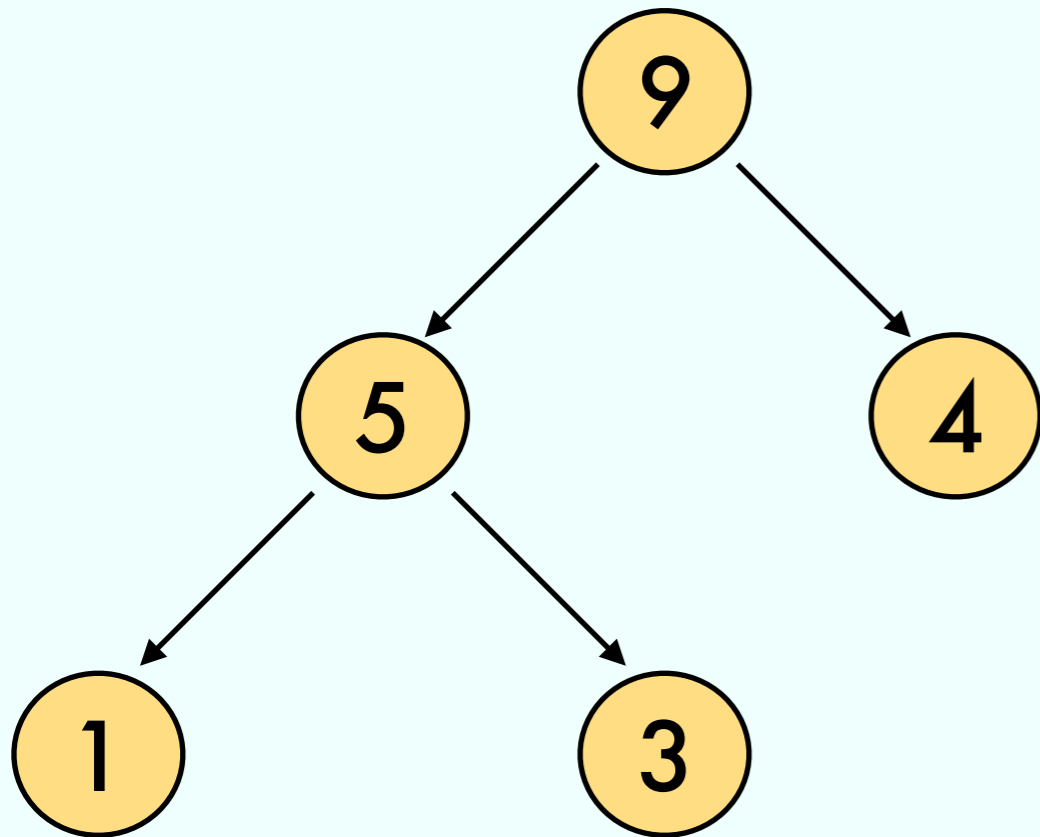


A[0] unused

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Binary Heap Recap



Operation	Time Complexity
findMax()	$O(1)$
deleteMax()	$O(\log N)$
insert(x)	$O(\log N)$

Exercise 6.2, 6.3

Draw the heap after each operation

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1. `for i in [10, 12, 1, 14, 6, 5, ...]
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Check your solution at <https://visualgo.net/heap>