Finite Automata Theory and Formal Languages

TMV026/TMV027/DIT321 – Responsible: Ana Bove

Tuesday 28 of May 2013
Total: 60 points

TMV027/DIT321 registration VT13 Exam valid 6hp	TMV026/DIT321 registration before VT13 Exam valid 7.5 hp
CTH: $\geqslant$ 27: 3, $\geqslant$ 40: 4, $\geqslant$50: 5 GU: $\geqslant$27: G, $\geqslant$45: VG	CTH: $\geqslant$ 33: 3, $\geqslant$ 43: 4, $\geqslant$53: 5 GU: $\geqslant$33: G, $\geqslant$50: VG

No help material but dictionaries to/from English or Swedish.

Write in English or Swedish, and as readable as possible (think that what we cannot read we cannot correct).

OBS: All answers should be well motivated. Points will be deduced when you give an unnecessarily complicated solution or when you do not properly justify your answer.

Good luck!

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1. (5pts) Prove that the words generated by the following grammar have always one more triangle (either up or down) than squares (either black or white):

   $$S \rightarrow \blacktriangledown \mid \blacktriangle \mid S \blacksquare S \mid \square S S$$

   Do not forget to clearly state which kind of induction you are using, the property you will prove, the base case(s) and the inductive hypothesis(es)!

2. (3.5pts) Construct a DFA which recognises the language generated by the regular expression $0(1+0)^*1 + 0(11+00)^*1$, without going via an ϵ-NFA.

3. Consider the following ϵ-NFA:

	0	1	ϵ
$\rightarrow q_0$	$\{q_0\}$	$\emptyset$	$\{q_1\}$
q_1	$\{q_2\}$	$\emptyset$	$\{q_3\}$
q_2	$\emptyset$	$\{q_1\}$	$\emptyset$
$^*q_3$	$\{q_3\}$	$\emptyset$	$\emptyset$

   (a) (1.5 pts) Use your intuition to give a regular expression generating exactly the same language as the one accepted by the automaton.

   (b) (4.5pts) Convert the ϵ-NFA into a DFA.

4. (5pts) Minimise the following automaton. Show the intermediate table and justify the construction of the new automaton.

	a	b
$\rightarrow q_0$	q_1	q_2
q_1	q_3	q_4
q_2	q_5	q_6
q_3	q_3	q_4
q_4	q_5	q_6
$^*q_5$	q_4	q_3
q_6	q_5	q_6

5. (a) (3pts) Show that $(aa^*b^*b)^* = \epsilon + a(a+b)^*b$.
   (b) (3pts) Explain why the languages $(0+1)^*01(0+1)^* + 1^*0^*$ and $(0+1)^*$ are the same.
   Hint: A possible way to go is to analyse what each language represents rather than to show double inclusion.

6. (a) (1pts) When is a language regular? Explain as much as you can.
   (b) (4.5pts) For each of the following languages, give a regular expression which generates the language or prove that the language is not regular.
      i. $\{0^i1^j2^i \mid i,j \geqslant 0\}$;
      ii. $\{0^i1^j2^k \mid i,j,k \geqslant 0\}$.

7. (a) (6pts) Give a context-free grammar that generates the language $\{a^ib^jc^k \mid j \neq i+k \text{ with } i+j+k > 0\}$.
   (b) (1pt) When is a grammar ambiguous?
   (c) (1.5pts) Is the grammar ambiguous? Justify.

8. Consider the following grammar with start symbol S:

$$
\begin{array}{ll}
S \to aAB \mid AbB \mid F & \\
A \to aA \mid C & C \to cC \mid \epsilon \\
B \to bB \mid D & D \to dD \mid \epsilon \\
F \to fF \mid Ff & E \to eE \mid \epsilon
\end{array}
$$

   (a) (2pts) Identify the nullable variables and eliminate ϵ-productions;
   (b) (2pts) Identify and eliminate unit productions in the grammar from (a);
   (c) (1.5pts) Identify and eliminate useless symbols in the grammar from (b);
   (d) (2.5pts) Use your intuition and describe, as formal as you can, the language generated by the grammar in (c);
   (e) (1pts) Is the language described in (d) regular? Justify;
   (f) (1.5pts) Put the grammar from (c) in Chomsky Normal Form.

9. (4pts) Consider the following grammar with start symbol S:

$$S \to AB \qquad A \to BB \mid a \qquad B \to AB \mid b$$

   Apply the CYK algorithm to determine if the string $aabba$ is generated by this grammar. Show the resulting table and justify your answer.

10. (a) For TMV026/DIT321 registration before VT2013
    (6pts) Construct a Turing machine for the language $\{0^i1^j \mid i > j\}$ by giving its transition function. Explain it.
    NOTE: You may chose to do part (b) instead if you prefer.
    (b) For TMV027/DIT321 registration VT2013
       i. (4pts) Give a high-level description of a Turing machine for the language $\{0^i1^j \mid i > j\}$.
       ii. (1pt) Explain what a Turing decider is.
       iii. (1pt) State whether your Turing machine is also a Turing decider or not. Justify.