Algorithms. Exercises from Week 6

1. Let n and m denote the number of nodes and edges, respectively, in a connected graph. Some author announces a time bound $O(m)$ for some graph algorithm. Another author announces an $O(n + m)$ time bound for the same algorithm. Is one of them mistaken?

2. Why can we write $O(m \log n)$ instead of $O(m \log m)$, if n and m denotes the number of nodes and edges, respectively, in a graph?

3. Suppose that we have executed depth-first search in an undirected graph and produced a DFS tree. Is the following claim true or false? (Explain.) "Every cycle in the graph only consists of tree edges and one back edge." Then, propose an $O(m + n)$ time algorithm that decides whether a given graph has a cycle, based either on this claim (if it is true), or on a suitable modification of this claim (if it is false).

4. We have seen how BFS can be used to test efficiently whether a given graph is bipartite. Devise an alternative algorithm for this problem, that uses DFS rather than BFS. Make sure that you prove correcntess of your proposed algorithm.

5. We run Kruskal's algorithm to get a clustering with maximum spacing. Are the following claims true or false? (Explain.) "The length of any selected edge inside a cluster is smaller than the spacing." – "The distance between any two nodes inside a cluster is smaller than the spacing."