Algorithms. Exercises from Week 5

The following exercises, except that last one, are only little questions that test understanding, rather than technical exercises asking to produce some mathematical object.

1. Regarding the time bound of a reduction of a problem X to a problem Y: Why do we count only the time for the transformation and simply ignore the time for actually solving problem Y? Explain in your own words.

2. "If a problem is NP-complete, then we know that it can be solved only in exponential time." Is this true? Motivate your answer.

3. "The Independent Set problem is NP-complete for arbitrary graphs. Interval graphs are graphs. Hence, Independent Set is NP-complete for interval graphs." Is this true? Motivate your answer.

4. A Boolean term consists of literals connected by $\wedge$. A disjunctive normal form (DNF) consists of terms connected by $\vee$. (In other words, a DNF looks like a CNF, where the roles of $\wedge$ and $\vee$ are exchanged.)
"The SAT problem is easy to solve for DNF: It suffices to make one term true, simply by setting all literals in this term true. Furthermore, we can easily transform every CNF into an equivalent DNF, using the distributive law for $\wedge$ and $\vee$. Therefore SAT should be easy for CNF as well. But we know that SAT for CNF is NP-complete. This is a contradiction." What, precisely, is the flaw in this reasoning?

5. In the Subset Sum problem we are given n integers w_i and another integer W, and the problem is to decide whether some subset has the sum W. Now we define the special case Half-Half Subset Sum, as the Subset Sum problem where $W = \sum_{i=1}^{n} w_i/2$. (In words: Can we split the given set of integers half-half?) Prove that Half-Half Subset Sum is still NP-complete, by a reduction from the usual Subset Sum problem. – Hint: Do not think complicated. Insert one extra item of suitable size in the given instance.