Surface Scattering
Surface Scattering

(not subsurface scattering)
Surface Scattering

- Ray tracing
- Materials
- BxDF Models
  - Bi-directional Distribution Function
  - BRDF: Reflectance
  - BTDF: Transmission
  - BSDF: Scattering ($= \text{BRDF} + \text{BTDF}$)
  - BSSRDF - Scattering-surface reflectance
    - Subsurface scattering
Ray Tracing

- Trace rays through plane
- Camera effects
- Sampling schemes
- Speed-up techniques
Ray Tracing

- Spheres
- Diffuse, specular, (glossy)
- Reflection, refraction
- (Textures)

Turner Whitted (1980)
“An improved illumination model for shaded display”

Figure 5: Jim Blinn (1978)
“Simulation of wrinkled surfaces” (Bump mapping)
Ray Tracing

- **Diffuse (or “Lambertian”)**
  - Uniform sampling
- **Specular**
  - Perfect reflection
- **(Glossy)**
  - “glossiness” parameter
Scale

Figure 11: Microscale Geometry for Velvet

Figure 1: Applicability of Techniques
Materials

- **Isotropic**
  - Reflectance invariant of rotation about the surface normal
  - Paints, papers, plastics, (sandblasted) metals

- **Anisotropic**
  - Not isotropic
  - Roughness **biased** in some direction
  - Materials with grains, fibers, grooves, crystalline structures
  - Wood, brushed metal, marble

[Heidrich-Seidel]
Materials

- Metals
  - Conductors
- Dielectrics
  - Insulators
  - Glass, plastic…
- (Semiconductors)
  - Rare in practice, often ignored

Specular reflectance by angle of incidence. Perfectly smooth surfaces.
Materials

- **Metals**
  - Prevents subsurface scattering
  - Absorbs all energy within a fraction of one wavelength

- **Dielectrics**
  - **Black dielectrics** absorb all energy, but over a longer distance
  - Other dielectrics reflect significant energy by subsurface scattering
  - “All non-metallic materials are translucent to some degree” - Henrik Wann Jensen
Materials

Wave property of light vs. Surface roughness

- X-rays (0.01-20 nm)
- Ultraviolet light (100-400 nm)
- Visible light (400-700 nm)
- Infrared (800-1000 nm)
- Microwave (1-1000 mm)
Materials: Smooth surfaces

- Surfaces with low roughness:
  - $\rho$: specular reflectance
  - $F - \rho$: non-specular reflectance
  - As the exponent approaches 0, all energy becomes specular reflectance

- Rough surfaces can look smooth
  - Effective roughness $\approx 0$

H Davies
The reflection of electromagnetic waves from a rough surface (1954)
Materials: Rough surfaces

- Roughness much greater than wavelength
- Rough surfaces:
  - “A set of faces with different slopes” or “microfacets”
  - Detectable with visible light (millimeter scale)
- Rough surface models
  - Phong, 1975
  - Cook & Torrance, 1982
  - Ward, 1992
  - Oren-Nayar, 1994
Reflectance models

- Lambertian (diffusely reflecting):
  \[ L_r = \frac{\rho}{\pi} \cos(\theta_i) E_0 \]

- Oren-Nayar:
  \[ L_r = \frac{\rho}{\pi} \cos(\theta_i) S E_0 \]
  \[ S = A + (B \max[0, \cos(\phi_i - \phi_r)] \sin \alpha \tan \beta) \]
  \[ A = 1 - 0.5 \frac{\sigma^2}{\sigma^2 + 0.57} \]
  \[ B = 0.45 \frac{\sigma^2}{\sigma^2 + 0.09} \]
  \[ \alpha = \max(\theta_i, \theta_r) \]
  \[ \beta = \min(\theta_i, \theta_r) \]
Models: BRDF

- **BRDF** (Bidirectional reflectance distribution function)
  - Assumes that light enters and leaves at the same point
  - Incoming direction $w_i$, outgoing direction $w_o$
  - BRDF: Ratio of reflected radiance exiting along $w_o$ to the irradiance incident to $w_i$

- **Physically plausible if...**
  - Always positive
  - Reciprocal: $w_i$, $w_o$ can be swapped for the same result
  - Energy-conserving: sum over the hemisphere is *one or less.*

- **Part of the rendering equation (…)**

* Error in reading material: “$\Sigma f \geq 1$"
The Rendering Equation

- The radiance is...
  - *Emitted* radiance, plus
  - *Reflected* radiance, over all incoming directions $w_i$ of the hemisphere $\Omega$ along $w_o$.

\[
L_o(x, \omega_o, \lambda) = L_e(x, \omega_o, \lambda) + \int_{\Omega} f(x, \omega_i, \omega_o, \lambda)L_i(x, \omega_i, \lambda) \cos \theta d\omega_i
\]
Models: $\text{BTDF}$, $\text{BSDF}$

- $\text{BTDF}$: Transmission
- $\text{BSDF}$: $\text{BRDF} + \text{BTDF}$
Models: BSSRDF

- BSSRDF (Bidirectional reflectance distribution function)
  - General case of BRDF
  - Entry and exit point do not have to be the same
  - Same laws (energy conservation, reciprocity)
Subsurface Scattering

- Not obvious to take into account
- Obvious when the effect is missing
Subsurface Scattering

- Translucent materials
  - Wax, milk, marble
- Different shapes and colors inside
  - Leaves, skin
- Light enters, scatters within, and exits
- Some light may be absorbed
Subsurface Scattering

- Single scattering approximation
  - A single bounce
  - Estimate number of photons reflected toward camera

- Depth map approximation
- Texture space diffusion
  - “General blurring of the diffuse lighting”
Subsurface Scattering - Multiple Layers

- Light Diffusion in Multi-Layered Translucent Materials *(2005)*
  - The skin is composed of *three layers*: epidermis, dermis, and the bloody dermis.
  - Scattering parameters for the layers are from measured data in medical literature.
  - The multilayered model gives the skin a **less waxy look** compared to the standard BSSRDF
- Reflectance and transmittance over radial distance
- Unique functions per layer

Figure 5: A buddha statuette sprayed with a thin layer of white paint. The first and third images are front-lit, the second and fourth back-lit.
Models: BRDF

- Many models
  - Difference in highlights
  - Size, fall-off

- Physically plausible
  - Cook-Torrance (1982)
  - GGX (2007)
  - Schlick (1994)
  - Oren-Nayar (non-simplified; 1994)

Chrome, GGX, Beckmann
Source: Physically-Based Shading at Disney (2012)
Cook-Torrance

- Two components
  - \( f = d f_{\text{diff}} + s f_{\text{spec}} \)
  - Weights: \( d + s = 1 \)

- Diffuse
  - “Body reflectance”
  - Independent of viewing angle; constant term
  - (Approximates subsurface scattering)

- Specular
  - “Surface reflectance”
  - Models \textit{microfacets}
  - \( h = \text{normalize}(l + v) \)
  - Contributes if \( h = n \)

Cook-Torrance: Microfacet phenomena

- **Shadowing**
  - Light-occlusion by microgeometry
  - “Self-shadowing”

- **Masking**
  - Visibility-occlusion by microgeometry
  - “Self-occlusion”

- **Interreflections**
  - When facets reflect onto other facets
  - *Not* taken into account

*Figure 2: On the left, occlusion of light source by microgeometry (shadowing). On the right, visibility occlusion of microgeometry (masking). Source: [1]*
Cook-Torrance:  
\[ f_{\text{spec}}(\omega_i, \omega_o) = \frac{F}{\pi} \frac{GD}{(n \cdot \omega_i)(n \cdot \omega_o)} \]

- **Reflectance:** \( f = df_{\text{diff}} + sf_{\text{spec}} \)
  - \( d + s = 1 \)
  - \( f_{\text{diff}} \) is constant (e.g. Lambertian)
- **\( f_{\text{spec}} \)**
  - F: Fresnel term
  - G: Geometric attenuation (shadowing, masking)
  - D: Normal distribution of facets oriented along \( h \)
Cook-Torrance: \( f_{\text{spec}}(\omega_i, \omega_o) = \frac{F}{\pi} \frac{GD}{(n \cdot \omega_i)(n \cdot \omega_o)} \)

Fresnel term \( F \)

\[
F(\theta) = \frac{1}{2} \frac{(g-c)^2}{(g+c)^2} \left( 1 + \frac{(c(g+c)-1)^2}{(c(g-c)+1)^2} \right)
\]

\( c = \cos \theta = v \cdot h \)
\( g = \sqrt{\eta^2 + c^2 - 1} \)

Geometric attenuation \( G \)

\[
G(\omega_i, \omega_o) = \min \left\{ 1, \frac{2(n \cdot h)(n \cdot \omega_o)}{(\omega_o \cdot h)}, \frac{2(n \cdot h)(n \cdot \omega_i)}{(\omega_o \cdot h)} \right\}
\]

Normal distribution \( D \) - (fraction of facets with normal = \( h \)) : (Phong, Heidrich-Seidel, GGX…)

Beckmann (rough surfaces):

\[
D_{\text{Beckmann}} = \frac{1}{m^2 \cos^4 \alpha} e^{-\left[ (\tan \alpha/m) \right]^2}
\]

Gaussian:

\[
D_{\text{Gaussian}} = c e^{-(\frac{\alpha}{m_2})^2}
\]
GGX

- Microfacet distribution model $D$
- Improvement over Beckmann distribution
  - Better fit to measured data (for some surfaces)
  - Derived from geometry term $G$
- BSDF
  - Handles transmittance

GGX

Cook-Torrance specular BRDF:

\[ f_{\text{spec}}(\omega_i, \omega_o) = \frac{F}{\pi} \frac{GD}{(n \cdot \omega_i)(n \cdot \omega_o)} \]

GGX BRDF:

\[ f(\omega_i, \omega_o) = \frac{FGD}{4(n \cdot \omega_i)(n \cdot \omega_o)} \]
GGX

Beckmann D (used in Cook-Torrance):

\[ D_{Beckmann} = \frac{1}{m^2 \cos^4 \alpha} e^{-[(\tan \alpha/m)^2} \]

GGX D:

\[ D = \frac{\alpha_g^2 \chi^+ (h \cdot n)}{\pi \cos^4 \theta_h (\alpha_g^2 + \tan^2 \theta_h)^2} \]
GGX

Cook-Torrance G:

\[ G(\omega_i, \omega_o) = \min \left\{ 1, \frac{2(n \cdot h)(n \cdot \omega_o)}{(\omega_o \cdot h)}, \frac{2(n \cdot h)(n \cdot \omega_i)}{(\omega_o \cdot h)} \right\} \]

GGX G (Smith):

\[ G(\omega_i, \omega_o) \approx G_1(\omega_i)G_1(\omega_o) \]

\[ G_1(v) = \chi^+ \left( \frac{v \cdot h}{v \cdot n} \right) \frac{2}{1 + \sqrt{1 + \alpha_g^2 \tan^2 \theta_v}} \]
GGX: Result

- Bigger tail
- Closer to real surfaces

(Measured chrome), GGX, Beckmann
Problems with BRDF models:

- Linear combination of specular, diffuse
  - Constant weights (Cook-Torrance: \( f = d_{\text{diff}} + s_{\text{spec}} \))
  - Instead, \( f \) should be a function of incident angle

- \( G \) only models reflection toward light
  - \( G \): the fraction of light not obstructed by microgeometry
  - Problem: the (unabsorbed) obstructed light \((1 - G)\) is reflected elsewhere in reality
  - Existing models do not account for this “re-emission” of light

- Physically plausible models are expensive
  - Schlick’s approximation
Schlick: Approximations

Fresnel term $F$ (Cook-Torrance):

$$F(\theta) = f_0 + (1 - f_0)(1 - \cos \theta)^5 \quad f_0 = \left(\frac{\eta_1 - \eta_2}{\eta_1 + \eta_2}\right)^2$$

Geometrical attenuation $G$ (Smith):

$$G(v) = \frac{v}{v - kv + k} \quad \text{with} \quad k = \sqrt{\frac{2m^2}{\pi}}$$

Distribution $D$ (Beckmann):

$$\forall t \in [1 - m, 1] \quad D(t) = \frac{m^3 x}{t (mx^2 - x^2 + m^2)^2} \quad \text{with} \quad x = t + m - 1$$
Approximations → faster Cook-Torrance

Two new BRDF models:
  - Homogeneous
    - metal, glass, paper, cotton
  - Heterogeneous (two layers)
    - opaque layer + translucent layer
    - varnished wood, lacquered glass

Layer properties
  - All values in [0, 1]:
  - $C_\lambda$: Reflection factor at wavelength $\lambda$
  - $r$: Roughness factor (0: perfectly specular; 1: perfectly diffuse)
  - $p$: Isotropy factor (0: perfectly anisotropic; 1: perfectly isotropic)
Summary: Cook-Torrance, GGX, Schlick

- Physically plausible?
  - Reciprocal: All
  - Energy-conserving: GGX, Schlick
  - Cook-Torrance: not energy-conserving under all angles
Summary: Cook-Torrance, GGX, Schlick

- **Cook-Torrance**
  - Suitable for: plastics, metals.
  - “Not as intuitive parameters. Experimentation required”
  - F, D, G are independent and can be replaced.

- **GGX**
  - Suitable for: developed for transmittance through rough surfaces (etched glass)
  - In general, “improved Cook-Torrance”
  - Many different versions of D, G

- **Schlick**
  - Suitable for: heterogeneous materials (translucent + opaque)
  - Continuums for isotropy, specular/diffuse
  - “Intuitive parameters”
### Overview

<table>
<thead>
<tr>
<th>Models</th>
<th>Physical</th>
<th>Plausible</th>
<th>Fresnel Eq.</th>
<th>Anisotropic</th>
<th>Sampling</th>
<th>Rel. Cost (cycles)</th>
<th>Material Type</th>
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<td>Ideal Specular</td>
<td>★</td>
<td>★</td>
<td>▼</td>
<td>▼</td>
<td>▼</td>
<td>x</td>
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<td>▼</td>
<td>▼</td>
<td>x</td>
<td>perfect diffuse</td>
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<td>▼</td>
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<td>▼</td>
<td>▼</td>
<td>▼</td>
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<tr>
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<td>▼</td>
<td>▼</td>
<td>▼</td>
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<td>★</td>
<td>▼</td>
<td>▼</td>
<td>▼</td>
<td>16.9x</td>
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<tr>
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<td>★</td>
<td>★</td>
<td>▼</td>
<td>…</td>
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<tr>
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<td>▼</td>
<td>★</td>
<td>★</td>
<td>▼</td>
<td>67x</td>
<td>clothes</td>
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<td>14.88x</td>
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<tr>
<td>He et al.</td>
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<td>★</td>
<td>★</td>
<td>★</td>
<td>▼</td>
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<td>★</td>
<td>★</td>
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<td>★</td>
<td>★</td>
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<td>★</td>
<td>▼</td>
<td>▼</td>
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<td>★</td>
<td>▼</td>
<td>▼</td>
<td>▼</td>
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<td>▼</td>
<td>▼</td>
<td>▼</td>
<td>…</td>
<td>metal, plastic</td>
</tr>
<tr>
<td>Lafortune</td>
<td>▼</td>
<td>★</td>
<td>★</td>
<td>▼</td>
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<tr>
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<td>★</td>
<td>★</td>
<td>▼</td>
<td>▼</td>
<td>17.65x</td>
<td>polished surf.</td>
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<td>★</td>
<td>★</td>
<td>▼</td>
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<tr>
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<td>▼</td>
<td>▼</td>
<td>▼</td>
<td>…</td>
<td>old-dirty metal</td>
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</tbody>
</table>

**Table 1:** Brief summary of the properties exhibited by the reviewed BRDFs. Legend: (★) if the BRDF has this property; (▼) if the BRDF does not; (…) unknown value.
Demo

**Lambertian, Oren-Nayar**

Roughness:

0 → 0.1 → 0.5 → 0.9
Demo

Lambertian, Oren-Nayar

Roughness:

0 → 0.1 → 0.5 → 0.9
Demo

Lambertian, Oren-Nayar

Roughness:

0 → 0.1 → 0.5 → 0.9
Demo

Lambertian, **Oren-Nayar**

Roughness:

0 → 0.1 → 0.5 → 0.9
Demo

Cook-Torrance (specular only)

\[ \text{ior} = 1.5 \text{ (typical glass)} \]

\( \Rightarrow f_0 = 0.04 \)

Roughness:

\[ 0.10 \rightarrow 0.25 \rightarrow 0.40 \rightarrow 0.55 \]
Demo

GGX (specular only)

$\text{ior} = 1.5$ (typical glass)

$(\Rightarrow f_0 = 0.04)$

Roughness:

$0.10 \rightarrow 0.25 \rightarrow 0.40 \rightarrow 0.55$
Demo

Cook-Torrance (specular only)

Random parameters:

\[ f_0 \text{ in } [0.02, 0.05] \]

( \Rightarrow \text{ ior in } \sim [1.33, 1.58] \ )

\[ \alpha \text{ in } [0, 1] \]

white specular color
Demo

GGX (specular only)

Random parameters:

- $f_0$ in $[0.02, 0.05]$
- ($\Rightarrow$ $\text{ior}$ in $\sim[1.33, 1.58]$)
- $\alpha$ in $[0, 1]$
- white specular color
The End
Extra
Microfacet distributions visualised

- Experimental Analysis of BRDF Models
  - [http://people.csail.mit.edu/addy/research/ngan05_brdf_eval.pdf](http://people.csail.mit.edu/addy/research/ngan05_brdf_eval.pdf)