LIGHT TRANSPORT

Advanced Computer Graphics 2017
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Before we start:

- Remember to choose a subject for your presentation soon.
- And your project.
- Student representatives:
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- Come for a quick talk with me during recess.
- Muddy Cards!
Light Transport Simulation

- Rendering an image is a matter of "simulating" how light propagates through a virtual scene and lands on a virtual camera film.
- Many algorithms exist, and the best one depends on many factors.
- For a long time, *Photon Mapping* and *Irradiance Caching* were extremely popular.
  - Trade correctness for speed.
  - Will cover these only very briefly.
Photon Mapping
Irradiance Caching
Path Tracing

- Path tracing is an *algorithm* for rendering images.
  - Introduced by James Kajiya in 1986 as a numerical solution to the Rendering Equation.
  - The algorithm is *convergent* and *unbiased*.
  - Has long been considered too noisy/slow to be used in industry.
- Today, almost all commercial renderers use some form of unbiased pathtracing (at least optionally).
  - Pixar (for example) only switched completely very recently.
- Why the sudden popularity?
Mental Ray (photon mapping) 32s

iRay (path tracing) 32s
Mental Ray (photon mapping) 2m8s

iRay (path tracing) 2m8s
Mental Ray (photon mapping) 2m8s
iRay (path tracing) 2m8s
iRay (path tracing) ~1h
**Mental Ray** 15m, 100M Photons, FG 1.0

- Artifacts never go away
- Scene specific tuning choices

**iRay** (path tracing) 15m

- Immediate response
- Much easier to parallelize
Where does an image come from?

Pinhole Camera
Where does an image come from?

Photon emitted
From some random point on the light.
Carries some energy E
Flies in some random direction.
Where does an image come from?

Pinhole Camera

Hits a surface
Some of the energy of some frequencies is absorbed.
The rest is reflected in some direction
Where does an image come from?

Pinhole Camera
Where does an image come from?

Pinhole Camera

Keeps bouncing on surfaces
Loosing energy at each hit
Where does an image come from?

- Pinhole Camera

- Until it leaves the scene…
  - Or just runs out of energy
Where does an image come from?

Pinhole Camera

Or happens to hit our film
Light Transport Equation

\[ L_o(p, \omega) = L_e(p, \omega) + \int_{\Omega} f(p, \omega, \omega') L_i(p, \omega') \cos(n, \omega') \, d\omega' \]

Radiance:
\[ W/(m^2 \text{sr}) \]
Light Transport Equation

\[ L_o(p, \omega) = L_e(p, \omega) + \int_{\Omega} f(p, \omega, \omega') L_i(p, \omega') \cos(n, \omega') \, d\omega' \]
Light Transport Equation

\[ L_o(p, \omega) = L_e(p, \omega) + \int_{\Omega} f(p, \omega, \omega') L_i(p, \omega') \cos(n, \omega') \, d\omega' \]

Integrate over all directions \( \omega' \) in the hemisphere around \( p \)
Light Transport Equation

\[ L_o(p, \omega) = L_e(p, \omega) + \int_\Omega f(p, \omega, \omega') L_i(p, \omega') \cos(n, \omega') d\omega' \]

*Integrate over all directions \( \omega' \) in the hemisphere around \( p \)

*The radiance incoming to point \( p \) from direction \( \omega' \)
Light Transport Equation

\[ L_o(p, \omega) = L_e(p, \omega) + \int_{\Omega} f(p, \omega, \omega') L_i(p, \omega') \cos(n, \omega') d\omega' \]

Integrate over all directions \( \omega' \) in the hemisphere around \( p \)

The radiance incoming to point \( p \) from direction \( \omega' \)

Convert between irradiance parallel to \( \omega \) and irradiance on surface.

\[ \cos(n, \omega') \]

Emitter radiance

Reflected radiance
Light Transport Equation

Integrate over all directions $\omega'$ in the hemisphere around $p$

The BRDF tells us the differential outgoing radiance in direction $\omega$ due to irradiance on surface.

Emissed radiance

$\mathbf{L}_o(p, \omega) = \mathbf{L}_e(p, \omega) + \int_{\Omega} f(p, \omega, \omega') \mathbf{L}_i(p, \omega') \cos(n, \omega') \, d\omega'$

Convert between irradiance parallel to $\omega'$ and irradiance on surface.

The radiance incoming to point $p$ from direction $\omega'$
Light Transport Equation

\[ L_i(p, \omega') = L_o(p', -\omega') \]

\[ = L_e(p', -\omega') + \int_{\Omega} f(p', -\omega', \omega'') L_i(p', \omega') \cos(n', \omega') \, d\omega' \]

\[ L_o(p, \omega) = L_e(p, \omega) + \int_{\Omega} f(p, \omega, \omega') L_i(p, \omega') \cos(n, \omega') \, d\omega' \]
Numerical Integration

\[ \text{Area} = \int_{0}^{X} f(x) \, dx \]
Monte Carlo Integration

- We can estimate any integral \( \int_{a}^{b} f(x)dx \) using MC:

\[
F_N = \frac{1}{N} \sum_{i=1}^{N} \frac{f(X_i)}{p(X_i)}
\]

- \( X_i \) is a random sample drawn with probability density function \( p(x) \)

- In this case, domain is \([0,1]\) and probability is constant, so \( p(x) = 1 \)
Monte Carlo Integration

\[ F_N = \frac{1}{N} \sum_{i=1}^{N} \frac{f(X_i)}{p(X_i)} = \frac{1}{1} \cdot 1.8 \]
Monte Carlo Integration

\[ F_N = \frac{1}{N} \sum_{i=1}^{N} f(X_i) \]

\[ p(X_i) = \frac{1}{1.8} \]

**Convergent:**
As \( N \) approaches infinity, \( F_N \) approaches \( \int_{0}^{X} f(x) \, dx \)

**Unbiased:**
Regardless of \( N \), the expected value \( E[F_N] = \int_{0}^{X} f(x) \, dx \).
Which means that just averaging the results of an infinite number of bad approximations will yield the correct value!
Monte Carlo Integration

$$F_N = \frac{1}{N} \sum_{i=1}^{N} \frac{f(X_i)}{p(X_i)} = \frac{1}{3} (1.8 + 1.0 + 0.2)$$
Monte Carlo Integration

$$F_N = \frac{1}{N} \sum_{i=1}^{N} \frac{f(X_i)}{p(X_i)} = \ldots$$

Area = \int_{0}^{x} f(x) \, dx
Monte Carlo Integration

\[ F_N = \frac{1}{N} \sum_{i=1}^{N} \frac{f(X_i)}{p(X_i)} = \ldots \]

\[ \text{Area} = \int_{0}^{1} f(x) \, dx \]
Monte Carlo Integration

\[ L_0(p, \omega) = L_e(p, \omega) + \int_{\Omega} f(p, \omega, \omega') L_i(p, \omega') \cos(n, \omega') d\omega' \]
Monte Carlo Integration

\[ L_o(p, \omega) \approx L_e(p, \omega) + \frac{1}{N} \sum_{i=0}^{N} \frac{f(p, \omega, \omega_i) L_i(p, \omega_i) \cos(n, \omega_i)}{p(\omega_i)} \]
Monte Carlo Integration

\[ L_0(p, \omega) = E \left[ L_e(p, \omega) + \frac{1}{N} \sum_{i=0}^{N} \frac{f(p, \omega, \omega_i)L_i(p, \omega_i)\cos(n, \omega_i)}{p(\omega_i)} \right] \]
Monte Carlo Integration

\[ L_o(p, \omega) = E \left[ L_e(p, \omega) + \frac{1}{1} \sum_{i=0}^{1} \frac{f(p, \omega, \omega_i)L_i(p, \omega_i)\cos(n, \omega_i)}{p(\omega_i)} \right] \]

Sample hemisphere uniformly:
\[ p(\omega_i) = \frac{1}{2\pi} \]
Monte Carlo Integration

\[ L_o(p, \omega) = E \left[ L_e(p, \omega) + \frac{f(p, \omega, \omega_i)L_i(p, \omega_i) \cos(n, \omega_i)}{p(\omega_i)} \right] \]

Sample hemisphere uniformly:
\[ p(\omega_i) = \frac{1}{2\pi} \]
Monte Carlo Integration

\[
L_o(p, \omega) = E[L_e(p, \omega) + 2\pi f(p, \omega, \omega_i)L_i(p, \omega_i)\cos(n, \omega_i)]
\]

Sample hemisphere uniformly:
\[
p(\omega_i) = \frac{1}{2\pi}
\]
Naive Pathtracing

\[ L_o(p, \omega) \approx L_e(p, \omega) + 2\pi f(p, \omega, \omega_i)L_i(p, \omega_i)\cos(n, \omega_i) \]
Naive Pathtracing

\[ L_o(p, \omega) \approx L_e(p, \omega) + 2\pi f(p, \omega, \omega_i)L_i(p, \omega_i)\cos(n, \omega_i) \]
Naive Pathtracing

\[ L_0(p, \omega) \approx 0 + 2\pi f(p, \omega, \omega_i)L_i(p, \omega_i) \cos(n, \omega_i) \]
Naive Pathtracing

\[ L_0(p, \omega) \approx 0 + 2\pi f(p, \omega, \omega_i)L_0(p', -\omega_i)\cos(n, \omega_i) \]
Naive Pathtracing

\[ L_o(p, \omega) \approx 0 + 2\pi f(p, \omega, \omega_i)L_i(p, \omega_i) \cos(n, \omega_i) \]
Naive Pathtracing

\[ L_0(p, \omega) \approx 0 + 2\pi f(p, \omega, \omega_i)[Le(p', -\omega_i) + 2\pi f(p', -\omega_i, \omega_j)L_i(p', \omega_j)\cos(n', \omega_j)]\cos(n, \omega_i) \]
Naive Pathtracing

\[ L_o(p, \omega) \approx 0 + 2\pi f(p, \omega, \omega_i)[0 + 2\pi f(p', -\omega_i, \omega_j)L_i(p', \omega_j)\cos(n', \omega_j)]\cos(n, \omega_i) \]
Naive Pathtracing

\[ L_o(p, \omega) \approx 0 + 2\pi f(p, \omega, \omega_i)[0 + 2\pi f(p', -\omega_i, \omega_j)L_i(p', \omega_j)\cos(n', \omega_j)]\cos(n, \omega_i) \]
Naive Pathtracing

\[ L_o(p, \omega) \approx 0 + 2\pi f(p, \omega, \omega_i)[0 + 2\pi f(p', -\omega_i, \omega_j)L_o(p'', -\omega_j)\cos(n', \omega_j)]\cos(n, \omega_i) \]
Naive Pathtracing

\[ L_o(p, \omega) \approx 0 + 2\pi f(p, \omega, \omega_i)[0 + 2\pi f(p', -\omega_i, \omega_j)L_e(p'', -\omega_i) \cos(n', \omega_j)] \cos(n, \omega_i) \]
Naive Pathtracing

What's so naïve about this?
Surface form of LTE

\[ L_0(p, \omega_0) = \int \frac{f(\omega_0, \omega_i) L_i(\omega_i) \cos(\theta_i)}{\cos\theta'} \frac{dA}{r^2} \]

\[ = \int_{A'} f(\omega_i, \omega_0) L_i(\omega_i) \cos(\theta_i) \frac{dA}{r^2} \]

Note: All visible surfaces

\[ dw = \frac{dA \cos\theta'}{r^2} \]
Separating Direct And Indirect Illumination

\[ L_o(p, \omega) = \int_A f(p, q \to p)L_i(p, q \to p)G(p, q)V(p, q)dA + \int_\Omega f(p, \omega, \omega')L_i(p, \omega')\cos(n, \omega')d\omega' \]
Importance Sampling

• So far we have sampled incoming light uniformly over the hemisphere. Why is that a bad idea?
• We want to shoot more samples where the function we are integrating is high!
Importance Sampling

• So far we have sampled incoming light uniformly over the hemisphere. Why is that a bad idea?
• We want to shoot more samples where the function we are integrating is high!
### Probability Density Function

In probability theory, a **probability density function (PDF)**, or **density** of a continuous random variable, is a function that describes the relative likelihood for this random variable to take on a given value. The probability of the random variable falling within a particular range of values is given by the integral of this variable’s density over that range—that is, it is given by the area under the density function but above the horizontal axis and between the lowest and greatest values of the range. The probability density function is nonnegative everywhere, and its integral over the entire space is equal to one.

The terms "probability distribution function" and "probability function" have also sometimes been used to denote the probability density function. However, this use is not standard among probabilists and statisticians. In other sources, "probability distribution function" may be used when the probability distribution is defined as a function over general sets of values, or it may refer to the cumulative distribution function, or it may be a probability mass function (PMF) rather than the density. Further confusion of terminology exists because density function has also been used for what is here called the "probability mass function" (PMF).

In general, though, the PMF is used in the context of discrete random variables (random variables that take values on a discrete set), while PDF is used in the context of continuous random variables.

The density function for a continuous random variable X is a function f(x) such that

\[ f(x) = \frac{dP(X)}{dx} \]

and

\[ \int_{-\infty}^{\infty} f(x) dx = 1 \]

#### Monte Carlo Integration

\[ F_N = \frac{1}{N} \sum_{i=1}^{N} f(X_i) \]

\[ p(X) \, dX = \frac{dP(X)}{dx} \]

\[ \int p(X) \, dX = 1 \]
Importance Sampling

Monte Carlo Integration

\[ F_N = \frac{1}{N} \sum_{i=1}^{N} \frac{f(X_i)}{p(X_i)} \]

\[ p(x) = \frac{1}{3} \]
Importance Sampling

Monte Carlo Integration

\[ F_N = \frac{1}{N} \sum_{i=1}^{N} \frac{f(X_i)}{p(X_i)} \]

\[ p(X) = \begin{cases} 
\frac{1}{6} & 0 < X < 1 \\
\frac{2}{3} & 1 < X < 2 \\
\frac{1}{6} & 2 < X < 3 
\end{cases} \]
Importance Sampling

\[ p(X) = \frac{2}{3} \]

\[ p(X) = \frac{1}{6} \]

Monte Carlo Integration

\[ F_N = \frac{1}{N} \sum_{i=1}^{N} \frac{f(X_i)}{p(X_i)} \]

\[ p(X) = \begin{cases} 
\frac{1}{6} & 0 < X < 1 \\
\frac{2}{3} & 1 < X < 2 \\
\frac{1}{6} & 2 < X < 3 
\end{cases} \]
Importance Sampling

Example:

\[ p(X) = a \, e^{-\frac{(x-1.5)^2}{2c^2}} \]
Importance Sampling

Example:

\[ p(X) = ae^{\frac{-(x-1.5)^2}{2c^2}} \]
\[ f(x) = x + 0.1 \times \sin(x^3) + \pi \]
\[ p(x) \propto x \Rightarrow p(x) = k \times x \]
\[ \int_0^x p(x) \, dx = 1 \Rightarrow p(x) = 2x \]

CDF: \[ P(x) = \int_0^x p(x') \, dx' = x^2 \]

\[ CDF^{-1}: P^{-1}(P(x)) = x \Rightarrow P^{-1}(x) = \sqrt{x} \]

---

What is prob. of choosing \( X_i < x \) ?

Which range \([0, x]\) should contain "x" of all samples?
\[ f(x) = x + 0.1 \times \sin(x^3 \times \pi) \]
\[ p(x) \propto x \Rightarrow p(x) = k \cdot x \]
\[ \int_0^x p(x) \, dx = 1 \Rightarrow p(x) = 2x \]

CDF: \( P(x) = \int_0^x p(x') \, dx' = x^2 \)

CDF\(^{-1}\): \( P^{-1}(P(x)) = x \Rightarrow P^{-1}(x) = \sqrt{x} \)

\[ p^{-1}(0.1) = 0.31 \]

What is the probability of choosing \( X_i < x \)?

Which range \([0 - x]\) should contain \( x\) of all samples?
$$f(x) = x + 0.1 \times \sin(x^3 \times 17 \pi)$$

$$p(x) \propto x \Rightarrow p(x) = k \cdot x$$

$$\int_0^x p(x) \, dx = 1 \Rightarrow p(x) = 2x$$

CDF: $$P(x) = \int_0^x p(x') \, dx' = x^2$$

$$CDF^{-1}: P^{-1}(P(x)) = x \Rightarrow P^{-1}(x) = \sqrt{x}$$

$$P^{-1}(0.2) = 0.44$$

What is prob. of choosing $$X_i < x$$?

Which range $$[0 - x]$$ should contain "x" of all samples?
\[ f(x) = x + 0.1 \times \sin(x^3 \pi) \]
\[ p(x) \propto x \quad \Rightarrow \quad p(x) = k \times x \]
\[ \int_0^x p(x) \, dx = 1 \quad \Rightarrow \quad p(x) = 2x \]

CDF: \[ P(x) = \int_0^x p(x') \, dx' = x^2 \]

CDF\(^{-1}\): \[ P^{-1}(P(x)) = x \quad \Rightarrow \quad P^{-1}(x) = \sqrt{x} \]

What is prob. of choosing \( X_i < x \) ?

Which range \([0, X]\) should contain \( x\) of all samples?
\[ f(x) = x + 0.1 \times \sin(x^3/17\pi) \]
\[ p(x) \propto x \Rightarrow p(x) = k \cdot x \]
\[ \int_0^x p(x) \, dx = 1 \Rightarrow p(x) = 2x \]
\[ CDF: P(x) = \int_0^x p(x') \, dx' = x^2 \]
\[ CDF^{-1}: P^{-1}(P(x)) = x \Rightarrow P^{-1}(x) = \sqrt{x} \]
Importance Sampling

- When evaluating the Rendering Equation, we do not know the function we want to integrate
  - Since it depends on the incoming light over the hemisphere
  - But we do know the BRDF, so we importance sample on that

\[
L_0(p, \omega) = \int_{\Omega} f(p, \omega, \omega') L_i(p, \omega') \cos(n, \omega') \, d\omega'
\]
Importance sampling Blinn MF BRDF

\[ f(\omega_o, \omega_i) = \frac{F(\omega_o) G(\omega_n) D(\omega_n)}{4 \cos \theta_o \cos \theta_i} \]

\[ D(\omega_n) = \left(\frac{(n+2)}{2\pi}\right) (\cos \theta_n)^n \]  
Need to sample \( \omega_n \)  
(and find \( \omega_o \) from that)

\( \Phi_h \) does not affect \( D \): \( \Phi_h = 2\pi \xi_2 \)

PDF must be normalized:

\[ D(\cos \theta_n) = (n+2)(\cos \theta_n)^n \]

\[ p_h(\cos \theta_n) = k D(\cos \theta_n) \]

\[ \int k D(\cos \theta_n) = 1 \]

\[ p_h(\cos \theta_n) = (n+1) \cos^n \theta_n \]

Power distribution in \( \cos \theta_n \):

\[ \cos \theta_n = \frac{n+1}{\xi_1} \]
Multiple Importance Sampling
Multiple Importance Sampling
Multiple Importance Sampling

\[ L_0(p, \omega) = \int_{\Omega} f(p, \omega, \omega') L_i(p, \omega') \cos(n, \omega') \, d\omega' \]
Multiple Importance Sampling

\[ L_0(p, \omega) = \int_{\Omega} f(p, \omega, \omega') L_i(p, \omega') \cos(n, \omega') \, d\omega' \]
Multiple Importance Sampling

\[ L_o(p, \omega) = \int_{\Omega} f(p, \omega, \omega') L_i(p, \omega') \cos(n, \omega') \, d\omega' \]
Multiple Importance Sampling
Multiple Importance Sampling
Multiple Importance Sampling

Series 1, Series 2, Series 3
Multiple Importance Sampling
Multiple Importance Sampling
Multiple Importance Sampling

![Graph showing Multiple Importance Sampling with Series 1, Series 2, Series 3, Series 4, and Series 5 represented in different colors.]
Multiple Importance Sampling
Multiple Importance Sampling

Need to estimate:

\[ \int f(x)g(x)dx \]

Could Use:

\[ 0.5 \left( \frac{f(X)g(X)}{p_f(X)} + \frac{f(Y)g(Y)}{p_g(Y)} \right) \]
Multiple Importance Sampling

Need to estimate:

\[ \int f(x)g(x)dx \]

Could Use:

\[
0.5 \left( \frac{f(X)g(X)}{p_f(X)} + \frac{f(Y)g(Y)}{p_g(Y)} \right)
\]

Better (MIS):

\[
0.5 \left( \frac{f(X)g(X)}{0.5(p_f(X) + p_g(X))} + \frac{f(Y)g(Y)}{0.5(p_f(Y) + p_g(Y))} \right)
\]
Multiple Importance Sampling

Need to estimate:

$$\int f(x) g(x) \, dx$$

Could Use:

$$0.5 \left( \frac{f(X)g(X)}{p_f(X)} + \frac{f(Y)g(Y)}{p_g(Y)} \right)$$

Better (MIS):

$$\frac{f(X)g(X)}{p_f(X) + p_g(X)} + \frac{f(Y)g(Y)}{p_f(Y) + p_g(Y)}$$
Multiple Importance Sampling
Multiple Importance Sampling

- Sample BRDF first
- Then light
- PDF of this sampling strategy is (weighted) sum
- Only very low if neither technique is likely to choose
Stratified Sampling

- Another standard variance reduction method
- When just choosing samples randomly over the domain, they may “clump” and take a long while to converge

It will converge to 0.5 after unlimited time.

But after four samples I still have prob=1/8 that the pixel will be considered all in shadow or completely unshadowed
Stratified Sampling

• Divide domain into “strata”
  • Don’t sample one strata again until all others have been sampled once.
Stratified Sampling

- If we know how many samples we want to take, we can get good stratification from “jittering”
- If not, we want any sequence of samples to have good stratification. We can use a *Low Discrepancy Sequence*
Are we done yet?
Pathtracing

1/25/2017
Advanced Computer Graphics - Path Tracing
Bidirectional Path Tracing
Bidirectional Path Tracing
Pathtracing 75SPP (~5min) 

Bidirectional Pathtracing 45SPP (~5min)
Are we done yet?
Metropolis Light Transport
Metropolis Light Transport
Metropolis Light Transport
Bidirectional Path Tracing
Metropolis Light Transport

MLT 5m

MLT 2h

MLT 17h35m, 9300 SPP
Bidirectional Path Tracing
Metropolis Light Transport
Further Reading