Advanced Algorithms 2016. Exercises 4-5

Exercise 4

In the baseball elimination application, we saw how to use a maxflow computation to decide if a given team was eliminated. By applying the algorithm repeatedly, we can determine with $|T|$ maxflow computations (where T is the set of all teams), the full set of eliminated teams. Show that it is possible to do this with only $\log |T|$ computations.

Hint: First show that if team k is eliminated and if $w_k + g_k \geq w_m + g_m$, then team m is also eliminated.

Exercise 5

(Carpooling). A group of n people decide to carpool for m days. Their schedule is given by a $n \times m$ matrix A such that $A(i, j) = 1$ if person i needs to go to work on day j. Each day one of the people who needs to go to work will do the driving. The objective is to assign the person to drive on each of the m days as "fairly" as possible. A natural measure of fairness is the following. On each day, we divide up the responsibility equally between the people going to work on that day (we assume that the car is big enough to carry all n people if needed.). So if k people are going to work on a given day and i is one of them, his obligation on that day is $o_{i,j} = 1/k$. If i is not going to work on day j, then of course $o_{i,j} = 0$.The total obligation of person i over all days is $o_i = \sum_j o_{i,j}$. Show by using a network flow argument that it is possible to assign the person driving on each day so that person i drives no more than $\lceil o_i \rceil$ days in total.