

## Advanced Algorithms 2016. Exercises 1-2

Section "Homework Exercises" on the course web provides detailed guidelines. In the first line, there is a link to the schedule of weekly exercises.

### Exercise 1.

An independent set in a graph is a subset of nodes without any edges between them. Consider the following rather intuitive greedy algorithm for the Maximum Independent Set problem (finding an independent set of maximum size): We take some node with smallest degree, put it in the solution, and remove this node and all its neighbors from the graph. We iterate this step until the graph is empty.

Prove that this algorithm returns an independent set of size at least  $1/\Delta$  of the maximum size, where  $\Delta$  denotes the largest degree of the nodes in the graph. (The only difficulty here is to do a logically conclusive comparison of the total sizes of both solutions.)

### Exercise 2.

A dominating set in a graph with  $n$  nodes is a subset  $D$  of nodes such that every node is in  $D$  or has at least one neighbor in  $D$ . The Dominating Set problem asks to find a dominating set with a minimum number of nodes in a given graph.

Propose an approximation algorithm that returns a dominating set being only  $O(\log n)$  times larger than a minimum solution. But do not create a new algorithm from scratch, instead make use of the known approximability result for Set Cover.