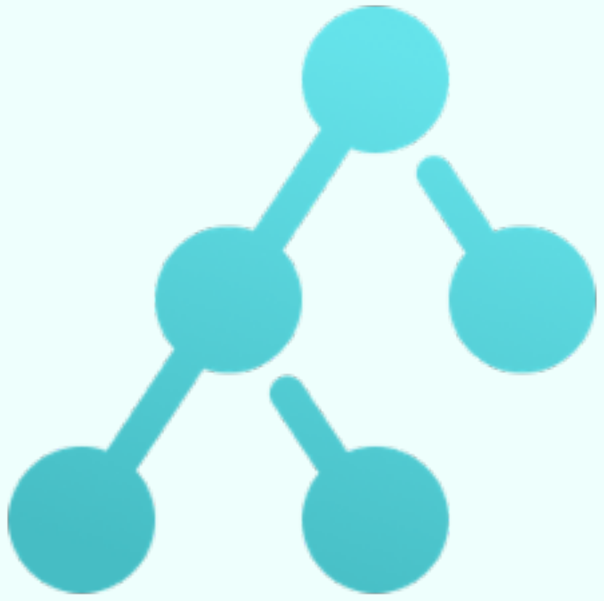
Data Structures

Exercise Session



Marco Vassena

Problem 3

Find if a list of words contains a reversed word

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cat
live
dog
evil
bird

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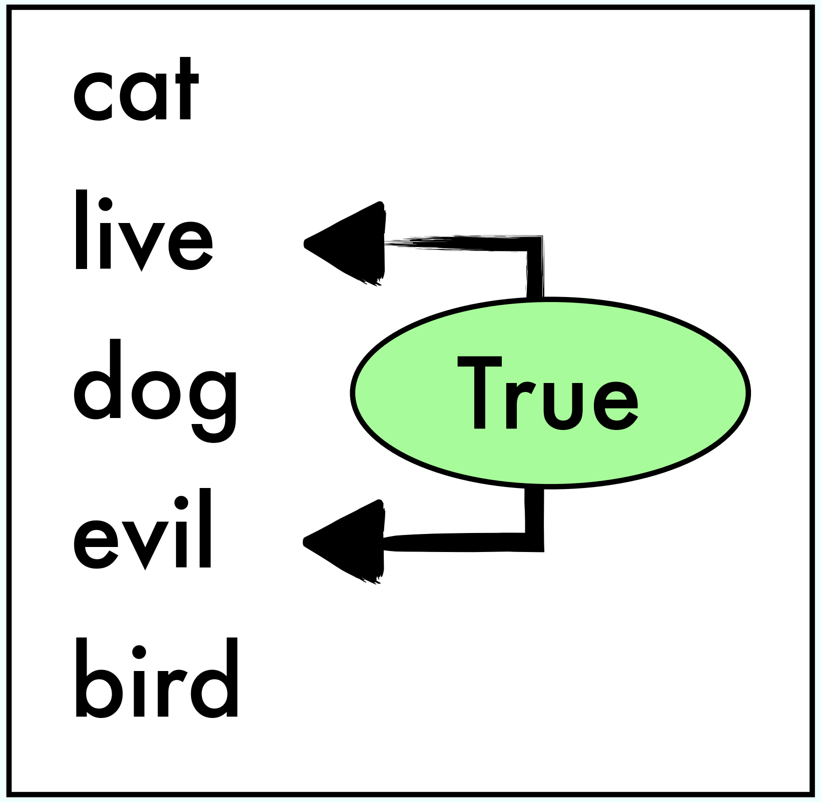
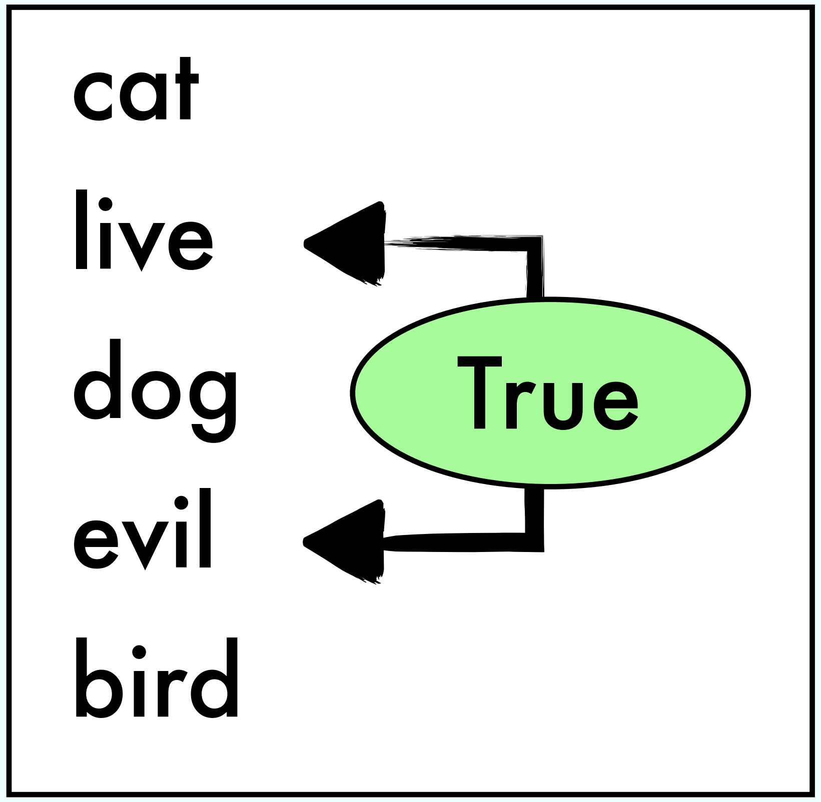
Find if a list of words contains a reversed word



Problem 3

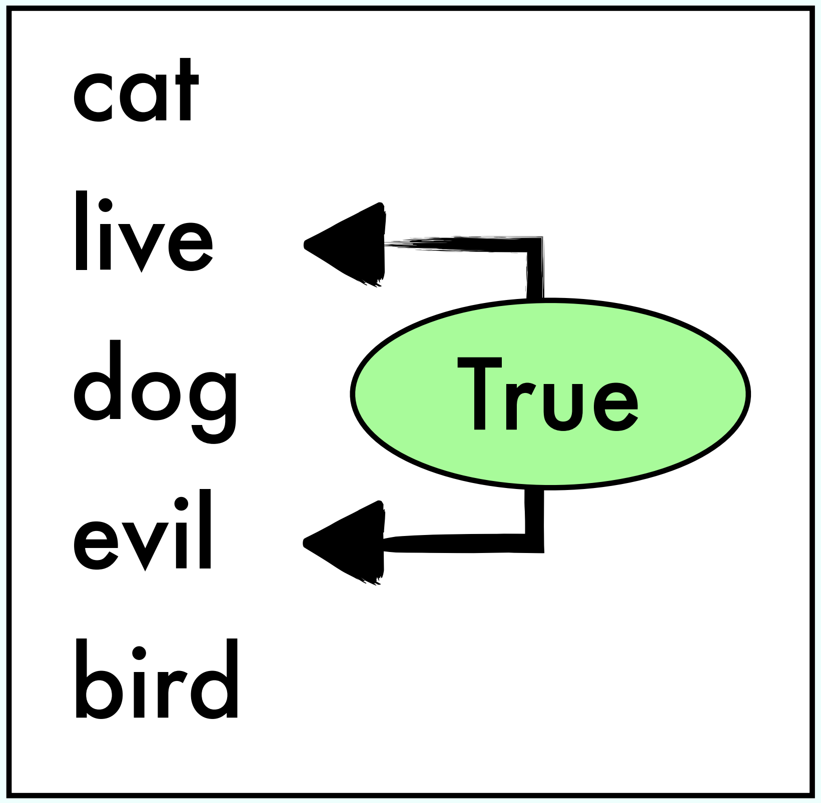
Find if a list of words contains a reversed word

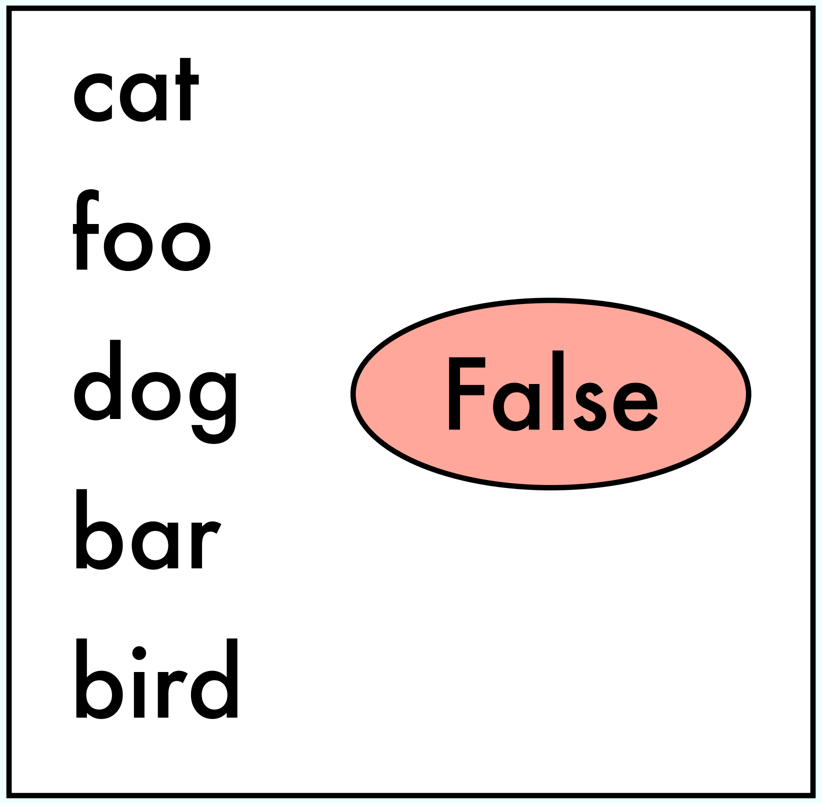


cat
foo
dog
bar
bird

Problem 3

Find if a list of words contains a reversed word





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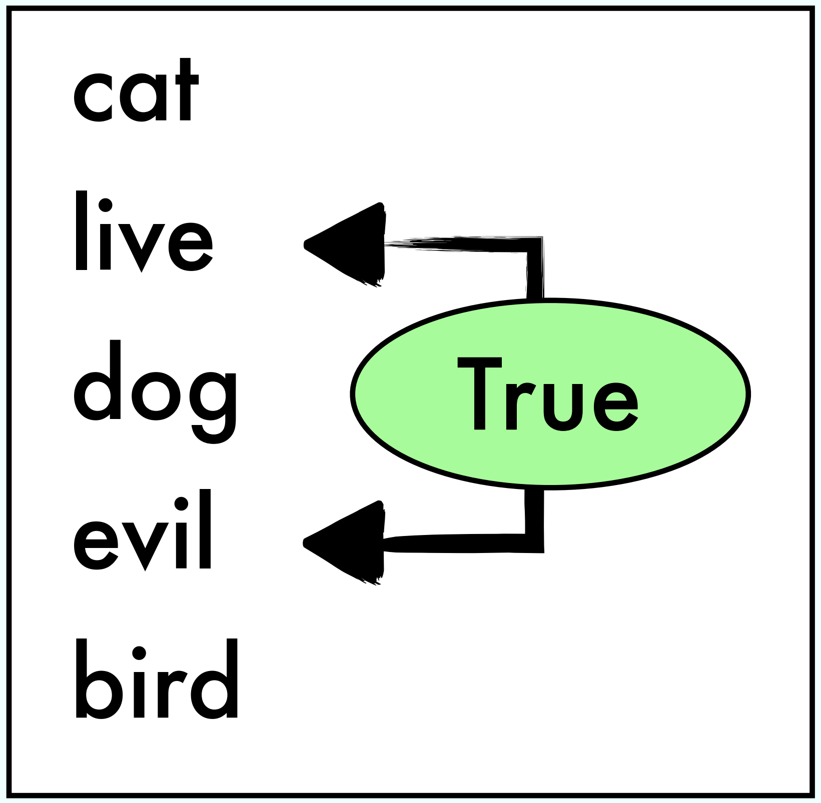
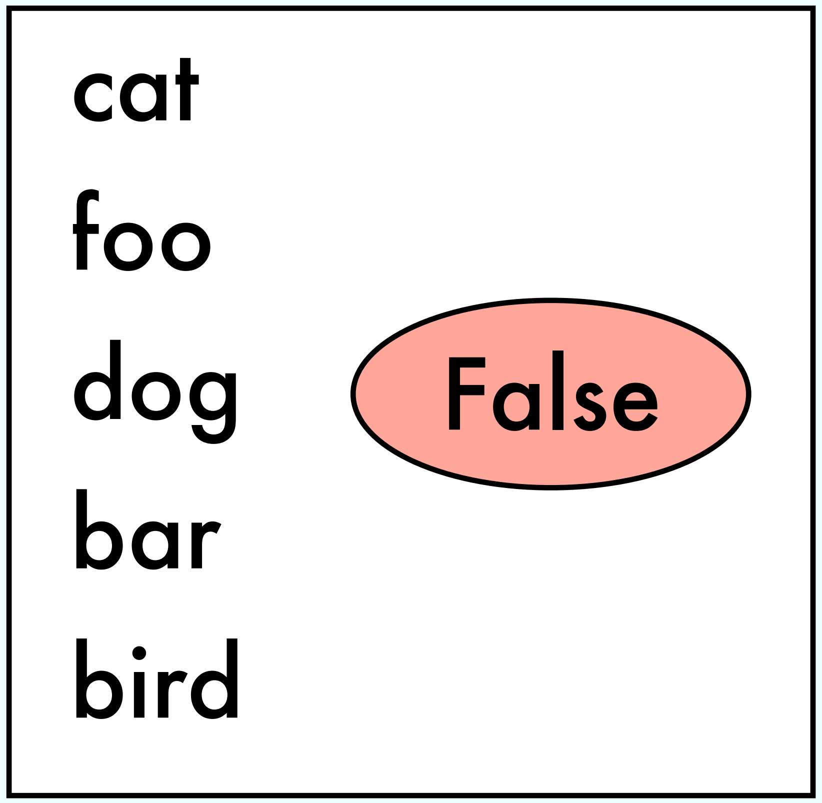


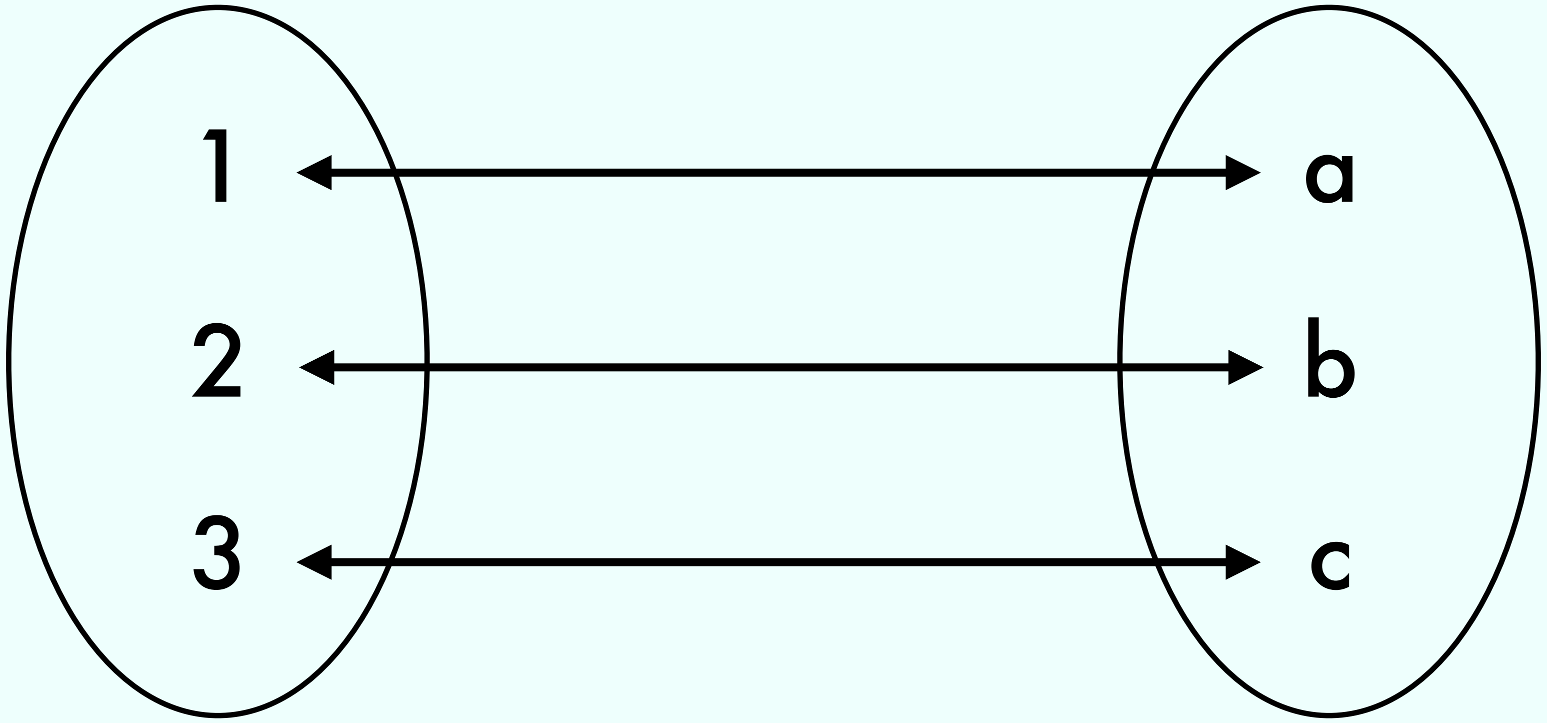


Use hash-based data structures

Analyze the time complexity

```
// $\sum_{w \in ws} |w| = N$
S = new HashSet();
for (String w : ws)
  if (reverse(w) ∈ S)
    then return true;
    else S.insert(w);
return false;
```

Bijection



Invariant

x → y if and only if x ← y

Exercise 2 from 12/8

Design a data structure for bijection

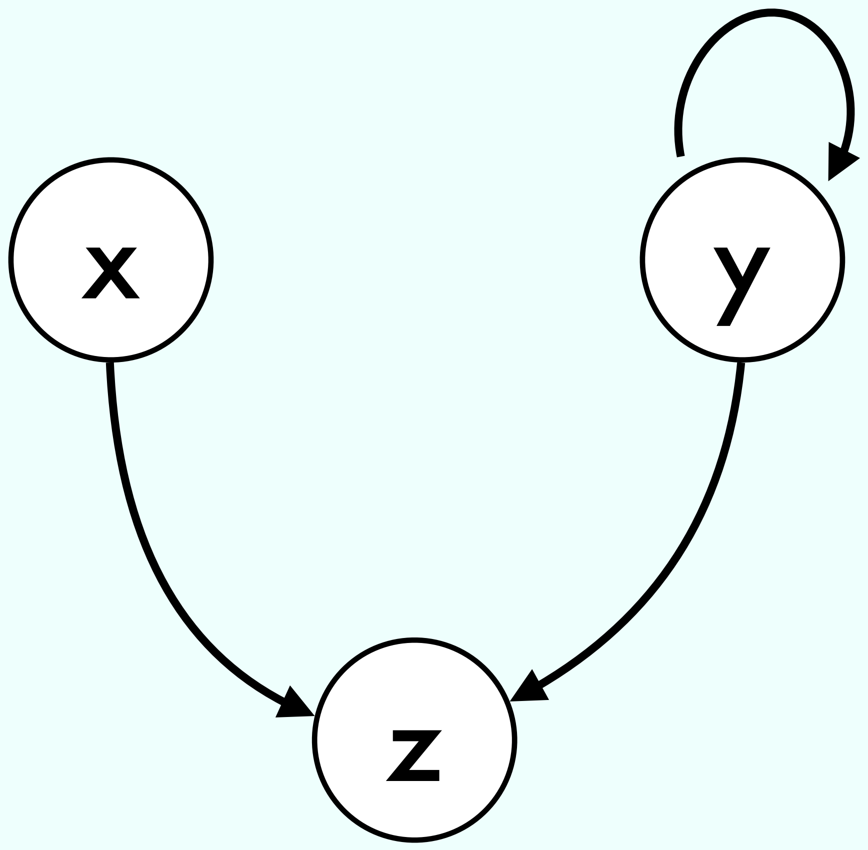
Operation	Time Complexity
`insert(x, y)`	O(log N)
`source(y)`	O(log N)
`target(x)`	O(log N)

Equivalence Relation

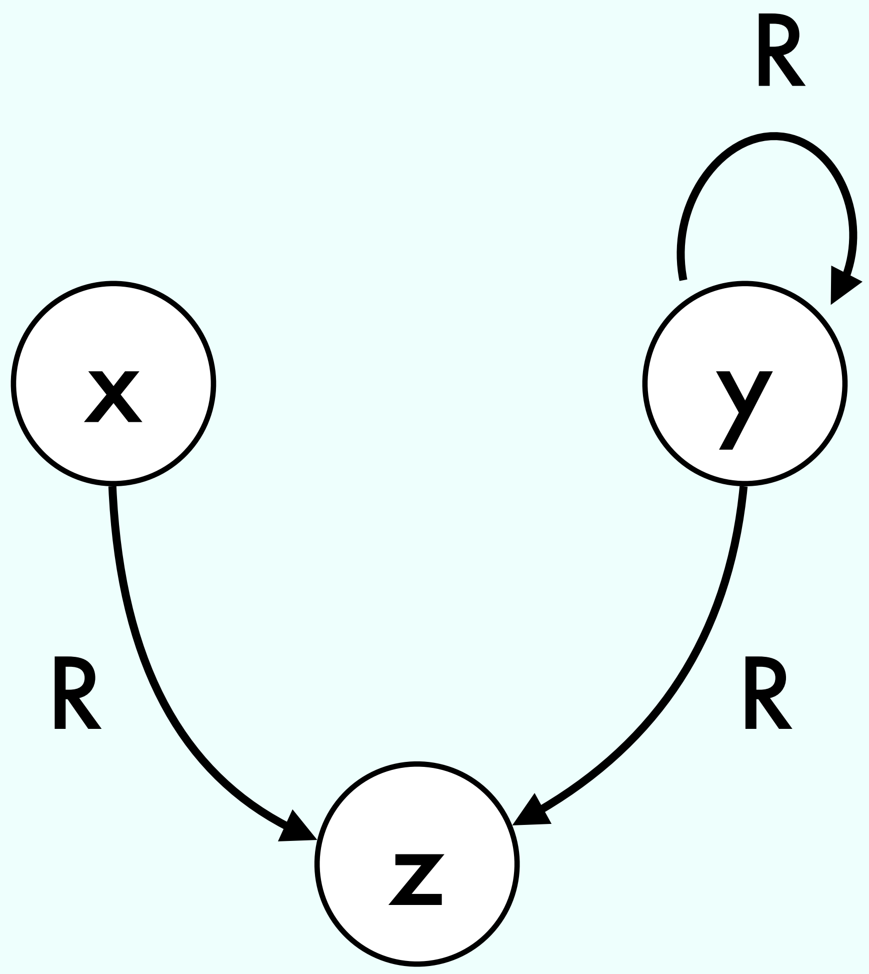
R is an equivalence relation, iff
$\forall$ x y z

`reflexive`	$x \mathrel{R} x$
`symmetric`	$x \mathrel{R} y \Rightarrow y \mathrel{R} x$
`transitive`	$x \mathrel{R} y,\ y \mathrel{R} z \Rightarrow x \mathrel{R} z$

A graph G = (V, E) represents a relation R

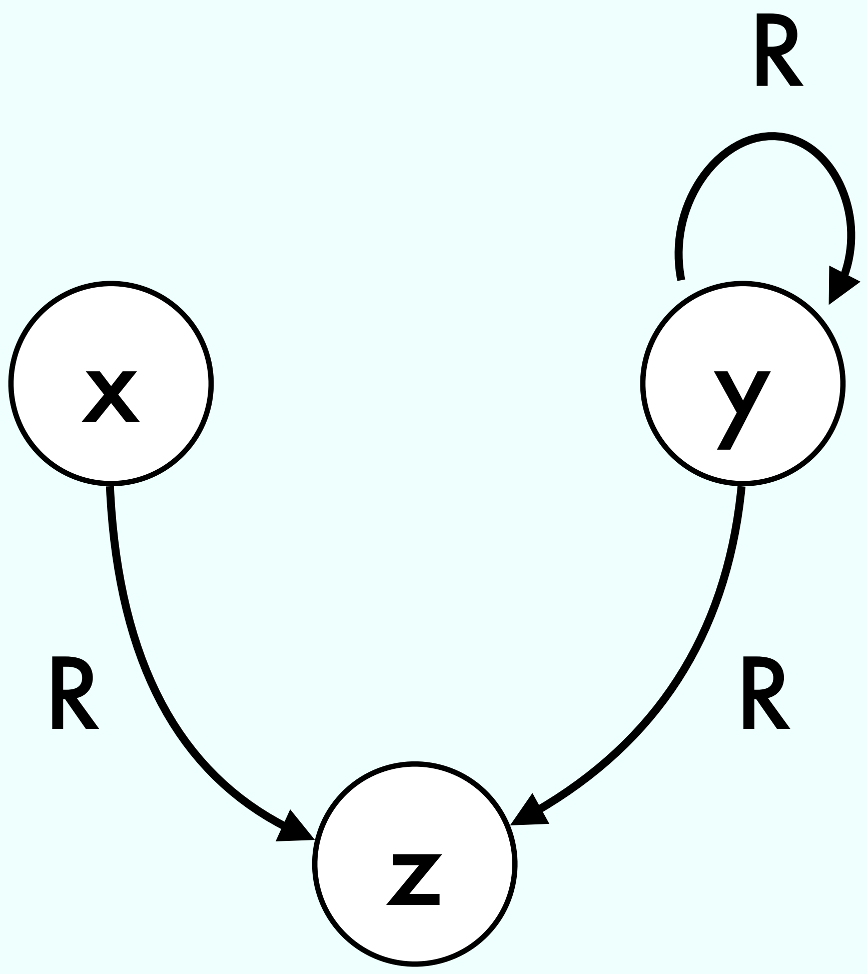


A graph G = (V, E) represents a relation R


$$\begin{tikzcd} x \arrow[dr, "R"'] & & y \arrow[dl, "R"] \arrow[loop above, "R"] \\ & z & \end{tikzcd}$$

A graph G = (V, E) represents a relation R

$$\begin{tikzcd}
x \arrow[dr, "R"'] & & y \arrow[dl, "R"] \arrow[loop above, "R"] \\
& z &
\end{tikzcd}$$



$\Leftrightarrow$

x R z

y R z

y R y

A graph G = (V, E) represents a relation R

$$\begin{tikzcd}
x \arrow[dr, bend right, "R"'] & & y \arrow[dl, bend left, "R"] \arrow[loop above, "R"] \\
& z &
\end{tikzcd}$$

$\Leftrightarrow$

$$\boxed{\begin{array}{c} x \; R \; z \\ y \; R \; z \\ y \; R \; y \end{array}}$$

$$(x,y) \in E \quad \Leftrightarrow \quad x \; R \; y$$

Problem 6

Does G represent
an equivalence relation?

$$\begin{tikzcd}
x \arrow[dr] & & y \arrow[loop above] \arrow[dl] \\
& z &
\end{tikzcd}$$

$\Leftrightarrow$

x R z

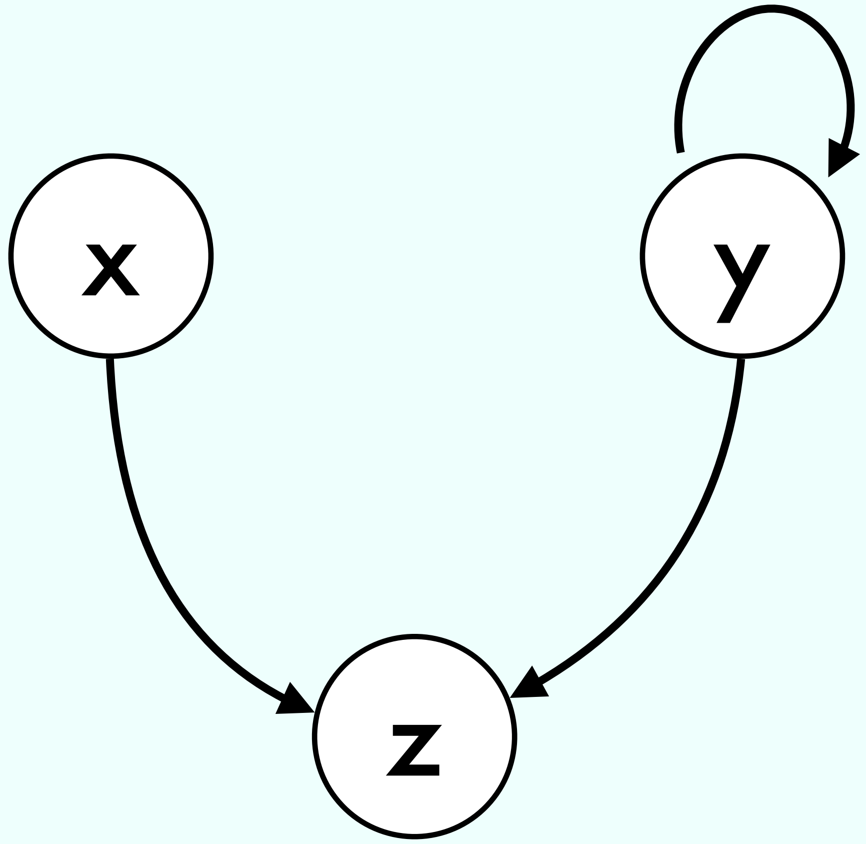
y R z

y R y

Problem 6

Does G represent
an equivalence relation?

False

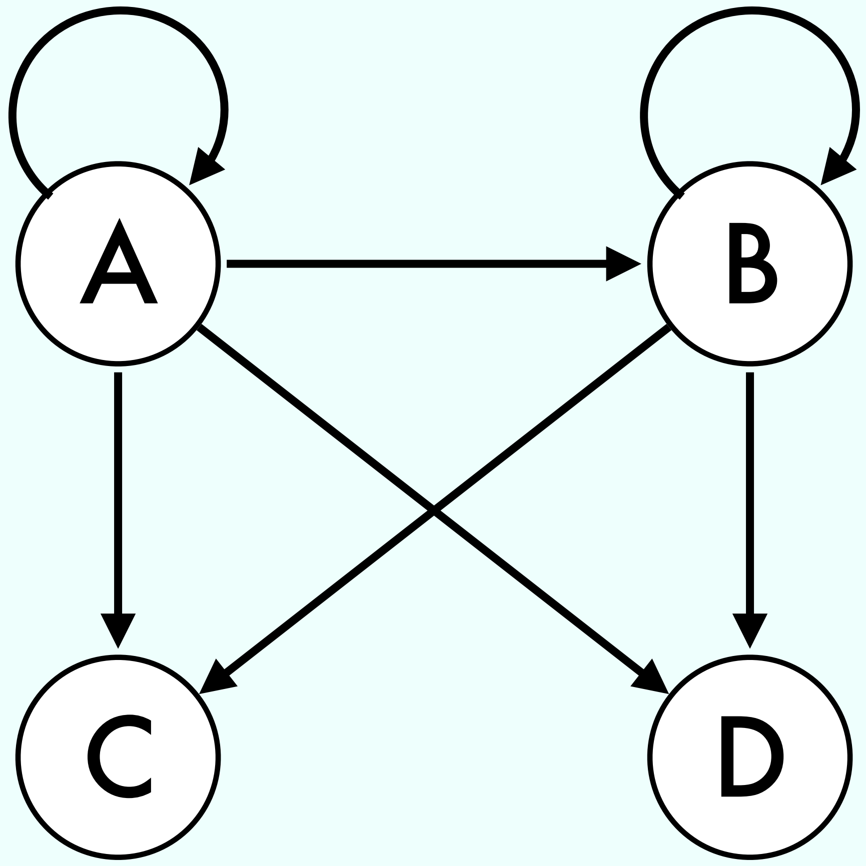


$\Leftrightarrow$

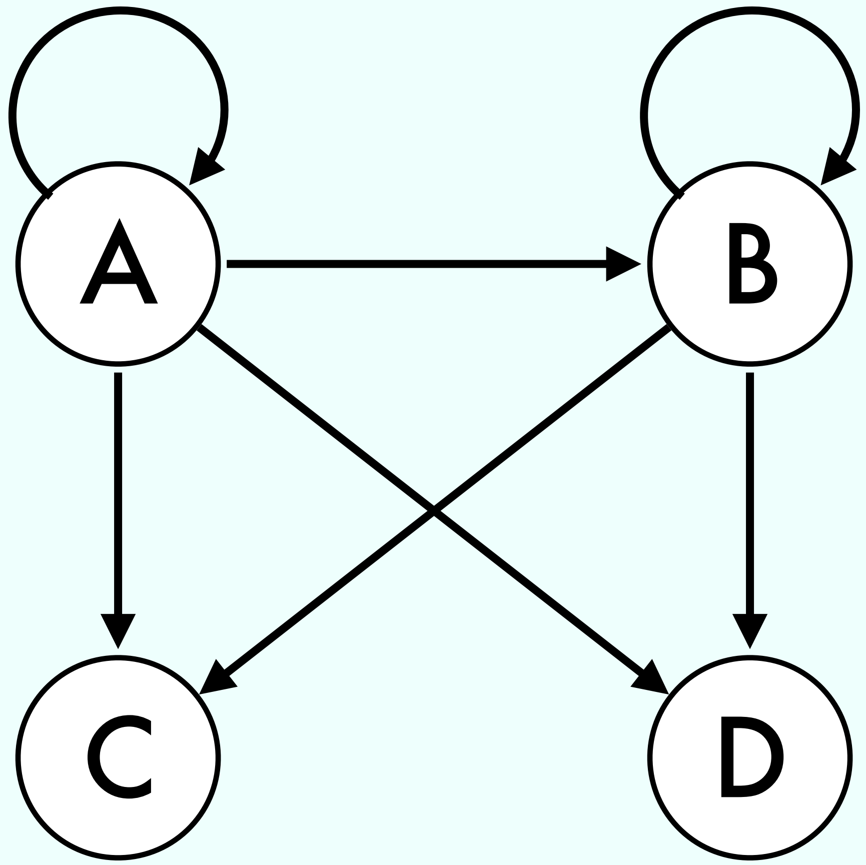
x R z

y R z

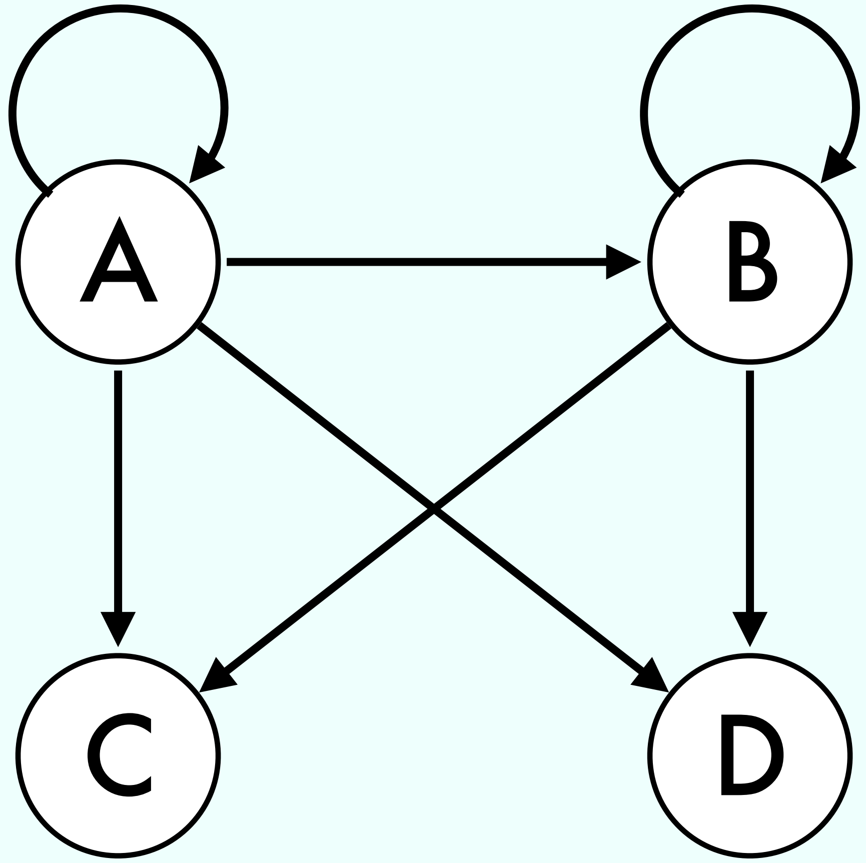
y R y



Path	Length



Path	Length
A - B	
B - C	
A - C	
A - A	
A - D	
B - D	
B - B	



Path	Length
A - B	1
B - C	1
A - C	1
A - A	0
A - D	1
B - D	1
B - B	0

Theorem

G reflexive

G transitive

$\Leftrightarrow$

$\forall$ u v $\in$ V

shortestPath(u, v) $\leq$ 1

Problem 5

Hamiltonian Path
Path that visits all nodes exactly once

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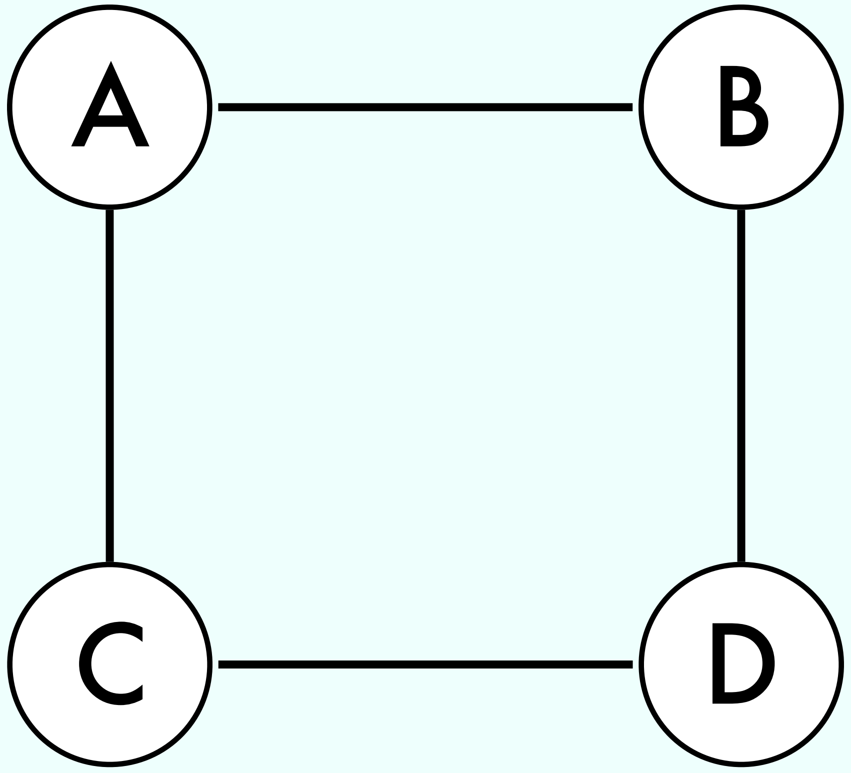
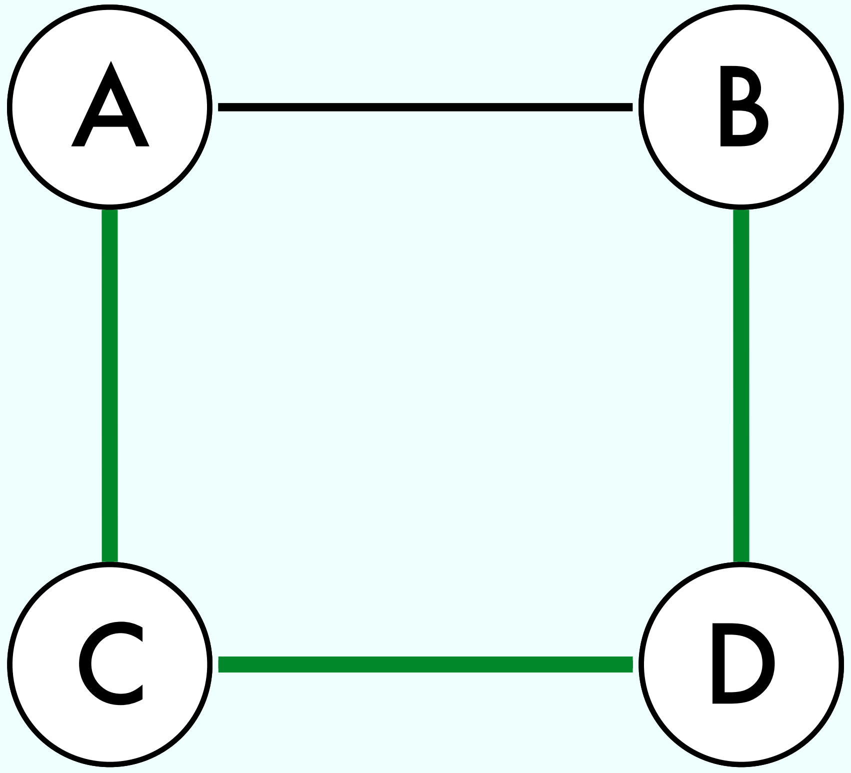
Path that visits all nodes exactly once



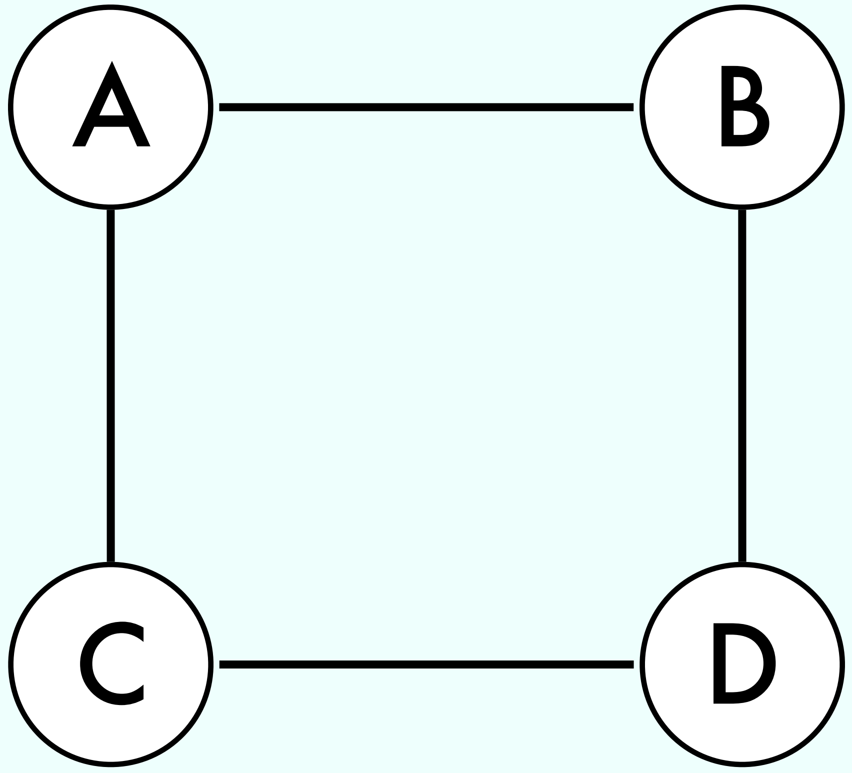
Problem 5

Hamiltonian Path
Path that visits all nodes exactly once



Problem 5

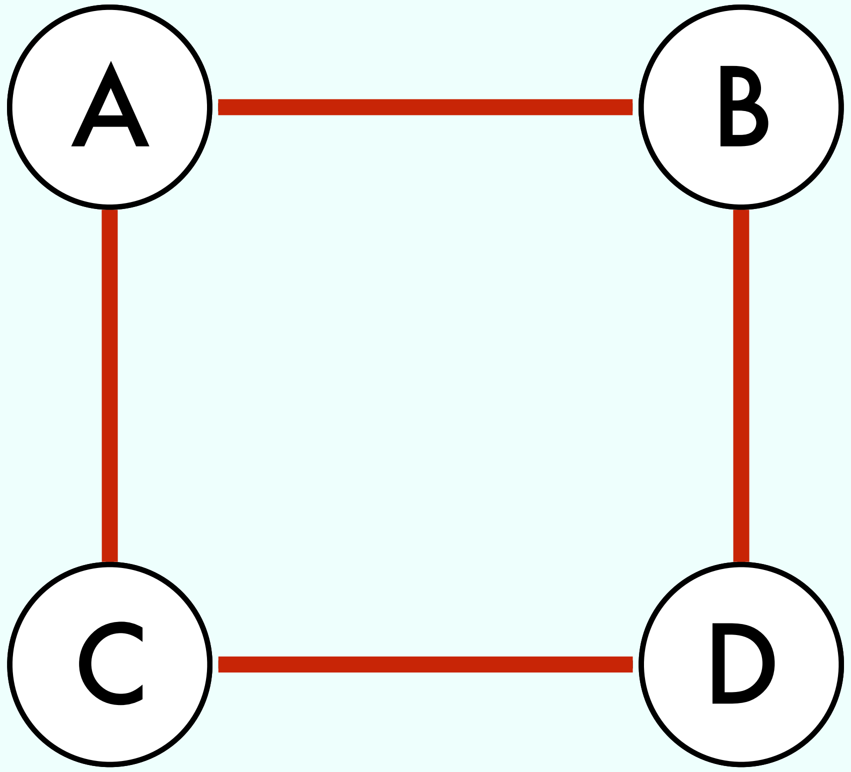
Hamiltonian Cycle
Hamiltonian path that is a cycle



Problem 5

Hamiltonian Cycle

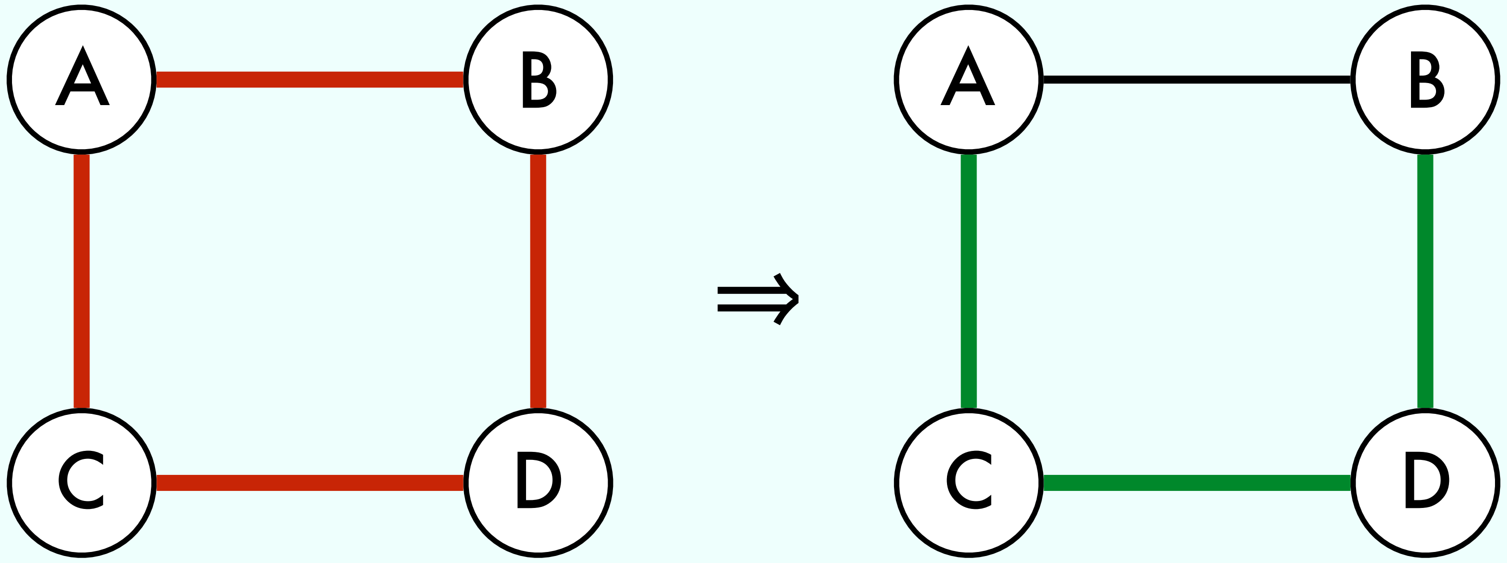
Hamiltonian path that is a cycle

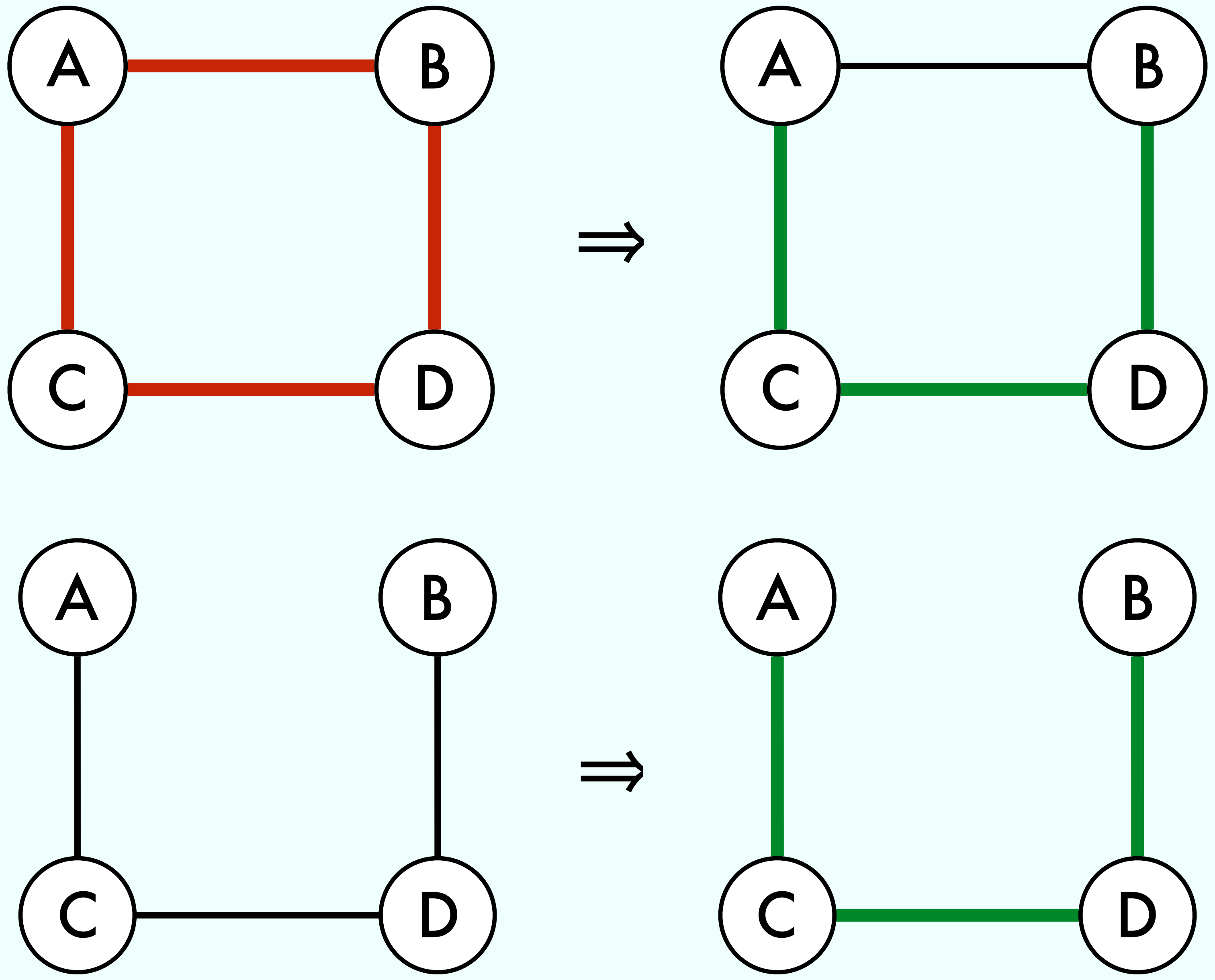


Problem 5

Assume you have hasHCycle(G)

Implement hasHPath(G)


A
B
C
D
⇒
A
B
C
D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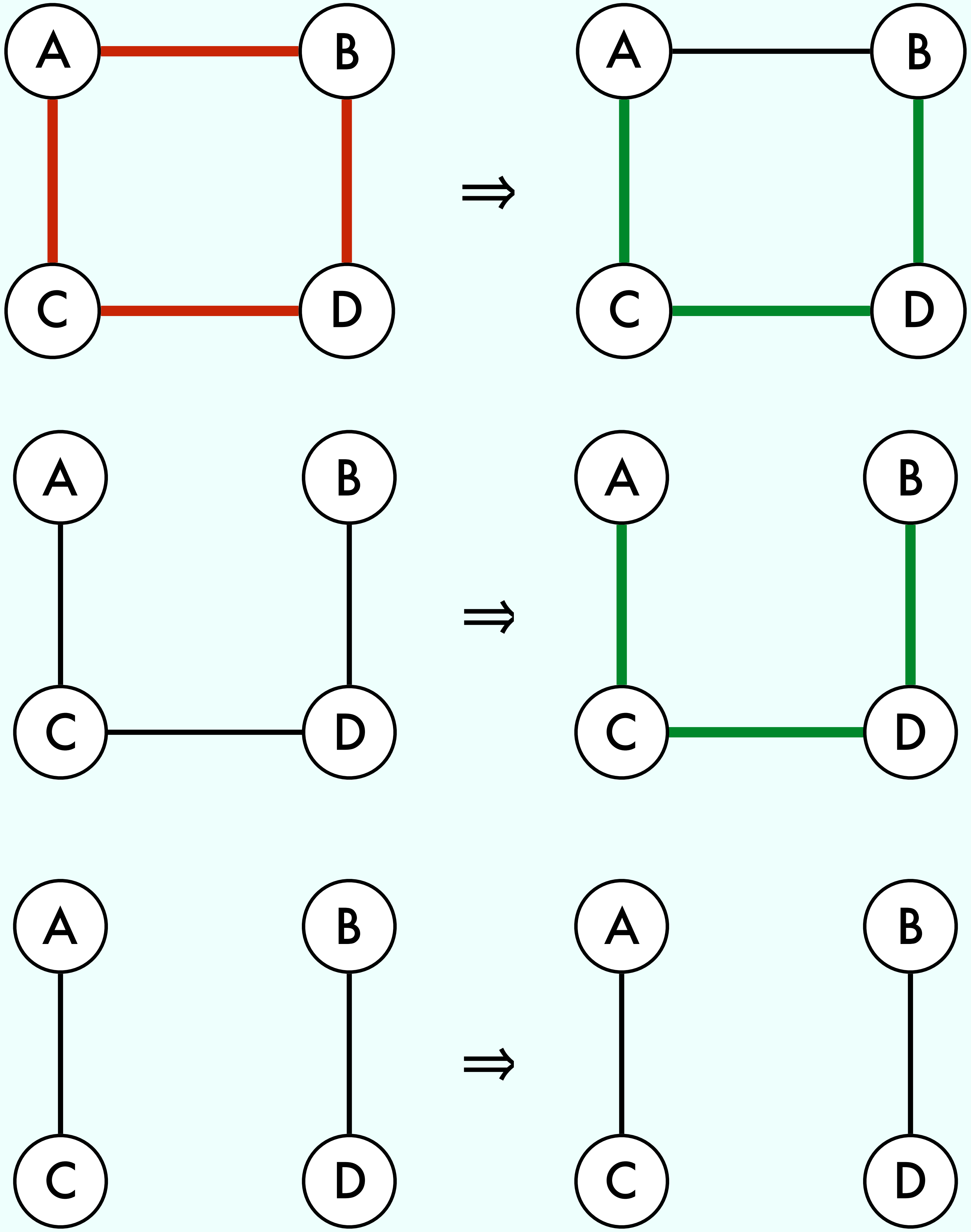
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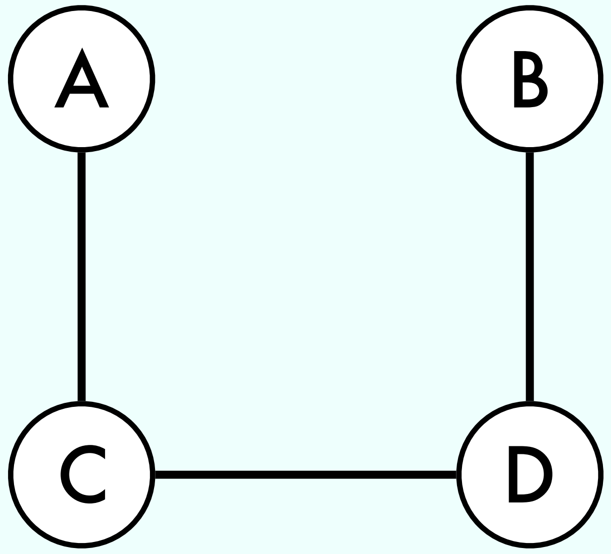
A
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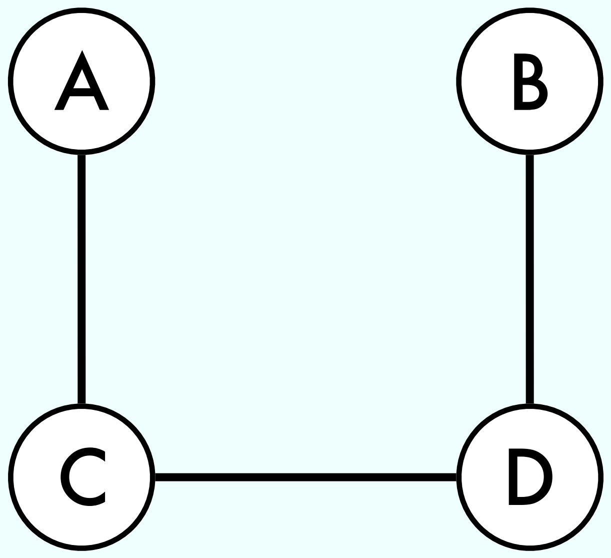
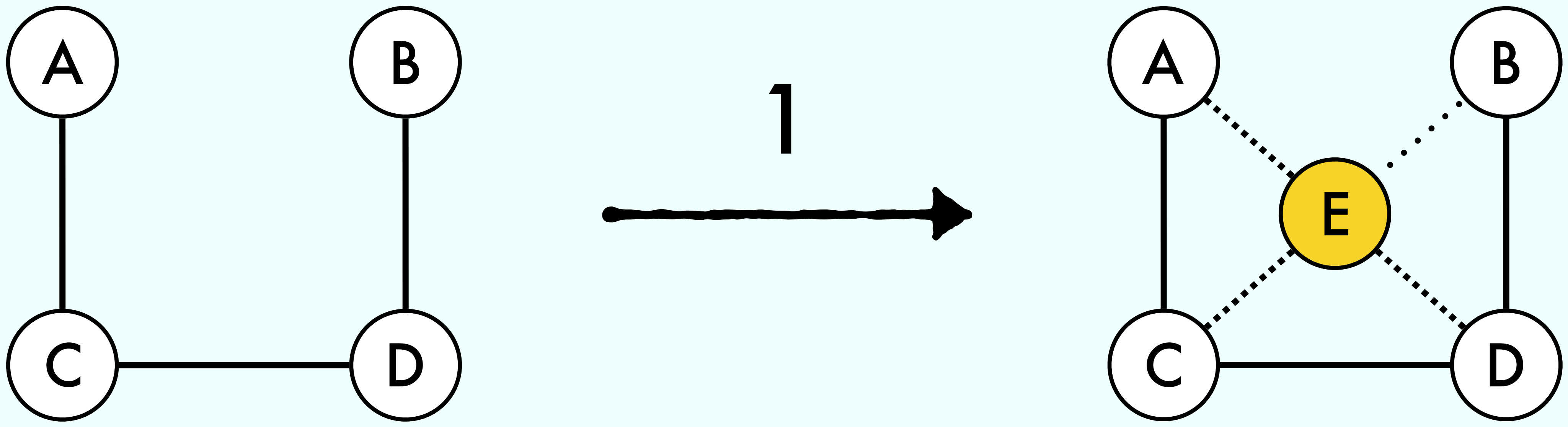
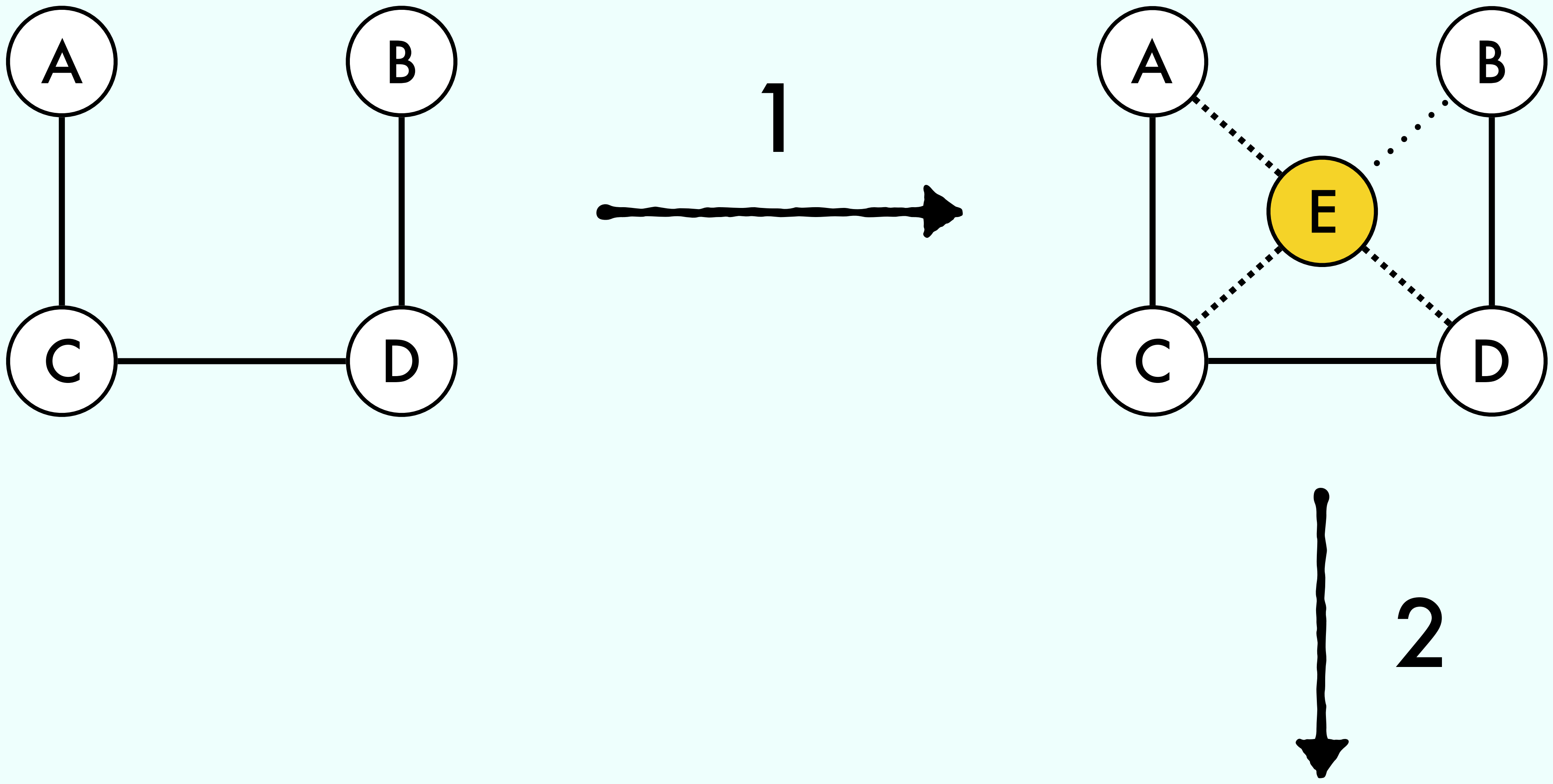
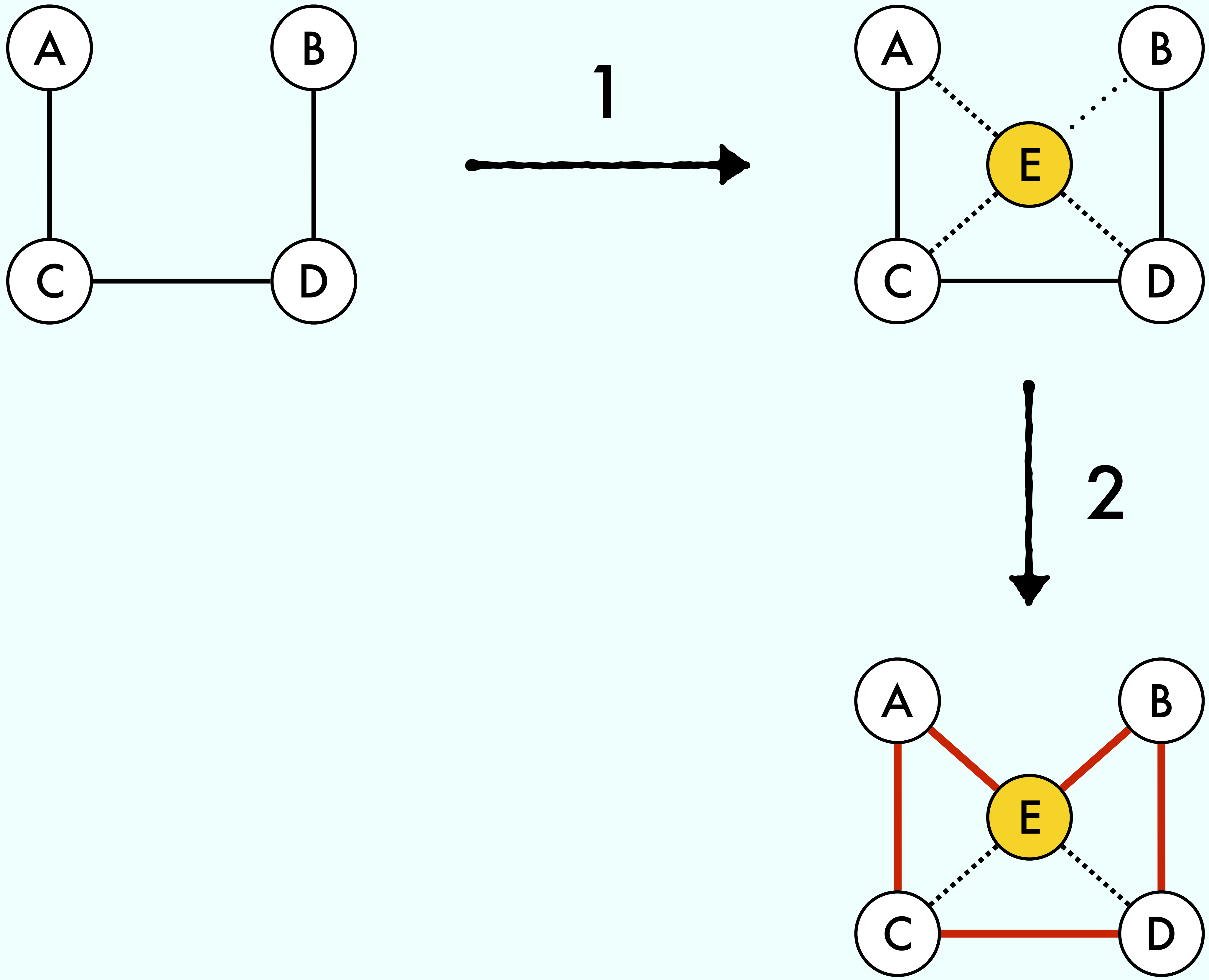


$$\xrightarrow{1}$$


A
B
C
D
1
A
B
E
C
D


A
B
C
D
1
A
B
E
C
D
2


A
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D
2
A
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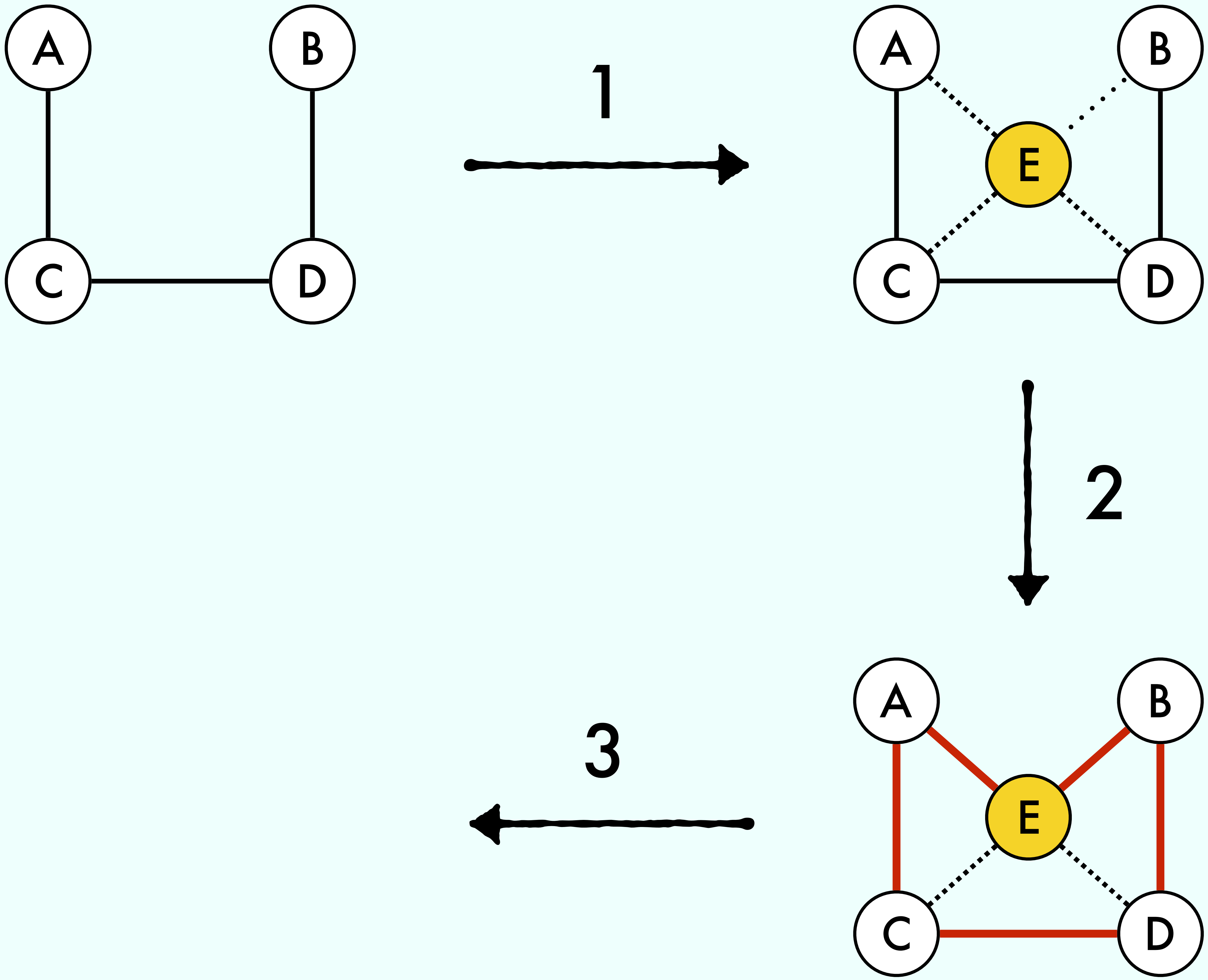
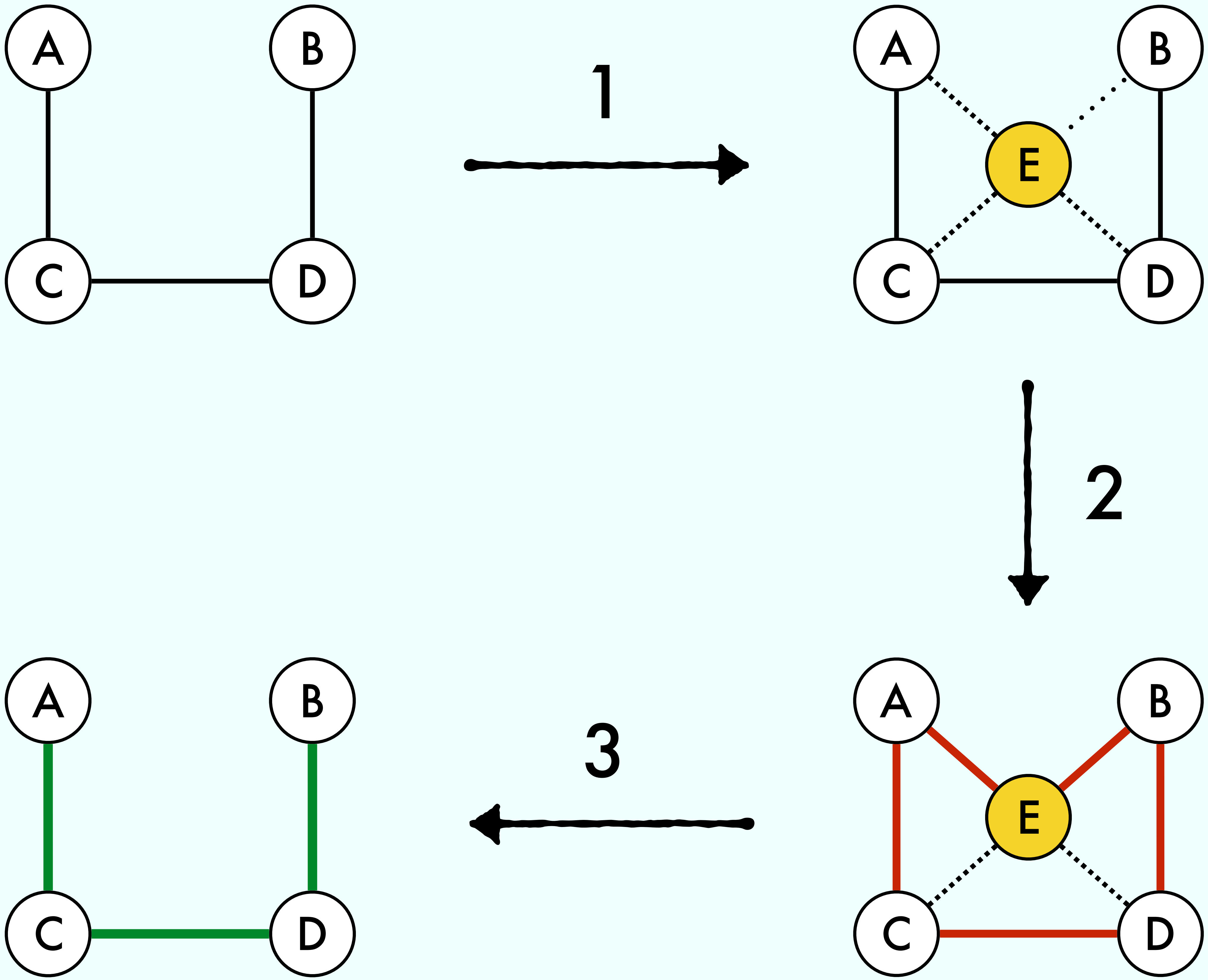
C
D


A
B
C
D
1
A
B
E
C
D
2
A
B
E
C
D
3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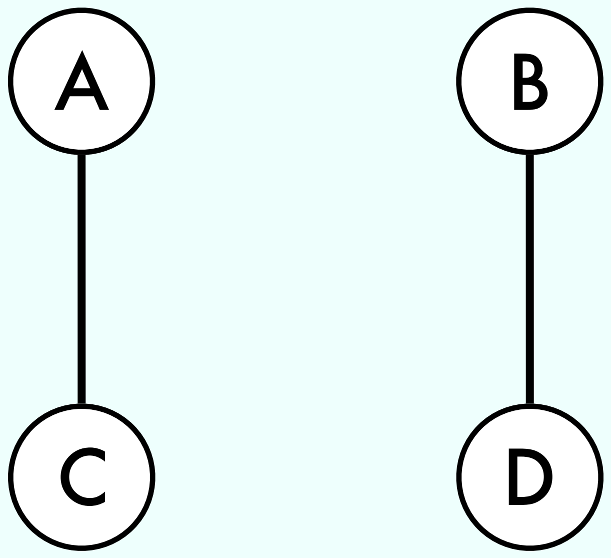
A
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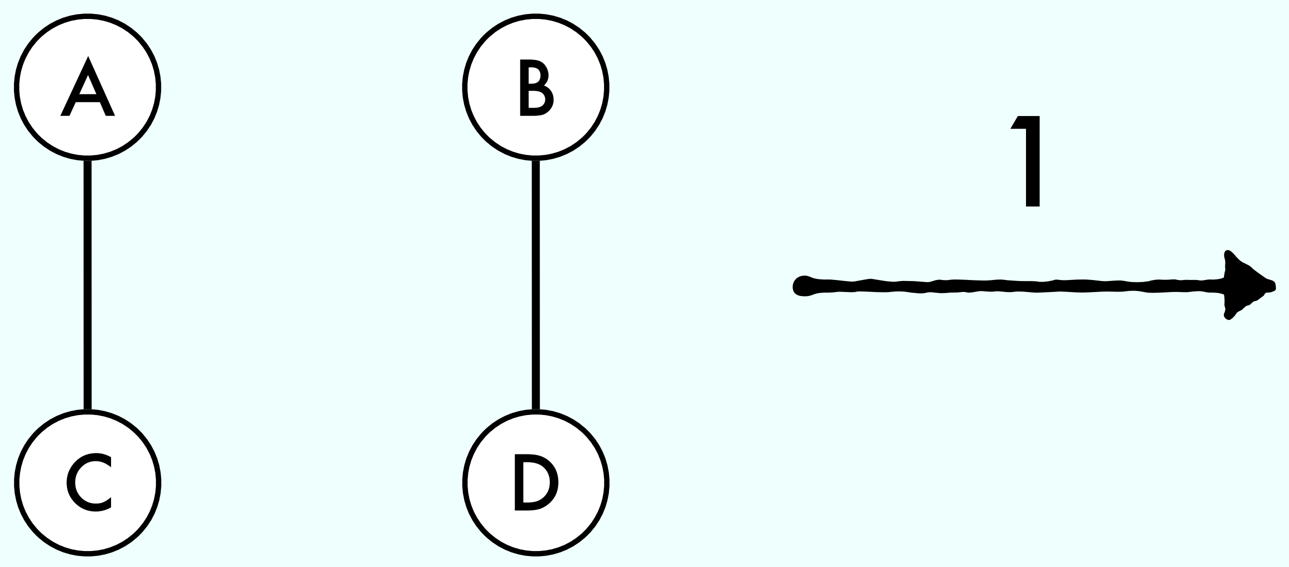
A
B
C
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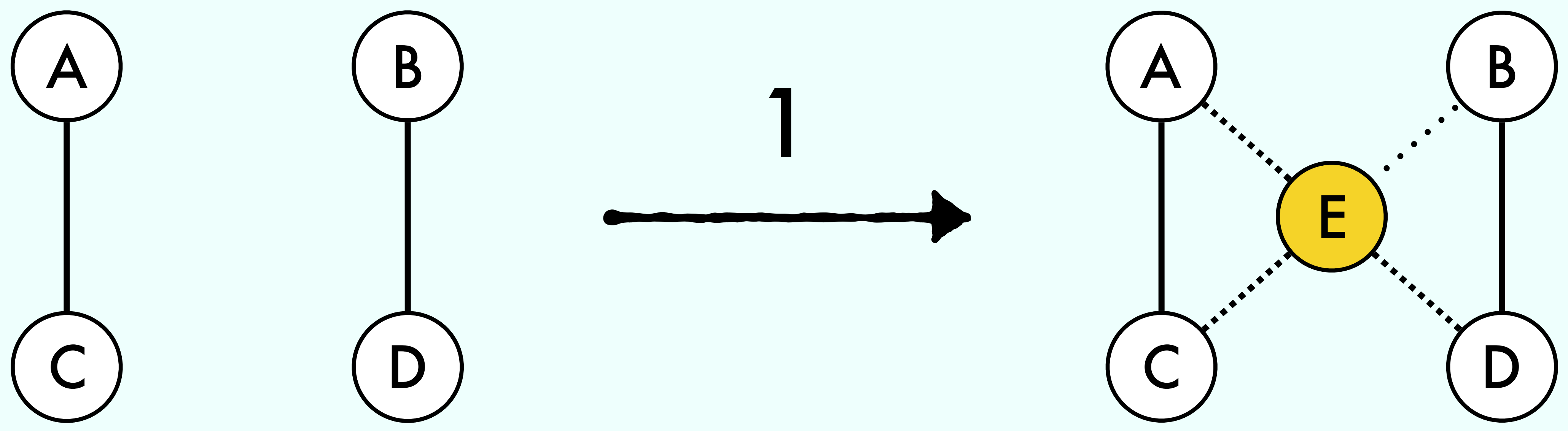
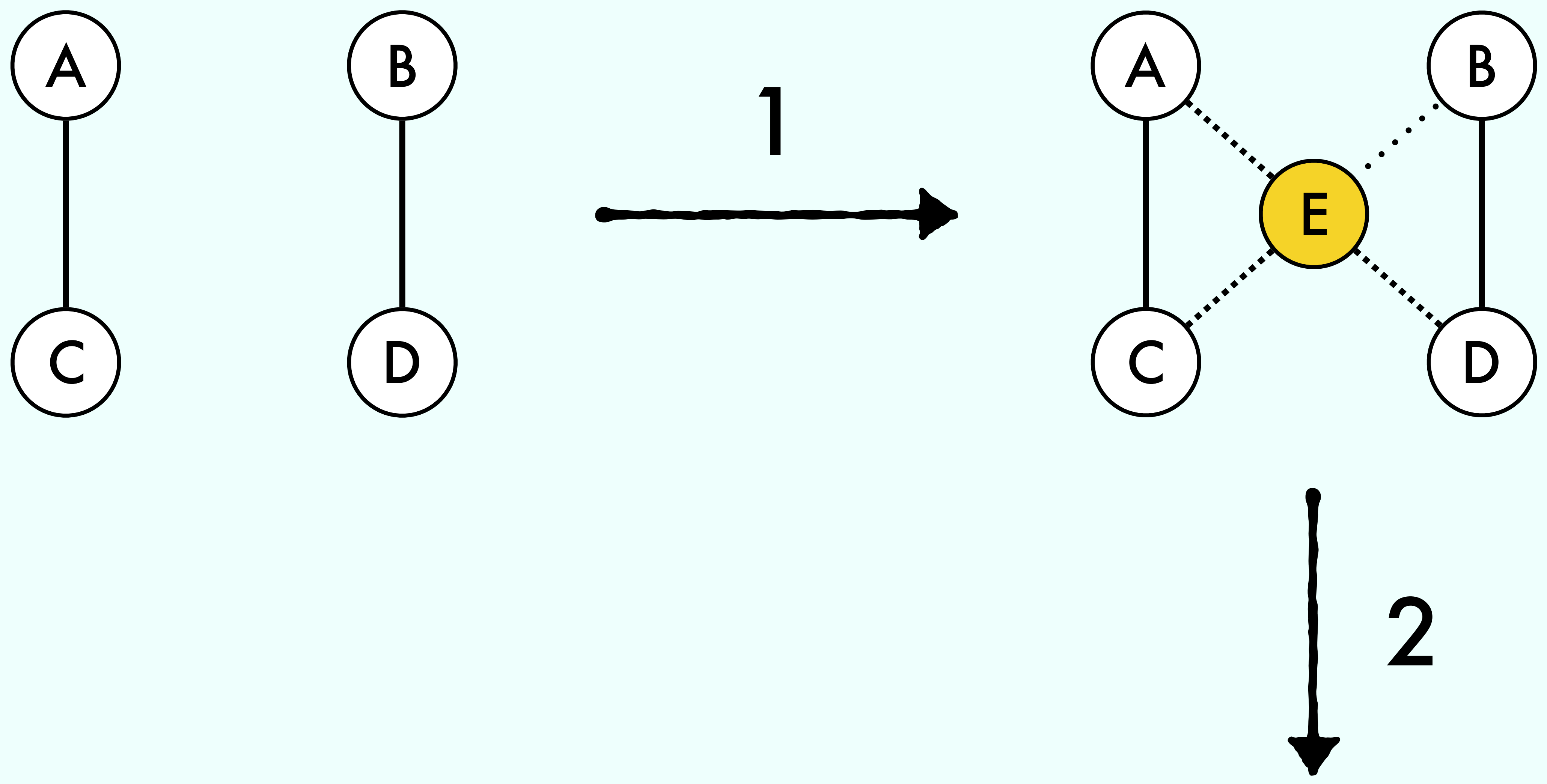
1


A
B
C
D
1
A
B
E
C
D


A
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C
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1
A
B
E
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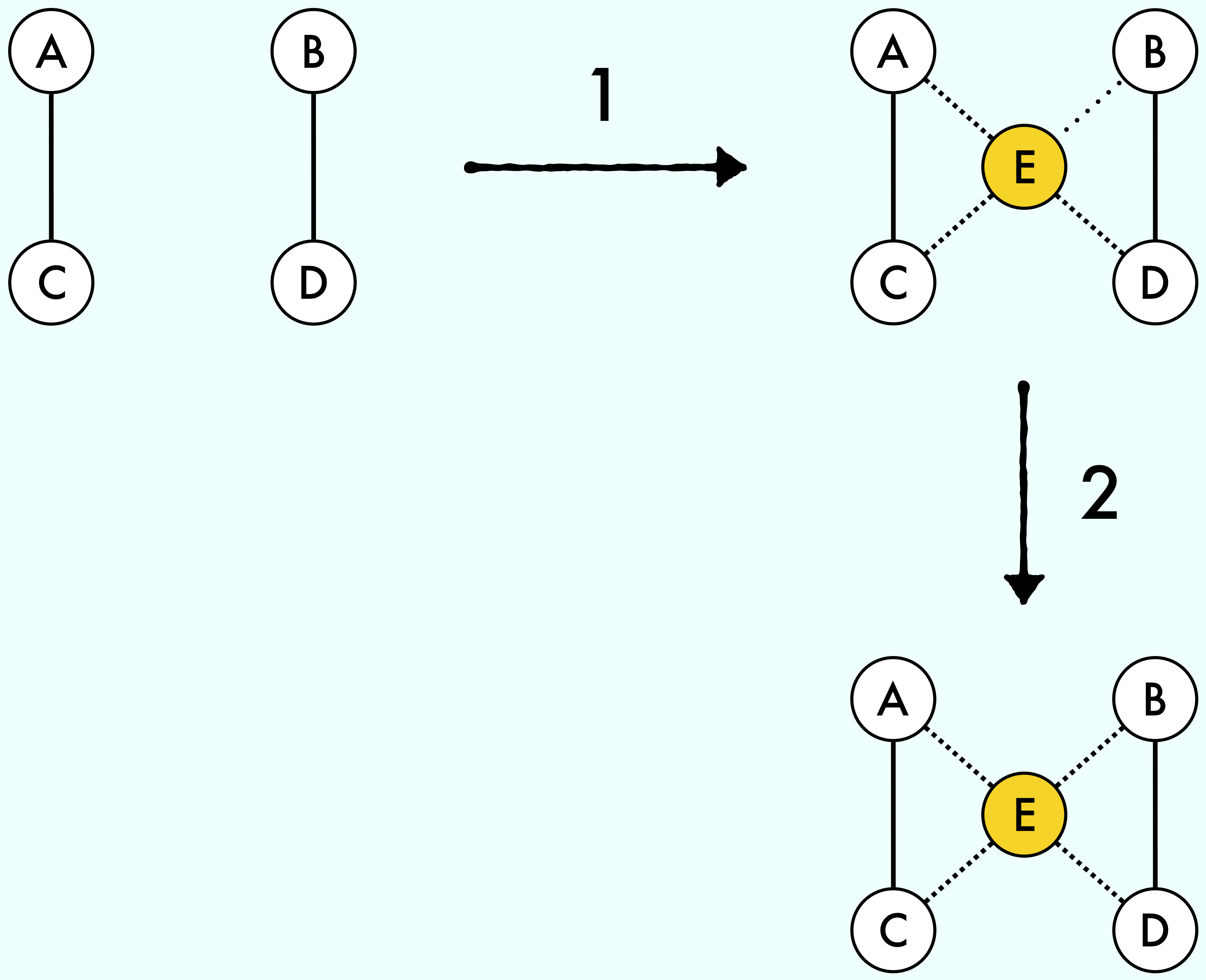
D
2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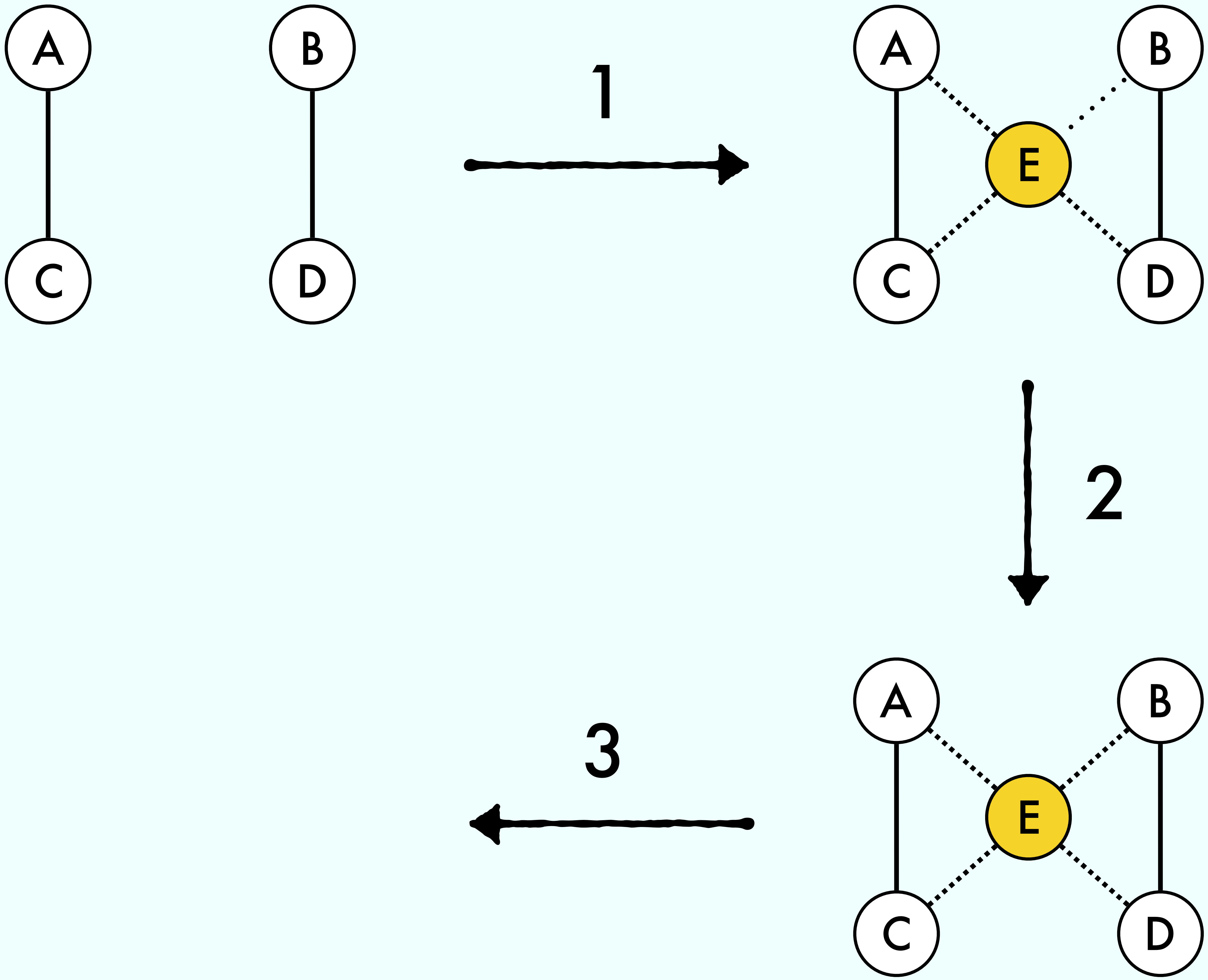
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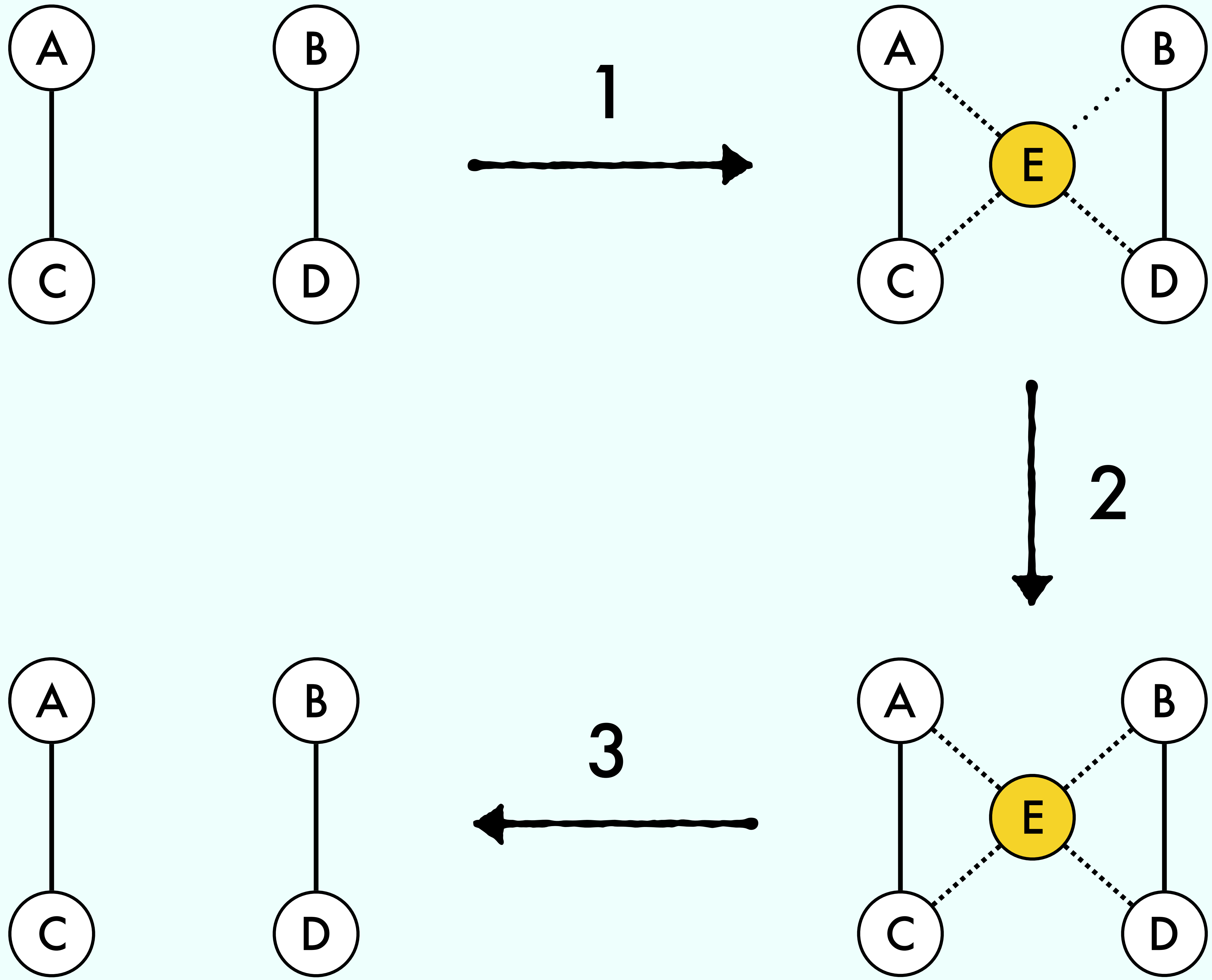
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Theorem

H. Path in G $\Leftrightarrow$ H. Cycle in G*