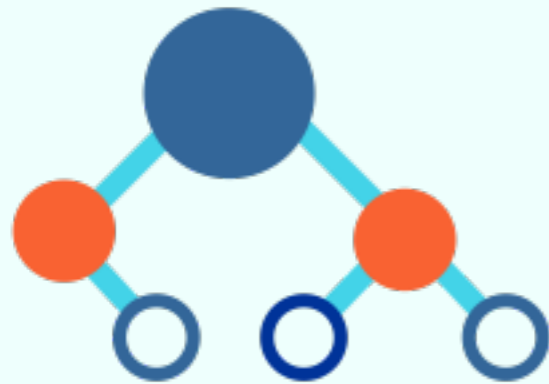


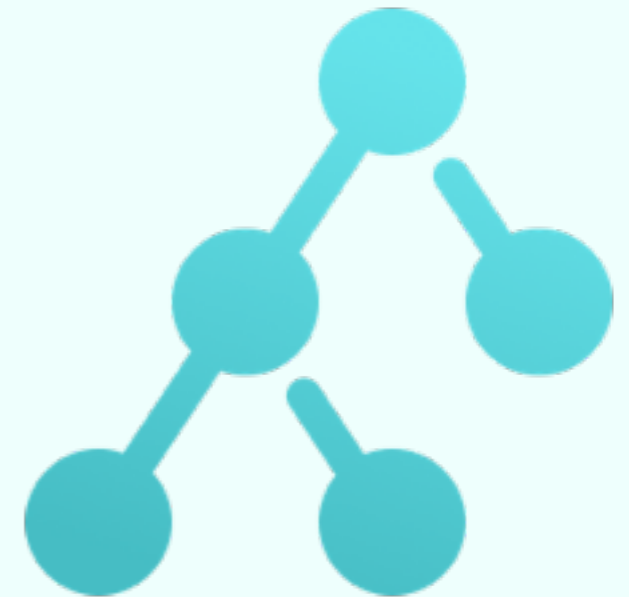


Data Structures

Exercise Session



Marco Vassena



Exercise 1.11 a

Prove:

$$\sum_{i=1}^{N-2} F_i = F_N - 2$$

Exercise 1.11 a

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$$\sum_{i=1}^{N-2} F_i = F_N - 2$$

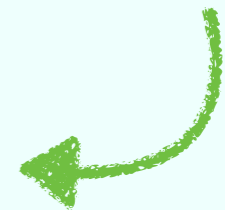
Where:

$$F_0 = 1$$

$$F_1 = 1$$

$$F_{k+1} = F_k + F_{k-1}$$

Fibonacci
number



Exercise 2.1

Order the following functions by growth rate:

$$N$$

$$2^N$$

$$N \log N$$

$$\text{SQRT}(N)$$

$$2^{N/2}$$

$$N \log \log N$$

$$N^{1.5}$$

$$37$$

$$N \log^2 N$$

$$N^2$$

$$N^2 \log N$$

$$N \log(N^2)$$

$$2/N$$

$$N^3$$

Exercise 2.1

Order the following functions by growth rate:

N	2^N	$N \log N$
$\text{SQRT}(N)$	$2^{N/2}$	$N \log \log N$
$N^{1.5}$	37	$N \log^2 N$
N^2	$N^2 \log N$	$N \log(N^2)$
$2/N$	N^3	

Which functions grow at the same rate?

Exercise 2.2

$$T_1(N) = O(f(N))$$

$$T_2(N) = O(f(N))$$

True or False?

A. $T_1(N) + T_2(N) = O(f(N))$

Exercise 2.2

$$T_1(N) = O(f(N))$$

$$T_2(N) = O(f(N))$$

True or False?

A. $T_1(N) + T_2(N) = O(f(N))$

B. $T_1(N) - T_2(N) = o(f(N))$

Exercise 2.2

$$T_1(N) = O(f(N))$$

$$T_2(N) = O(f(N))$$

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A. $T_1(N) + T_2(N) = O(f(N))$

B. $T_1(N) - T_2(N) = o(f(N))$

C. $T_1(N) / T_2(N) = O(1)$

Exercise 2.2

$$T_1(N) = O(f(N))$$

$$T_2(N) = O(f(N))$$

True or False?

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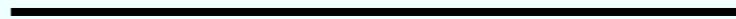
C. $T_1(N) / T_2(N) = O(1)$

D. $T_1(N) = O(T_2(N))$

Exercise 2.10

1 2 3 4 +

9 8 7 6 =



Exercise 2.10

$$\begin{array}{cccccc} 1 & 2 & 3 & 4 & + & \\ 9 & 8 & 7 & 6 & = & \\ \hline \end{array}$$

Exercise 2.10

1 2 3 4 +

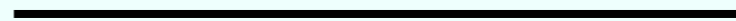
9 8 7 6 =

0

Exercise 2.10

1 2 3 4 +

9 8 7 6 =



0


+1

Exercise 2.10

$$\begin{array}{cccccc} 1 & 2 & 3 & 4 & + & \\ 9 & 8 & 7 & 6 & = & \\ \hline & & & & & 0 \\ & & & & & \curvearrowright \\ & & & & & +1 \end{array}$$

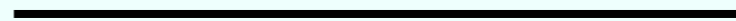
Exercise 2.10

$$\begin{array}{cccccc} 1 & 2 & 3 & 4 & + & \\ 9 & 8 & 7 & 6 & = & \\ \hline & & 1 & 0 & & \\ & & \text{+1} & & & \end{array}$$

Exercise 2.10

1 2 3 4 +

9 8 7 6 =



1 0


+1

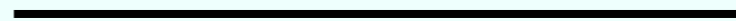
Exercise 2.10

$$\begin{array}{rcccccc} & 1 & 2 & 3 & 4 & + \\ & 9 & 8 & 7 & 6 & = \\ \hline & 1 & 1 & 0 & & \end{array}$$

Exercise 2.10

1 2 3 4 +

9 8 7 6 =



1 1 0


+1

Exercise 2.10

$$\begin{array}{rcccc} 1 & 2 & 3 & 4 & + \\ 9 & 8 & 7 & 6 & = \\ \hline 1 & 1 & 1 & 0 & \\ +1 & & & & \end{array}$$

Exercise 2.10

1 2 3 4 +

9 8 7 6 =

1 1 1 1 0



+1

Exercise 2.10

$$\begin{array}{rcccccc} & 1 & 2 & 3 & 4 & + & \\ & 9 & 8 & 7 & 6 & = & \\ \hline 1 & 1 & 1 & 1 & 0 & & \end{array}$$

Exercise 2.10

Find the running time for adding two N-digit integers:

$$\begin{array}{rcccc} x_N & \dots & x_1 & + \\ y_N & \dots & y_1 & = \\ \hline \end{array}$$

Exercise 2.10

Find the running time for adding two N -digit integers:

$$\begin{array}{rcccc} x_N & \dots & x_1 & + \\ y_N & \dots & y_1 & = \\ \hline z_{N+1} & z_N & \dots & z_1 \end{array}$$

Exercise 2.10

Find the running time for multiplying two N -digit integers:

$x_N \quad \dots \quad x_1 \quad \times$

$y_N \quad \dots \quad y_1 \quad =$

Exercise 2.10

Find the running time for multiplying two N -digit integers:

$$\begin{array}{r} x_N \quad \dots \quad x_1 \quad \times \\ y_N \quad \dots \quad y_1 \quad = \\ \hline z_{2N} \quad \dots \quad z_N \quad \dots \quad z_1 \end{array}$$

Exercise 2.15

Given an integer array A



Such that

$$A_1 < A_2 < \dots < A_{N-1} < A_N$$

Find the index i , such that

$$A_i = i$$