markov

Exercises for HMM lecture 1

Conditional probabilities and Markov chains

1. Consider an occasionally dishonest casino that uses two types of dice. Of the dice 99% are fair but 1% are loaded so that a six comes up 50% of the time. Assume that you pick up a die from a table at random. Calculate the following probabilities:
   a) P(six | die loaded)
   b) P(six , die loaded)
   c) P(six , die fair)
   d) P(six)

2. A Markov chain $\{X_n,\ n = 1, 2, \ldots\}$ has state space {1, 2, 3 } and transition matrix

$$\mathbf{P} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & \frac{3}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

   Given that the Markov chain starts in state 2 at time 1, what is the probability that $X_4 = 3$?

3. An insurance company has estimated that the conditional probability for a driver to be involved in at least one accident next year is 10% given that he/she was not involved in any accident this year, and it is 40% given that he/she was involved in at least one accident this year. Their statistics also show that during the very first year of driving (year 0), 80% of the drivers are not involved in any accidents. Let

$$X_n = \begin{cases} 0 & \text{if no accidents year } n, \\ 1 & \text{if at least one accident year } n, \end{cases}$$

   for $n \geq 1$.
   Assume $\{X_n,\ n = 1, 2, \ldots\}$ is a Markov chain.

   a) Write the transition matrix for $\{X_n,\ n = 1, 2, \ldots\}$. What is the initial distribution of $\{X_n\}$, (that is $P(X_1 = 0)$ and $P(X_1 = 1)$)?
   b) Calculate $P(X_4 = 0 \mid X_1 = 1)$.
   c) Determine the probability distribution of X_2, that is calculate $P(X_2 = 0)$ and $P(X_2 = 1)$.

An example of a simple HMM

4. Read the article *What is a hidden Markov model* by Sean Eddy.

   For the toy HMM presented there, specify the following:

   a) The state space of the Markov chain (also called the state path).
   b) The observable sequence, and its state space.
   c) The set of emission probabilities, and the set of transition probabilities.

Literature for the HMM part of Sequence Bioinformatics

Parts of chapter 2 and 3 in the book **Comparative gene finding**, Springer 2010, written by Marina Axelsson-Fisk.

Chapter 2: Single Species gene finding.
Chapter 3: Sequence alignment.

Available online:

http://www.springer.com/computer/bioinformatics/book/978-1-84996-103-5

lecture 1.

2.1.1
2.1.2, 2.1.3, 2.1.4, 2.1.5, 2.1.6