

# Feasibility test Guarantee bound for liquid tasks: • For time-independent scheduling on a uniprocessor ... ... deadline-monotonic scheduling is optimal for liquid aperiodic tasks in the sense that it maximizes the guarantee bound

monotonic scheduling of periodic tasks (by Liu & Layland)
... the real utilization of admitted tasks can be much higher than
the synthetic guarantee bound, often close to 100% ⇒ using
the (seemingly low) bound will not underutilize the processor

... the guarantee bound is lower than the bound for rate-

(note that feasibility tests for periodic tasks do not exhibit such a property)



#### **Feasibility test**

Synthetic utilization:

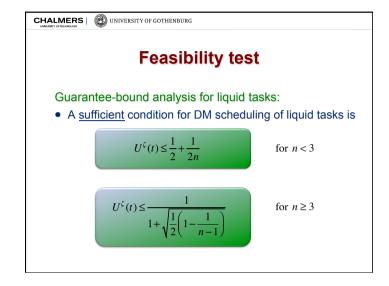
Let  $V^{\zeta}(t)$  be the set of current tasks at time t , that is:

$$V^{\zeta}(t) = \left\{ \tau_i \mid a_i \le t \le a_i + D_i \right\}$$

The <u>synthetic utilization</u>  $U^{\zeta}(t)$  is then defined as:

$$U^{\zeta}(t) = \sum_{\tau_i \in V^{\zeta}(t)} \frac{C_i}{D_i}$$

Note that the synthetic utilization is a function of time!

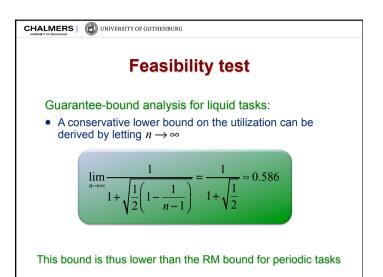


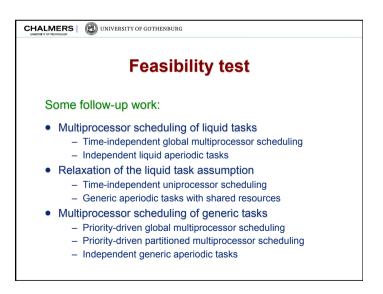
Feasibility test

Synthetic utilization (improved definition):

If the processor becomes idle due to a task completing before its deadline, the synthetic utilization can be immediately reset to zero (since tasks that precede the idle time will have no effect on the schedulability of future task arrivals.)

The synthetic utilization  $U^{\zeta}(t)$  can therefore be redefined as:  $U^{\zeta}(t) = \begin{cases} \sum_{\tau_i \in V^{\zeta}(t)} C_i/D_i & \text{if the processor is busy} \\ 0 & \text{if the processor is idle} \end{cases}$ The average synthetic utilization will thus generally be lower than the average real utilization of admitted tasks





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#### **Feasibility test**

Multiprocessor scheduling of liquid tasks: (Abdelzaher et al, 2002)

- Time-independent global multiprocessor scheduling
- Deadline-monotonic scheduling is also optimal among time-independent multiprocessor scheduling policies
- Synthetic guarantee bound is identical to the uniprocessor case, and is independent of the number of processors
- Synthetic utilization is redefined as: Note that synthetic utilization for multiprocessors is defined

as a per-processor average

$$U^{\zeta}(t) = \begin{cases} \frac{1}{m} \sum_{\tau_i \in V^{\zeta}(t)} C_i / D_i & \text{if all processors are busy} \\ 0 & \text{otherwise} \end{cases}$$

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#### Feasibility test

Relaxation of liquid task assumption: (Abdelzaher & Sharma, 2003)

- Time-independent uniprocessor scheduling
- Presents a generalized synthetic guarantee bound that is a function of parameters that depend on the scheduling policy used:
  - Preemptable deadline ratio
  - Resource blocking ratio
- For deadline-monotonic scheduling, the synthetic guarantee bound reduces to the optimal bound for liquid tasks

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### **Feasibility test**

Relaxation of liquid task assumption:

Let  $D_{max}^k$  be the longest relative deadline among the tasks with priority equal to or higher than task  $\tau_{\iota}$ .

The preemptable deadline ratio  $\alpha$  is then defined as:

$$\alpha = \min_{\forall k} \frac{D_k}{D_{\max}^k}$$

Note that  $\alpha = 1$  for deadline-monotonic scheduling since tasks with higher priorities have shorter deadlines.

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#### Feasibility test

Relaxation of liquid task assumption:

Let  $B_{\iota}$  be the longest critical region being used by tasks with a priority lower than task  $\tau_{\nu}$ , and that calls critical regions with a ceiling priority equal to or higher than the priority of  $\tau_k$  (that is, PCP is assumed).

The resource blocking ratio  $\gamma$  is then defined as:

$$\gamma = \max_{\forall k} \frac{B_k}{D_k}$$

#### Feasibility test

Relaxation of liquid task assumption:

The generalized synthetic guarantee bound  $U_{IB}^{\xi}$  for time-independent uniprocessor scheduling is then defined as:

$$U_{LB}^{\zeta} = 1 + \alpha - \sqrt{1 + \gamma + \alpha^2}$$

A <u>sufficient</u> condition for time-independent uniprocessor scheduling of aperiodic tasks is thus:

$$U^{\zeta}(t) \leq 1 + \alpha - \sqrt{1 + \gamma + \alpha^2}$$

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#### **Feasibility test**

Multiprocessor scheduling of generic tasks: (Andersson et al, 2003)

- Global multiprocessor scheduling with m processors:
  - The EDF-US{m/(2m-1)} scheduling policy is extended to handle aperiodic tasks
  - Added difficulty: the number of "heavy" tasks is not necessarily the same at all times (as is the case with periodic tasks) ⇒ the number of processors available for "light" tasks may vary with time
  - It is shown that the aperiodic EDF-US{m/(2m-1)} has a synthetic quarantee bound of at least m/(2m-1)
  - It is shown that no priority-driven global scheduler can have a synthetic guarantee bound higher than 0.5 + 0.5/m

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#### **Feasibility test**

Relaxation of liquid task assumption:

For deadline-monotonic scheduling ( $\alpha = 1$ ) of independent tasks ( $\gamma = 0$ ), the guarantee bound evaluates to

$$1 + \alpha - \sqrt{1 + \gamma + \alpha^2} = 2 - \sqrt{2} = \frac{2(1 + \sqrt{\frac{1}{2}}) - \sqrt{2}(1 + \sqrt{\frac{1}{2}})}{1 + \sqrt{\frac{1}{2}}} = \frac{1}{1 + \sqrt{\frac{1}{2}}}$$

The bound for DM thus reduces to that of the liquid task case

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### **Feasibility test**

Global multiprocessor scheduling of aperiodic tasks:

- Priority-driven multiprocessor scheduling:
  - For every pair of tasks, one task has higher priority than the other task in the pair, and these relative priority orderings never change (i.e., not pfair scheduling)
- · Synthetic utilization is redefined as:

$$U^{\zeta}(t) = \begin{cases} \frac{1}{m} \sum_{\tau_i \in V^{\zeta}(t)} C_i / D_i & \text{if } \exists \text{ a busy processor} \\ 0 & \text{otherwise} \end{cases}$$

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#### **Feasibility test**

Global multiprocessor scheduling of aperiodic tasks:

 A <u>sufficient</u> condition for global scheduling of aperiodic tasks on m processors using EDF-US{m/(2m-1)} is

$$U^{\zeta}(t) \le \frac{m}{2m-1}$$

EDF-US{m/(2m-1)} thus has close-to-optimal performance (recall that maximum achievable bound is 0.5+0.5/m). For an infinite number of processors, EDF-US{m/(2m-1)} is optimal among priority-driven global schedulers for aperiodic tasks.

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#### **Feasibility test**

Partitioned multiprocessor scheduling of aperiodic tasks:

The <u>synthetic utilization</u>  $U_p^{\zeta}(t)$  on processor p is defined as:

$$U_p^{\zeta}(t) = \sum_{\tau_i \in V_p^{\zeta}(t)} \frac{C_i}{D_i}$$

where

$$V_p^{\zeta}(t) = \left\{ \left. \tau_i \right| a_i \le t \le a_i + D_i \wedge (\tau_i \text{ is assigned to processor } p) \right\}$$

The total synthetic utilization is still  $U^{\zeta}(t) = \frac{1}{m} \sum_{\tau_i \in V^{\zeta}(t)} C_i/D_i$ 

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#### Feasibility test

Multiprocessor scheduling of generic tasks: (Andersson et al, 2003)

- Partitioned multiprocessor scheduling with *m* processors:
  - An EDF-FF scheduling policy is proposed to handle aperiodic tasks
  - Added difficulty: a task "disappears" as soon as its deadline has expired ⇒ "first-fit" bin-packing must be redefined
  - It is shown that the aperiodic EDF-FF scheduler has a synthetic guarantee bound of at least 0.31
  - It is also shown that no analysis of guarantee bounds for the aperiodic EDF-FF scheduler can achieve a guarantee bound higher than  $3/7 \approx 0.428$

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#### **Feasibility test**

Partitioned multiprocessor scheduling of aperiodic tasks:

A processor is said to be <u>occupied</u> at time *t* if there is at least one task that is both current at time *t* and that is assigned to the processor. A processor that is not occupied is called empty.

Let  $transition_p(t)$  be the latest time  $\leq t$  such that processor p makes a transition from being empty to being occupied.

If a processor p has never been occupied, then  $transition_n(t) = -\infty$ 

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#### **Feasibility test**

Partitioned multiprocessor scheduling of aperiodic tasks:

The EDF-FF algorithm:

When a task  $\tau_i$  arrives it is assigned to the occupied processor with the earliest  $transition_n(a_i)$  for which

$$U_p^{\zeta}(t) = \sum_{\tau_k \in V_p^{\zeta}(t) \cup \tau_i} \frac{C_k}{D_k} \le 1$$

If no occupied processor passes the test, the task is assigned to an arbitrary empty processor (if no empty processor exists, EDF-FF declares failure.)

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#### Server-less approach

Forwarding tasks in a multiprocessor system:

- Immediate drop:
  - If an aperiodic task is not centrally schedulable, it is dropped.
- Focused addressing and bidding:
  - If an aperiodic task is not locally schedulable, it is forwarded to a suitable processor candidate based on statistics and bids.
- Load sharing:
  - If an aperiodic task is not locally schedulable, it is forwarded to a suitable processor candidate based on load thresholds and "buddy" processors.

#### Feasibility test

Partitioned multiprocessor scheduling of aperiodic tasks:

 A <u>sufficient</u> condition for partitioned scheduling of aperiodic tasks on *m* processors using EDF-FF is

 $U^{\zeta}(t) \leq 0.31$ 

EDF-FF thus has no tight guarantee bound (it is known from earlier work that maximum achievable bound for partitioned multiprocessor scheduling is 0.5). Recall, however, that no analysis of EDF-FF can achieve a guarantee bound higher than 3/7.

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#### Task forwarding

Focused addressing and bidding: (Ramamritham et al, 1989)

- Each processor maintains a table of currently-guaranteed tasks. It also maintains a table of the <u>surplus</u> computational capacity at every other processor. The surplus capacity is expressed as fractions of a (future) time window of a common size.
- If a processor cannot guarantee an aperiodic task locally, it consults its surplus table and selects the processor (<u>focused processor</u>) that is most likely to successfully schedule the task.



#### Task forwarding

Focused addressing and bidding:

- Because of possible out-of-date entries in the surplus table, the processor might also send out requests-for-bids to other lightly-loaded processors. These bids are then sent to the focused processor.
- The focused processor determines whether to schedule locally or pass the task on to the highest bidder. Tasks that cannot be guaranteed locally, or through focused addressing and bidding, are rejected.

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#### Task forwarding

Load sharing: (Shin & Chang, 1989)

- Each processor has three states of processor loading:
  - Underloaded: the processor is judged to be in a position to accept and execute tasks from other processors.
  - Fully loaded: the processor will neither accept tasks from other processors, nor offload tasks onto other processors.
  - Overloaded: the processor looks for other processors on which to offload some tasks.
- The processor load is derived from the number of task instances awaiting service in the processor's ready queue

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Focused addressing and bidding:

- Important overhead factors that must be taken into account during the bidding process:
  - task deadline and task execution time
  - task transfer time (between processors)
  - time taken by the focused processor to make a decision
  - time taken to respond with a bid (schedulability test)
  - surplus time available at bidder



#### Task forwarding

#### Load sharing:

- To update processor state, the load is compared against a set of load thresholds corresponding to the loading states.
- When a processor makes a transition into and out of the underloaded state, it broadcasts an announcement to its <u>buddy set</u> which is a limited subset of the processors chosen mainly based on the nature of the interconnection network.



- Each processor is aware of whether any members in its buddy set are in the underloaded state. An overloaded processor chooses an underloaded member (if any) in its buddy set on which to offload a task.
- Each processor has an ordered list of preferred processors. Careful design of the lists will reduce the risk of "flooding" underloaded processors.