Software Engineering using Formal Methods

Reasoning about Programs with Dynamic Logic

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Part I

Where are we?

Where Are We?

before specification of JAVA programs with JML

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now dynamic logic (DL) for resoning about JAVA programs

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after that generating DL from JML+JAVA

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before specification of JAVA programs with JML

now dynamic logic (DL) for resoning about JAVA programs

after that generating DL from JML+JAVA

$+$ verifying the resulting proof obligations

Motivation

Consider the method

```
public void doubleContent(int[] a) {
  int i = 0;
  while (i < a.length) {
    a[i] = a[i] * 2;
    i++;
  }
}
```

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  }
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```

We want a logic/calculus allowing to express/prove properties like, e.g.:

> *If* `a` $\neq$ `null`
> *then* `doubleContent` terminates normally
> *and* afterwards all elements of `a` are twice the old value

Motivation (contd.)

One such logic is dynamic logic (DL).

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The above statemet in DL would be:

$$
\begin{array}{l}
\quad\ \ \mathtt{a} \neq \mathtt{null} \\
\quad \land\, \mathtt{a} \neq \mathtt{b} \\
\quad \land\, \forall \mathtt{int}\ \mathtt{i};((0 \leq \mathtt{i} \land \mathtt{i} < \mathtt{a.length}) \rightarrow \mathtt{a}[\mathtt{i}] = \mathtt{b}[\mathtt{i}]) \\
\rightarrow \langle \mathtt{doubleContent(a);} \rangle \\
\quad \forall \mathtt{int}\ \mathtt{i};((0 \leq \mathtt{i} \land \mathtt{i} < \mathtt{a.length}) \rightarrow \mathtt{a}[\mathtt{i}] = 2 * \mathtt{b}[\mathtt{i}])
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- DL combines first-order logic (FOL) with programs

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- Theory of DL extends theory of FOL

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- DL combines first-order logic (FOL) with programs
- Theory of DL extends theory of FOL
- Necessary to look closer at FOL at first
- Then extend towards DL

Today

introducing dynamic logic for JAVA

- recap first-order logic (FOL)
- semantics of FOL
- dynamic logic = extending FOL with
  - dynamic interpretations
  - programs to describe state change

Repetition: First-Order Logic

Signature

A first-order signature Σ consists of

- a set T_Σ of types
- a set F_Σ of function symbols
- a set P_Σ of predicate symbols

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Type Declarations

- $\tau\ x$; 'variable x has type τ'
- $p(\tau_1, \ldots, \tau_r)$; 'predicate p has argument types $\tau_1, \ldots, \tau_r$'
- $\tau\ f(\tau_1, \ldots, \tau_r)$; 'function f has argument types $\tau_1, \ldots, \tau_r$ and result type τ'

Part II

First-Order Semantics

First-Order Semantics

From propositional to first-order semantics

- In prop. logic, an interpretation of variables with $\{T, F\}$ sufficed
- In first-order logic we must assign meaning to:
  - function symbols (incl. constants)
  - predicate symbols
- Respect typing: **int** `i`, `List l` must denote different elements

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- Respect typing: `int i`, `List l` must denote different elements

What we need (to interpret a first-order formula)

1. A collection of typed universes of elements
2. A mapping from variables to elements
3. For each function symbol, a mapping from arguments to results
4. For each predicate symbol, a set of argument tuples where that predicate holds

First-Order Domains/Universes

1. A collection of typed universes of elements

Definition (Universe/Domain)

A non-empty set $\mathcal{D}$ of elements is a universe or domain.
Each element of $\mathcal{D}$ has a fixed type given by $\delta : \mathcal{D} \to T_\Sigma$

- Notation for the domain elements of type $\tau \in T_\Sigma$:
  $\mathcal{D}^\tau = \{d \in \mathcal{D} \mid \delta(d) = \tau\}$
- Each type $\tau \in T_\Sigma$ must 'contain' at least one domain element:
  $\mathcal{D}^\tau \neq \emptyset$

First-Order States

3. For each function symbol, a mapping from arguments to results
4. For each predicate symbol, a set of argument tuples where that predicate holds

Definition (First-Order State)

Let $\mathcal{D}$ be a domain with typing function δ.

For each f be declared as $\tau\ f(\tau_1, \ldots, \tau_r)$;

and each p be declared as $p(\tau_1, \ldots, \tau_r)$;

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Then $\mathcal{S} = (\mathcal{D}, \delta, \mathcal{I})$ is a first-order state

First-Order States Cont'd

Example

Signature: $\textbf{int}$ `i`; $\textbf{int}$ `j`; $\textbf{int}$ `f(`$\textbf{int}$`)`; `Object obj`; `<(`$\textbf{int}$`,`$\textbf{int}$`)`;
$\mathcal{D} = \{17, 2, o\}$

First-Order States Cont'd

Example

Signature: `int i; int j; int f(int); Object obj; <(int,int);`
$\mathcal{D} = \{17,\ 2,\ o\}$
The following $\mathcal{I}$ is a possible interpretation:

$$\mathcal{I}(i) = 17$$
$$\mathcal{I}(j) = 17$$
$$\mathcal{I}(\texttt{obj}) = o$$

$\mathcal{D}^{\mathbf{int}}$	$\mathcal{I}(f)$
2	2
17	2

$\mathcal{D}^{\mathbf{int}} \times \mathcal{D}^{\mathbf{int}}$	in $\mathcal{I}(<)$?
$(2, 2)$	*no*
$(2, 17)$	*yes*
$(17, 2)$	*no*
$(17, 17)$	*no*

One of uncountably many possible first-order states!

Semantics of Reserved Signature Symbols

Definition

Reserved predicate symbol for equality: $=$

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Reserved predicate symbol for equality: $=$

Interpretation is fixed as $\mathcal{I}(=) = \{(d, d) \mid d \in \mathcal{D}\}$

Exercise: write down all elements of the set $\mathcal{I}(=)$ for example domain

Signature Symbols vs. Domain Elements

- Domain elements different from the terms representing them
- First-order formulas and terms have no access to domain

Example

Signature: `Object obj1, obj2;`
Domain: $\mathcal{D} = \{o\}$

Signature Symbols vs. Domain Elements

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Signature: `Object obj1, obj2;`
Domain: $\mathcal{D} = \{o\}$

In this state, necessarily $\mathcal{I}(\texttt{obj1}) = \mathcal{I}(\texttt{obj2}) = o$

Variable Assignments

2. A mapping from variables to domain elements

Definition (Variable Assignment)

A variable assignment β maps variables to domain elements
It respects the variable type, i.e., if x has type τ then $\beta(x) \in \mathcal{D}^\tau$

Semantic Evaluation of Terms

Given a first-order state $\mathcal{S}$ and a variable assignment β
it is possible to evaluate first-order terms under $\mathcal{S}$ and β

Definition (Valuation of Terms)

$val_{\mathcal{S},\beta}$: Term $\rightarrow \mathcal{D}$ such that $val_{\mathcal{S},\beta}(t) \in \mathcal{D}^{\tau}$ for $t \in \text{Term}_{\tau}$:

- $val_{\mathcal{S},\beta}(x) =$

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- $val_{\mathcal{S},\beta}(f(t_1, \ldots, t_r)) = \mathcal{I}(f)(val_{\mathcal{S},\beta}(t_1), \ldots, val_{\mathcal{S},\beta}(t_r))$

Semantic Evaluation of Terms Cont'd

Example

Signature: **int** `i`; **int** `j`; **int** `f(`**int**`)`;

$\mathcal{D} = \{17, 2, o\}$ Variables: `Object obj`; **int** `x`;

$\mathcal{I}(\mathtt{i}) = 17$
$\mathcal{I}(\mathtt{j}) = 17$

$\mathcal{D}^{\mathbf{int}}$	$\mathcal{I}(\mathtt{f})$
2	17
17	2

Var	β
`obj`	o
`x`	17

- $val_{\mathcal{S},\beta}(\mathtt{f(f(i))})$?
- $val_{\mathcal{S},\beta}(\mathtt{f(f(x))})$?
- $val_{\mathcal{S},\beta}(\mathtt{obj})$?

Preparing for Semantic Evaluation of Formulas

Definition (Modified Variable Assignment)

Let y be variable of type τ, β variable assignment, $d \in \mathcal{D}^\tau$:

$$\beta_y^d(x) := \begin{cases} \beta(x) & x \neq y \\ d & x = y \end{cases}$$

Semantic Evaluation of Formulas

Definition (Valuation of Formulas)

$val_{\mathcal{S},\beta}(\phi)$ for $\phi \in For$

- $val_{\mathcal{S},\beta}(p(t_1, \ldots, t_r)) = T \quad$ iff $\quad (val_{\mathcal{S},\beta}(t_1), \ldots, val_{\mathcal{S},\beta}(t_r)) \in \mathcal{I}(p)$

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- $val_{\mathcal{S},\beta}(\phi \land \psi) = T$ iff $val_{\mathcal{S},\beta}(\phi) = T$ and $val_{\mathcal{S},\beta}(\psi) = T$
- ... as in propositional logic

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- $val_{\mathcal{S},\beta}(\forall\, \tau\ x;\ \phi) = T$ iff $val_{\mathcal{S},\beta_x^d}(\phi) = T$ for all $d \in \mathcal{D}^\tau$

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- $val_{\mathcal{S},\beta}(\forall\, \tau\; x;\, \phi) = T \quad$ iff $\quad val_{\mathcal{S},\beta_x^d}(\phi) = T$ for all $d \in \mathcal{D}^\tau$
- $val_{\mathcal{S},\beta}(\exists\, \tau\; x;\, \phi) = T \quad$ iff $\quad val_{\mathcal{S},\beta_x^d}(\phi) = T$ for at least one $d \in \mathcal{D}^\tau$

Semantic Evaluation of Formulas Cont'd

Example

Signature: `int j; int f(int); Object obj; <(int,int);`

$\mathcal{D} = \{17,\ 2,\ o\}$, $\mathcal{D}^{\mathtt{int}} = \{17,\ 2\}$, $\mathcal{D}^{\mathtt{Object}} = \{o\}$

$$\mathcal{I}(j) = 17$$
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$\mathcal{D}^{\mathtt{int}}$	$\mathcal{I}(f)$
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$\mathcal{D}^{\mathtt{int}} \times \mathcal{D}^{\mathtt{int}}$	in $\mathcal{I}(<)$?
$(2, 2)$	F
$(2, 17)$	T
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$(17, 17)$	F

Semantic Evaluation of Formulas Cont'd

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- $val_{\mathcal{S},\beta}(f(j) < j)$?
- $val_{\mathcal{S},\beta}(\exists\, \texttt{int}\ x;\ f(x) = x)$?
- $val_{\mathcal{S},\beta}(\forall\, \texttt{Object}\ o1;\ \forall\, \texttt{Object}\ o2;\ o1 = o2)$?

Semantic Notions

Definition (Satisfiability, Truth, Validity)

$val_{\mathcal{S},\beta}(\phi) = T$			($\mathcal{S}, \beta$ **satisfies** ϕ)
$\mathcal{S} \models \phi$	iff	for all β : $val_{\mathcal{S},\beta}(\phi) = T$	(ϕ is **true** in $\mathcal{S}$)
$\models \phi$	iff	for all $\mathcal{S}$: $\mathcal{S} \models \phi$	(ϕ is **valid**)

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Example

- $f(j) < j$ is true in $\mathcal{S}$
- $\exists\, \mathbf{int}\ x;\ i = x$ is valid
- $\exists\, \mathbf{int}\ x;\ \neg(x = x)$ is not satisfiable

Part III

Towards Dynamic Logic

Type Hierarchy

First, we *refine the type system* of FOL:

Definition (Type Hierarchy)

- T_Σ is set of types
- Given subtype relation '$\sqsubseteq$', with top element '*any*'
- $\tau \sqsubseteq \mathit{any}$ for all $\tau \in T_\Sigma$

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Example (A Minimal Type Hierarchy)

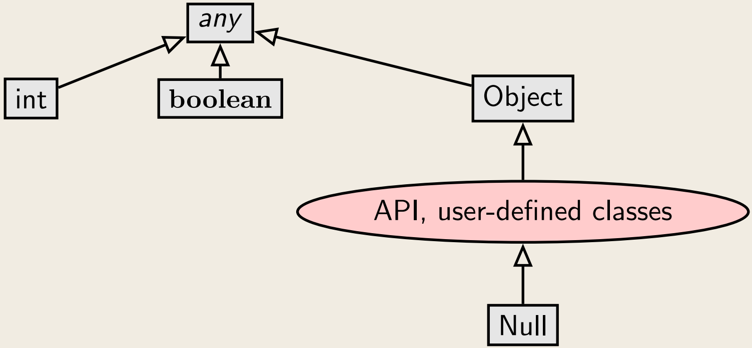
$\mathcal{T} = \{any\}$
All signature symbols have same type *any*.

Example (Type Hierarchy for Java)

(see next slide)

Modelling Java in FOL: Fixing a Type Hierarchy

Signature based on Java's type hierarchy (simplified)



Each class in API and target program is a type, with appropriate subtyping.

Modelling Classes and Fields in FOL

Modeling instance fields

Person
`int age` `int id`
`int setAge(int newAge)` `int getId()`

- domain of all Person objects: $\mathcal{D}^{\text{Person}}$

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- each $o \in \mathcal{D}^{\mathsf{Person}}$ has associated age value
- $\mathcal{I}(\mathsf{age})$ is mapping from $\mathcal{D}^{\mathsf{Person}}$ to $\mathcal{D}^{\mathbf{int}}$

Modelling Classes and Fields in FOL

Modeling instance fields

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Person
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int age
int id
------------------------------
int setAge(int newAge)
int getId()
```

- domain of all Person objects: $\mathcal{D}^{\mathsf{Person}}$
- each $o \in \mathcal{D}^{\mathsf{Person}}$ has associated age value
- $\mathcal{I}$(age) is mapping from $\mathcal{D}^{\mathsf{Person}}$ to $\mathcal{D}^{\mathbf{int}}$
- for each class C with field τ a:

  FSym declares function τ a(C);

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Field Access

Signature FSym: $\texttt{int}$ age(Person); Person p;

Java/JML expression `p.age >= 0`

Typed FOL age(p)>=0

KeY postfix notation for FOL `p.age >= 0`

Navigation expressions in KeY look exactly as in JAVA/JML

Dynamic View

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- Values of fields in a certain range

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Considers only one state at a time.

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- ...

Considers only one state at a time.

Goal: Express functional properties of a program, e.g.

> If method `setAge` is called on an object o of type `Person`
> and the method argument `newAge` is positive
> then afterwards field `age` has same value as `newAge`.

Observation

Need a logic that allows us to

- relate different program states, i.e., before and after execution, within one formula

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Dynamic Logic meets the above requirements.

Dynamic Logic

(JAVA) Dynamic Logic

Typed FOL

- $+$ programs $\mathtt{p}$

Dynamic Logic

(Java) Dynamic Logic

Typed FOL

- $+$ programs $\mathtt{p}$
- $+$ modalities $\langle \mathtt{p} \rangle \phi$, $[\mathtt{p}]\phi$ ($\mathtt{p}$ program, ϕ DL formula)

Dynamic Logic

(JAVA) Dynamic Logic

Typed FOL

- $+$ programs $\mathtt{p}$
- $+$ modalities $\langle \mathtt{p} \rangle \phi$, $[\mathtt{p}]\phi$ ($\mathtt{p}$ program, ϕ DL formula)
- $+$ ... (later)

Dynamic Logic

(JAVA) Dynamic Logic

Typed FOL

- $+$ programs $\texttt{p}$
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- $+$ ... (later)

An Example

$$\texttt{i} > 5 \;\rightarrow\; [\texttt{i = i + 10;}]\texttt{i} > 15$$

Meaning?

Dynamic Logic

(JAVA) Dynamic Logic

Typed FOL

- $+$ programs $\mathtt{p}$
- $+$ modalities $\langle \mathtt{p} \rangle \phi$, $[\mathtt{p}]\phi$ ($\mathtt{p}$ program, ϕ DL formula)
- $+$ ... (later)

An Example

$$\mathtt{i} > 5 \;\rightarrow\; [\mathtt{i = i + 10;}]\mathtt{i} > 15$$

Meaning?

If program variable `i` is greater than 5, then after executing `i = i + 10;`, `i` is greater than 15.

Program Variables

Dynamic Logic = Typed FOL + ...

$$\mathtt{i} > 5 \;\rightarrow\; [\mathtt{i\ =\ i\ +\ 10;}]\mathtt{i} > 15$$

Program variable `i` refers to different values before and after execution of a program.

- Program variables like `i` are state-dependent constant symbols.
- Value of state dependent symbols changeable by program.

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- Program variables like `i` are state-dependent constant symbols.
- Value of state dependent symbols changeable by program.

Three words one meaning: flexible, state-dependent, non-rigid

Rigid versus Flexible Symbols

Signature of dynamic logic defined as in FOL, but:
In addition there are flexible symbols

Rigid versus Flexible

- Rigid symbols, same interpretation in all program states
  - First-order variables (aka logical variables)
  - Built-in functions and predicates such as `0,1,...,+,*,...,<,...`
- Flexible (or non-rigid) symbols, interpretation depends on state

  Capture side effects on state during program execution
  - Functions modeling program variables and fields are flexible

Any term containing at least one flexible symbol is also flexible

Signature of Dynamic Logic

Definition (Dynamic Logic Signature)

$\Sigma = (\mathsf{PSym}_r, \mathsf{FSym}_r, \mathsf{FSym}_f, \alpha), \quad \mathsf{FSym}_r \cap \mathsf{FSym}_f = \emptyset$

Rigid Predicate Symbols	$\mathsf{PSym}_r = \{>, >=, \ldots\}$
Rigid Function Symbols	$\mathsf{FSym}_r = \{+, -, *, 0, 1, \ldots\}$
Flexible Function Symbols	$\mathsf{FSym}_f = \{i, j, k, \ldots\}$

Standard typing: **boolean** `TRUE`; `<(`**int**`,`**int**`)`; etc.

Flexible constant/function symbols FSym_f used to model

- program variables (flexible constants) and
- fields (flexible unary functions)

Dynamic Logic Signature - KeY input file

```
\sorts {
  // only additional sorts (predefined:  int/boolean/any)
}
\functions {
 // only additional rigid functions
 // (arithmetic functions like +,- etc.  predefined)
}
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Empty sections can be left out.

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}
\predicates {  /* same as for functions */  }

\programVariables { // flexible functions
   int i, j;
   boolean b;
}
```

Empty sections can be left out.

Variables

Logical Variables

Typed logical variables (rigid), declared locally in quantifiers as `T x;`

Program Variables

Flexible constants **int** `i;` **boolean** `p;` used as program variables

Dynamic Logic Programs

Dynamic Logic = Typed FOL + programs ...

Programs here: any legal sequence of JAVA statements.

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Example

Signature for FSym_f: int r; int i; int n;

Signature for FSym_r: int 0; int +(int,int); int -(int,int);

Signature for PSym_r: <(int,int);

```
i=0;
r=0;
while (i<n) {
  i=i+1;
  r=r+i;
}
r=r+r-n;
```

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```
i=0;
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```

Which value does the program compute in `r`?

Relating Program States: Modalities

DL extends FOL with two additional (mix-fix) operators:

- $\langle \mathtt{p} \rangle \phi$ (diamond)
- $[p]\phi$ (box)

with $\mathtt{p}$ a program, ϕ another DL formula

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- $\langle \mathtt{p} \rangle \phi$: $\mathtt{p}$ terminates and formula ϕ holds in final state (total correctness)

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Attention: JAVA programs are deterministic, i.e., if a JAVA program terminates then exactly one state is reached from a given initial state.

Dynamic Logic - Examples

Let `i`, `j`, `old_i`, `old_j` denote program variables.
Give the meaning in natural language:

1. $\mathtt{i} = \mathtt{old\_i} \rightarrow \langle\mathtt{i\ =\ i\ +\ 1;}\rangle \mathtt{i} > \mathtt{old\_i}$

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   `p` and `q` are equivalent concerning termination and the final value of `i`.

Dynamic Logic - KeY input file

KeY

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\programVariables {  // Declares global program variables
       int i, j;
       int old_i, old_j;
}
```

KeY

Dynamic Logic - KeY input file

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        i = old_i -> \<{   i = i + 1;  }\> i > old_i
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KeY

Visibility: Program variables declared

- global can be accessed anywhere in the formula.
- inside modality like $pre \rightarrow \langle$`int j; p`$\rangle post$ only visible in `p`

Dynamic Logic Formulas

Definition (Dynamic Logic Formulas (DL Formulas))

- Each FOL formula is a DL formula
- If p is a program and ϕ a DL formula then $\left\{ \begin{matrix} \langle \mathrm{p} \rangle \phi \\ [\mathrm{p}]\phi \end{matrix} \right\}$ is a DL formula
- DL formulas closed under FOL quantifiers and connectives

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- Each FOL formula is a DL formula
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- DL formulas closed under FOL quantifiers and connectives

- Program variables are flexible constants: never bound in quantifiers
- Program variables need not be declared or initialized in program
- Programs contain no logical variables
- Modalities can be arbitrarily nested

Dynamic Logic Formulas Cont'd

Example (Well-formed? If yes, under which signature?)

- $\forall\,\mathbf{int}\ y; ((\langle \texttt{x = 2;}\rangle \texttt{x} = y) \leftrightarrow (\langle \texttt{x = 1; x++;}\rangle \texttt{x} = y))$

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  Well-formed if FSym_f contains $\mathbf{int}\ \texttt{x;}$
  program formulas can be nested

Dynamic Logic Semantics: States

First-order state can be considered as program state

- Interpretation of flexible symbols can vary from state to state (eg, program variables, field values)
- Interpretation of rigid symbols is the same in all states (eg, built-in functions and predicates)

Program states as first-order states

From now, consider program state s as first-order state $(\mathcal{D}, \delta, \mathcal{I})$

- Only interpretation $\mathcal{I}$ of flexible symbols in FSym_f can change
- *States* is set of all states s

Kripke Structure

Definition (Kripke Structure)

Kripke structure or Labelled transition system $K = (States, \rho)$

- State (=first-order model) $s = (\mathcal{D}, \delta, \mathcal{I}) \in States$
- Transition relation $\rho : Program \to (States \rightharpoonup States)$

$$\rho(\mathrm{p})(s1) = s2$$

iff.

program p executed in state $s1$ terminates and its final state is $s2$, otherwise undefined.

- ρ is the semantics of programs $\in Program$
- $\rho(\mathrm{p})(s)$ can be undefined ('$\rightharpoonup$'):
  p may not terminate when started in s
- Our programs are deterministic (unlike PROMELA):
  $\rho(\mathrm{p})$ is a function (at most one value)

Semantic Evaluation of Program Formulas

Definition (Validity Relation for Program Formulas)

- $s \models \langle \mathrm{p} \rangle \phi$ iff $\rho(\mathrm{p})(s)$ is defined and $\rho(\mathrm{p})(s) \models \phi$

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- Duality: $\langle \mathrm{p} \rangle \phi$ iff $\neg[\mathrm{p}]\neg\phi$
  Exercise: justify this with help of semantic definitions
- Implication: if $\langle \mathrm{p} \rangle \phi$ then $[\mathrm{p}]\phi$
  Total correctness implies partial correctness
  - converse is false
  - holds only for deterministic programs

More Examples

valid?
meaning?

Example

$\forall\, \tau\ y;\ (((\langle \mathtt{p} \rangle \mathtt{x} = y)\ \leftrightarrow\ (\langle \mathtt{q} \rangle \mathtt{x} = y))$

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Not valid in general

Programs $\mathtt{p}$ behave $\mathtt{q}$ equivalently on variable τ $\mathtt{x}$

More Examples

valid?
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Example

$\forall\, \tau\ y; ((\langle \mathrm{p} \rangle \mathrm{x} = y) \ \leftrightarrow\ (\langle \mathrm{q} \rangle \mathrm{x} = y))$

Not valid in general

Programs p behave q equivalently on variable τ x

Example

$\exists\, \tau\ y; (\mathrm{x} = y \ \rightarrow\ \langle \mathrm{p} \rangle \mathbf{true})$

More Examples

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Example

$\forall\, \tau\ y; ((\langle \mathrm{p} \rangle \mathrm{x} = y) \leftrightarrow (\langle \mathrm{q} \rangle \mathrm{x} = y))$

Not valid in general

Programs p behave q equivalently on variable τ x

Example

$\exists\, \tau\ y; (\mathrm{x} = y \rightarrow \langle \mathrm{p} \rangle \mathbf{true})$

Not valid in general

Program p terminates if initial value of x is suitably chosen

Semantics of Programs

In labelled transition system $K = (\mathit{States}, \rho)$:
$\rho : \mathit{Program} \rightarrow (\mathit{States} \rightharpoonup \mathit{States})$ is semantics of programs $\mathrm{p} \in \mathit{Program}$

ρ defined recursively on programs

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Example (Semantics of assignment)

States s interpret flexible symbols f with $\mathcal{I}_s(f)$

$\rho(\mathtt{x=t;})(s) = s'$ where s' identical to s except $\mathcal{I}_{s'}(x) = val_s(t)$

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Very tedious task to define ρ for JAVA. $\Rightarrow$ Not in this course.
Next lecture, we go directly to calculus for program formulas!

Literature for this Lecture

KeY Book Verification of Object-Oriented Software (see course web page), Chapter 10: Using KeY

KeY Book Verification of Object-Oriented Software (see course web page), Chapter 3: Dynamic Logic (Sections 3.1, 3.2, 3.4, 3.5, 3.6.1, 3.6.3, 3.6.4)