Parallel Functional Programming
Lecture 8
Data Parallelism I

Mary Sheeran

http://www.cse.chalmers.se/edu/course/pfp

Data parallelism

Introduce parallel data structures and make operations on them parallel

Often data parallel arrays

Canonical example : NESL (NESted-parallel Language) (Blelloch)

NESL

concise (good for specification, prototyping)

allows programming in familiar style (but still gives parallelism)

allows nested parallelism   (see later)

associated language-based cost model

gave decent speedups on wide-vector parallel machines of the day

Hugely influential!

http://www.cs.cmu.edu/~scandal/nesl.html

NESL

Parallelism without concurrency!

Completely deterministic (modulo floating point noise)

No threads, processes, locks, channels, messages, monitors, barriers, or even futures, at source level

Based on Blelloch's thesis work: Vector Models for Data-Parallel Computing, MIT Press 1990

NESL

NESL is a sugared typed lambda calculus with a set of array primitives and an explicit parallel map over arrays

To be useful for analyzing parallel algorithms, NESL was designed with rules for calculating the work (the total number of operations executed) and depth (the longest chain of sequential dependence) of a computation.

Quotes are from ICFP'96 paper

A Provable Time and Space Efficient Implementation of NESL

Guy E. Blelloch and John Greiner
Carnegie Mellon University
{blelloch,jdg}@cs.cmu.edu

Abstract

In this paper we prove time and space bounds for the implementation of the programming language NESL on various parallel machine models. NESL is a sugared typed λ-calculus with a set of array primitives and an explicit parallel map over arrays. Our results extend previous work on provable implementation bounds for functional languages by considering space and by including arrays. For modeling the cost of NESL we augment a standard call-by-value operational semantics to return two cost measures: a DAG representing the sequential dependences in the computation, and a measure of the space taken by a sequential implementation. We show that a NESL program with w work (nodes in the DAG), d depth (levels in the DAG), and s sequential space can be implemented on a p processor butterfly network, hypercube, or CRCW PRAM using $O(w/p + d \log p)$ time and $O(s + dp \log p)$ reachable space.[1] For programs with sufficient parallelism these bounds are optimal in that they give linear speedup and use space within a constant factor of the sequential space.

The idea of a provably efficient implementation is to add to the semantics of the language an accounting of costs, and then to prove a mapping of these costs into running time and/or space of the implementation on concrete machine models (or possibly to costs in other languages). The motivation is to assure that the costs of a program are well defined and to make guarantees about the performance of the implementation. In previous work we have studied provably time efficient parallel implementations of the λ-calculus using both call-by-value [3] and speculative parallelism [18]. These results accounted for work and depth of a computation using a profiling semantics [29, 30] and then related work and depth to running time on various machine models.

This paper applies these ideas to the language NESL and extends the work in two ways. First, it includes sequences (arrays) as a primitive data type and accounts for them in both the cost semantics and the implementation. This is motivated by the fact that arrays cannot be simulated efficiently in the λ-calculus without arrays (the simulation of an array of length n using recursive types requires a $\Omega(\log n)$ slowdown). Second, it augments the profiling semantics with

Quotes

This paper adds the accounting of costs to the semantics of the language and proves a mapping of those costs into running time / space on concrete machine models

A Provable Time and Space Efficient Implementation of NESL

Guy E. Blelloch and John Greiner
Carnegie Mellon University
{blelloch,jdg}@cs.cmu.edu

Abstract

In this paper we prove time and space bounds for the implementation of the programming language NESL on various parallel machine models. NESL is a sugared typed λ-calculus with a set of array primitives and an explicit parallel map over arrays. Our results extend previous work on provable implementation bounds for functional languages by considering space and by including arrays. For modeling the cost of NESL we augment a standard call-by-value operational semantics to return two cost measures: a DAG representing the sequential dependences in the computation, and a measure of the space taken by a sequential implementation. We show that a NESL program with w work (nodes in the DAG), d depth (levels in the DAG), and s sequential space can be implemented on a p processor butterfly network, hypercube, or CRCW PRAM using $O(w/p + d \log p)$ time and $O(s + dp \log p)$ reachable space.[1] For programs with sufficient parallelism these bounds are optimal in that they give linear speedup and use space within a constant factor of the sequential space.

The idea of a provably efficient implementation is to add to the semantics of the language an accounting of costs, and then to prove a mapping of these costs into running time and/or space of the implementation on concrete machine models (or possibly to costs in other languages). The motivation is to assure that the costs of a program are well defined and to make guarantees about the performance of the implementation. In previous work we have studied provably time efficient parallel implementations of the λ-calculus using both call-by-value [3] and speculative parallelism [18]. These results accounted for work and depth of a computation using a profiling semantics [29, 30] and then related work and depth to running time on various machine models.

This paper applies these ideas to the language NESL and extends the work in two ways. First, it includes sequences (arrays) as a primitive data type and accounts for them in both the cost semantics and the implementation. This is motivated by the fact that arrays cannot be simulated efficiently in the λ-calculus without arrays (the simulation of an array of length n using recursive types requires a $\Omega(\log n)$ slowdown). Second, it augments the profiling semantics with

Image: © Thinking Machines Corporation, 1986.
Photo: Steve Grohe.

http://www.venturenavigator.co.uk/content/152

Connection Machine

First commercial massively parallel machine

65k processors

can see CM-1 and CM-5 (from 1993) at Computer History Museum, Mountain View

NESL array operations

```
function factorial(n) =
  if (n <= 1) then 1
  else n*factorial(n-1);

{factorial(i) : i in [3, 1, 7]};
```

apply to each = parallel map (works with user-defined functions
=> load balancing)

list comprehension style notation

Online interpreter ☺

The result of:

```
function factorial(n) =

    if (n <= 1) then 1
    else n*factorial(n-1);


{factorial(i) : i in [3, 1, 7]};
```

is:

```
factorial = fn : int -> int

it = [6, 1, 5040] : [int]

Bye.
```

http://www.cs.cmu.edu/~scandal/nesl/tutorial2.html

apply to each (multiple sequencs)

The result of:

{a + b : a in [3, -4, -9]; b in [1, 2, 3]};

is:

it = [4, -2, -6] : [int]

Bye.

apply to each (multiple sequencs)

The result of:

{a + b : a in [3, -4, -9]; b in [1, 2, 3]};

is:

it = [4, -2, -6] : [int]

Bye.

Qualifiers in comprehensions are zipping rather than nested as in Haskell
Prelude> [a + b | a <- [3,-4,-9], b <- [1,2,3]]
[4,5,6,-3,-2,-1,-8,-7,-6]

Filtering too

The result of:

```
{a * a : a in [3, -4, -9, 5] | a > 0};
```

is:

```
it = [9, 25] : [int]

Bye
```

scan (Haskell first)

```
*Main> scanl1 (+) [1..10]
[1,3,6,10,15,21,28,36,45,55]

*Main> scanl1 (*) [1..10]
[1,2,6,24,120,720,5040,40320,362880,3628800]
```

1 2 3 4 5 6 7 8 9 10

1 2 3 4 5 6 7 8 9 10

3

1 2 3 4 5 6 7 8 9 10

3

6

1 2 3 4 5 6 7 8 9 10

3

6

10

1 2 3 4 5 6 7 8 9 10

3

6

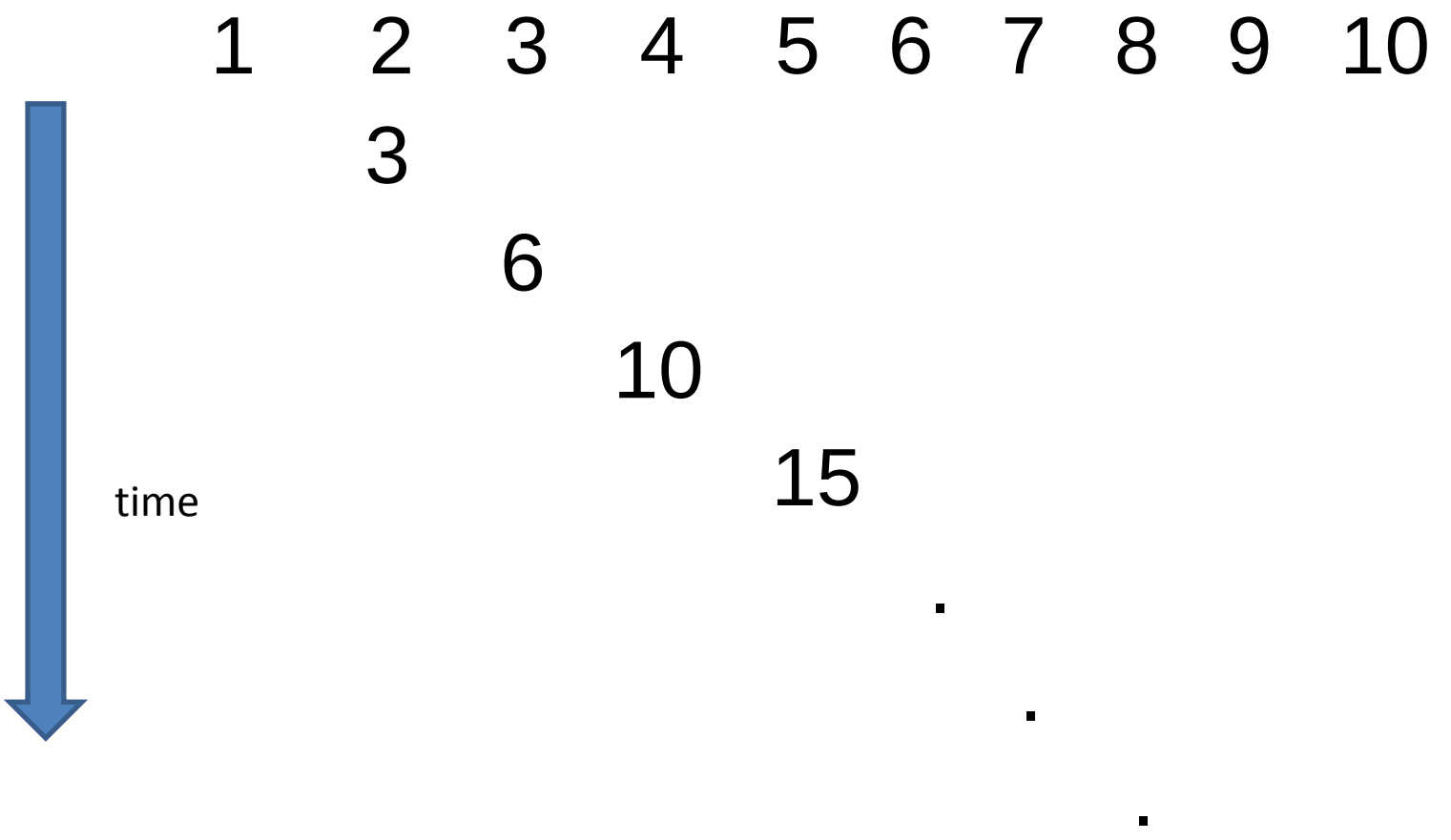
10

15

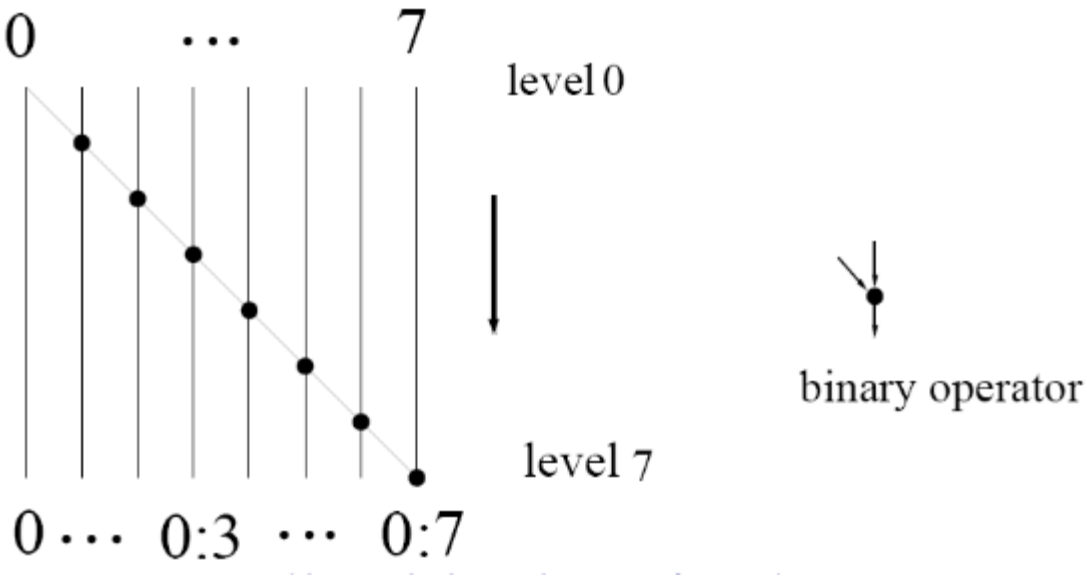
.

.

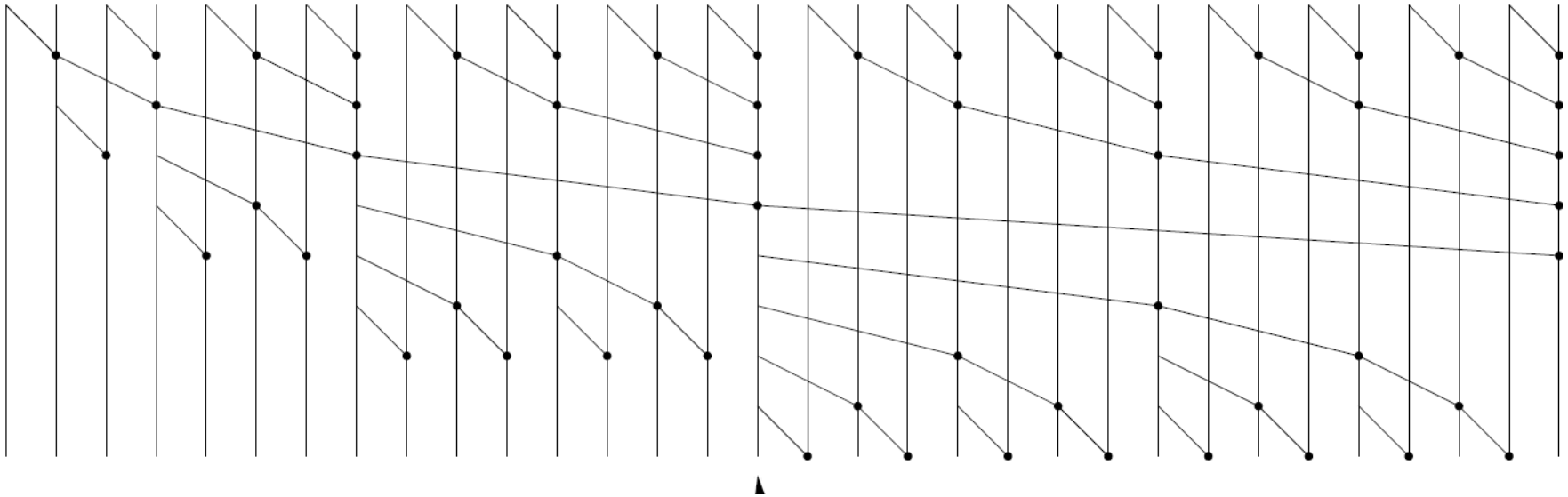
.


1 2 3 4 5 6 7 8 9 10
3
6
10
15
time

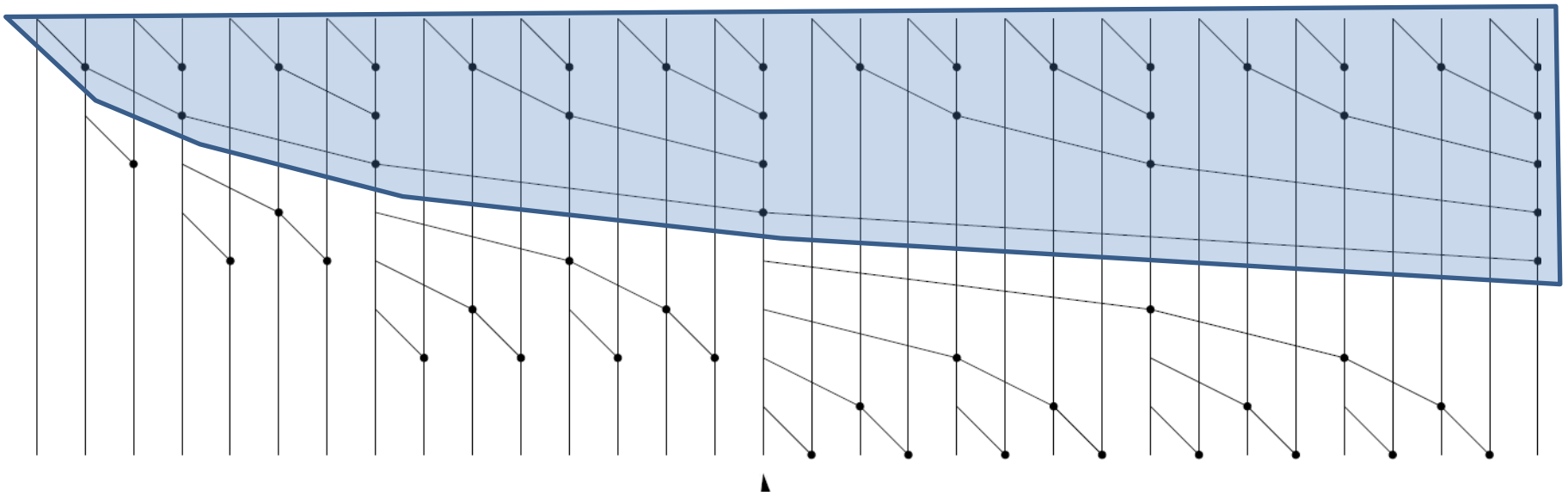
scan diagram



Brent Kung (’79)

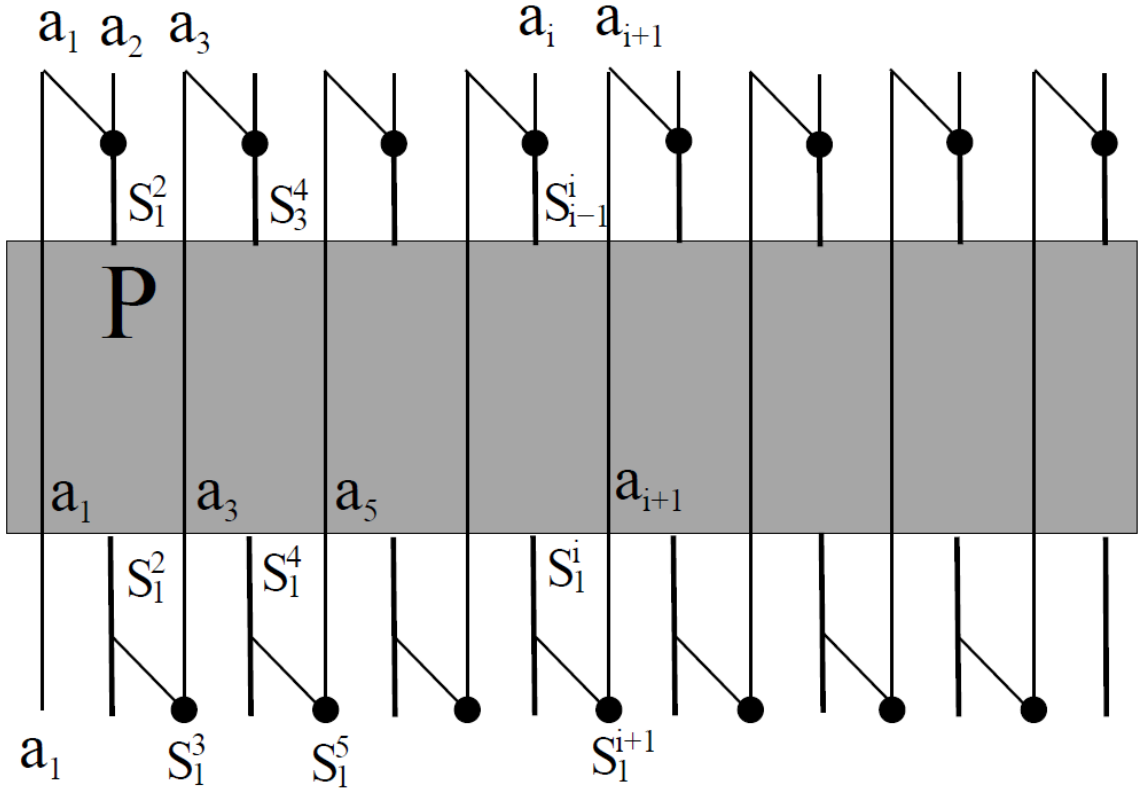


Brent Kung



forward tree + several reverse trees

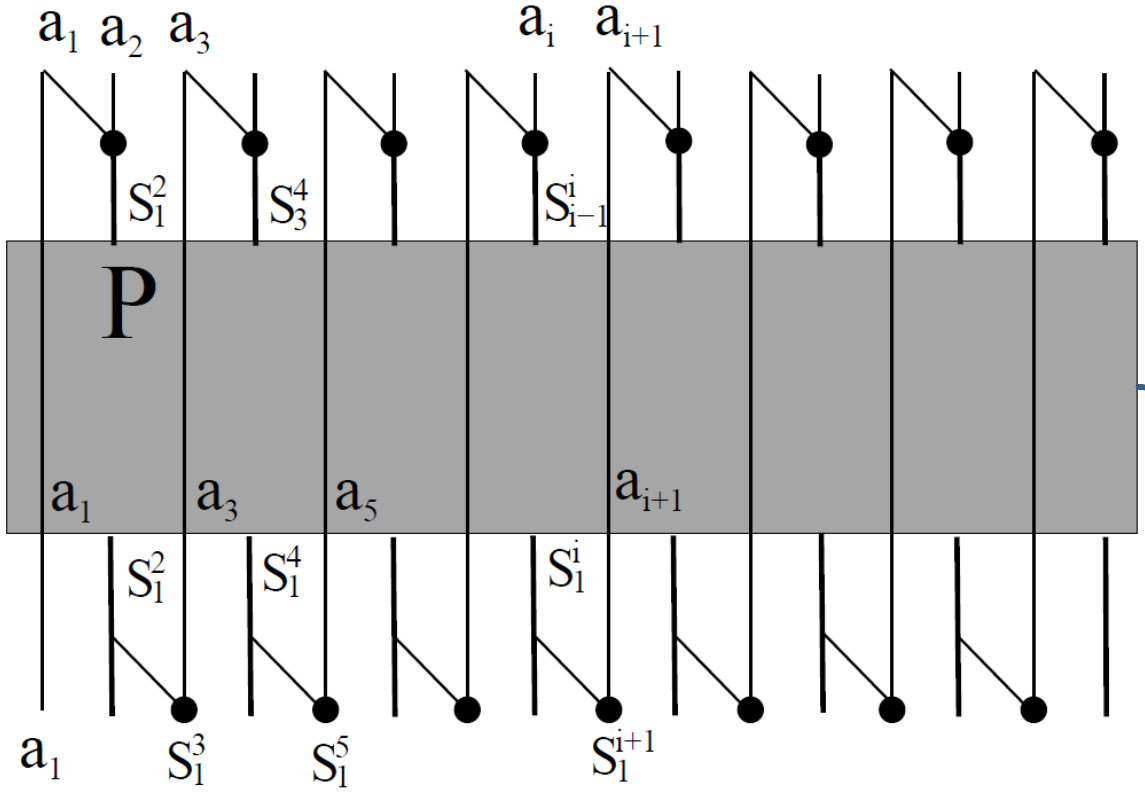
recursive decomposition



$S_i^j = a_i * a_{i+1} * \ldots * a_j$

indices from 1 here

recursive decomposition



$S_i^j = a_i * a_{i+1} * \ldots * a_j$

one recursive call on n/2 inputs

divide
conquer
combine

prescan

scan ”shifted right by one”

prescan of

[a_1 , a_2, ,a_3, ,a_4, . . . , a_n]

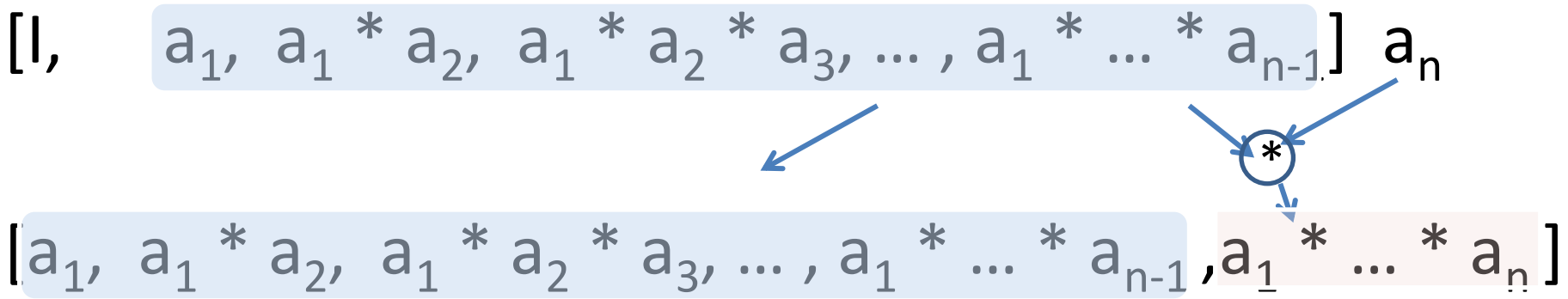
is

[I, a_1, $a_1 * a_2$, $a_1 * a_2 * a_3$, … , $a_1 * … * a_{n-1}$]

identity element

scan from prescan

easy (constant time)



the power of scan

Blelloch pointed out that once you have scan
you can do LOTS of interesting algorithms, inc.

To lexically compare strings of characters. For example, to determine that "strategy" should appear before "stratification" in a dictionary

To evaluate polynomials

To solve recurrences. For example, to solve the recurrences

$$x_i = a_i x_{i-1} + b_i x_{i-2} \text{ and } x_i = a_i + b_i / x_{i-1}$$

To implement radix sort

To implement quicksort

To solve tridiagonal linear systems

To delete marked elements from an array

To dynamically allocate processors

To perform lexical analysis. For example, to parse a program into tokens

and many more

http://www.cs.cmu.edu/~blelloch/papers/Ble93.pdf

prescan in NESL

```
function scan_op(op,identity,a) =
if #a == 1 then [identity]
else
  let e = even_elts(a);
      o = odd_elts(a);
      s = scan_op(op,identity,{op(e,o): e in e; o in o})
  in interleave(s,{op(s,e): s in s; e in e});
```

prescan in NESL

```
function scan_op(op,identity,a) =
if #a == 1 then [identity]
else
  let e = even_elts(a);
      o = odd_elts(a);
      s = scan_op(op,identity,{op(e,o): e in e; o in o})
  in interleave(s,{op(s,e): s in s; e in e});
```

zipWith op e o
zipWith op s e

prescan

```
function scan_op(op,identity,a) =
if #a == 1 then [identity]
else
  let e = even_elts(a);
      o = odd_elts(a);
      s = scan_op(op,identity,{op(e,o): e in e; o in o})
  in interleave(s,{op(s,e): s in s; e in e});
```

```
scan_op('+, 0, [2, 8, 3, -4, 1, 9, -2, 7]);

is:

scan_op = fn : ((b, b) -> b, b, [b]) -> [b] :: (a in any; b in any)

it = [0, 2, 10, 13, 9, 10, 19, 17] : [int]
```

prescan

```
function scan_op(op,identity,a) =
if #a == 1 then [identity]
else
  let e = even_elts(a);
      o = odd_elts(a);
      s = scan_op(op,identity,{op(e,o): e in e; o in o})
  in interleave(s,{op(s,e): s in s; e in e});
```

```
scan_op(max, 0, [2, 8, 3, -4, 1, 9, -2, 7]);

is:

scan_op = fn : ((b, b) -> b, b, [b]) -> [b] :: (a in any; b in any)


it = [0, 2, 8, 8, 8, 8, 9, 9] : [int]
```

Exercise

Try to write scan (as distinct from prescan)

Call it oscan (as scan is built in (gives both prescan list and the final element))

Note that apply-to-each on two sequences demands that the two sequences have equal length (unlike zipWith)

Assume first that the sequence has length a power of two

Type your answer into one of the boxes on
http://www.cs.cmu.edu/~scandal/nesl/tutorial2.html

Outline of one possible solution

```
function init is = take(is,#is-1);

function tail is = drop(is,1);

function oscan(op,v) =
  if #v == 1 then v
  else let es = even_elts(v);
           os = odd_elts(v);
           is = oscan(…);
           us = …
  in interleave … ;
```

Outline of one possible solution

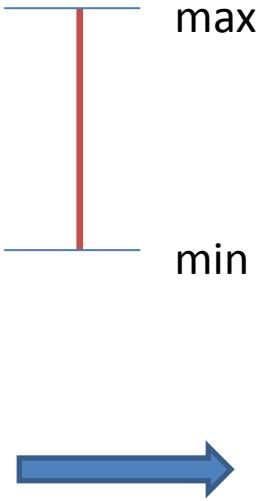
```
function init is = take(is,#is-1);

function tail is = drop(is,1);

function oscan(op,v) =
  if #v == 1 then v
  else let es = even_elts(v);
           os = odd_elts(v);
           is = os
           us = …
  in interleave …
```

```
interleave([1,2,3],[4,5,6]);

it = [1, 4, 2, 5, 3, 6] : [int]

interleave([1,2,3],[4,5]);

it = [1, 4, 2, 5, 3] : [int]


interleave([1,2,3],[4]);

RUNTIME ERROR: Length mismatch for function INTERLEAVE.
```

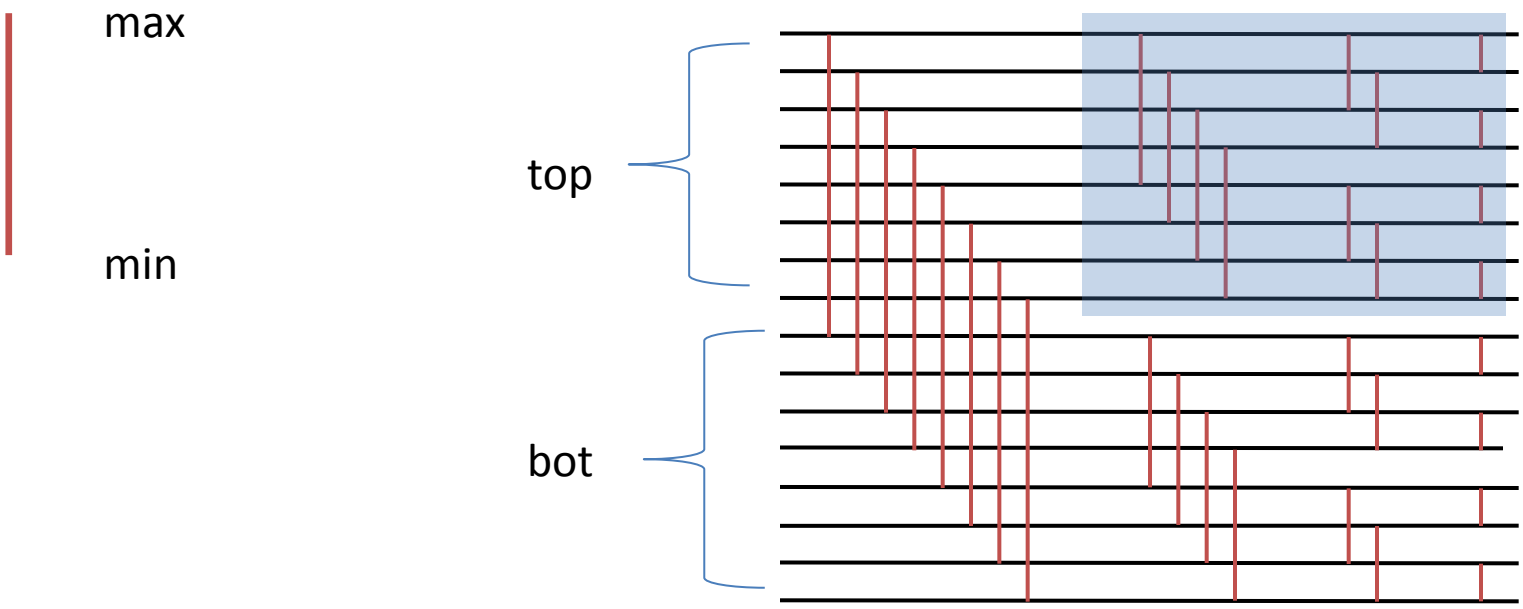
Batcher’s bitonic merge

```
function bitonic_sort(a) =
if (#a == 1) then a
else
    let
            bot = subseq(a,0,#a/2);
            top = subseq(a,#a/2,#a);
            mins = {min(bot,top):bot;top};
            maxs = {max(bot,top):bot;top};
    in flatten({bitonic_sort(x) : x in [mins,maxs]});
```

bitonic_sort (merger)



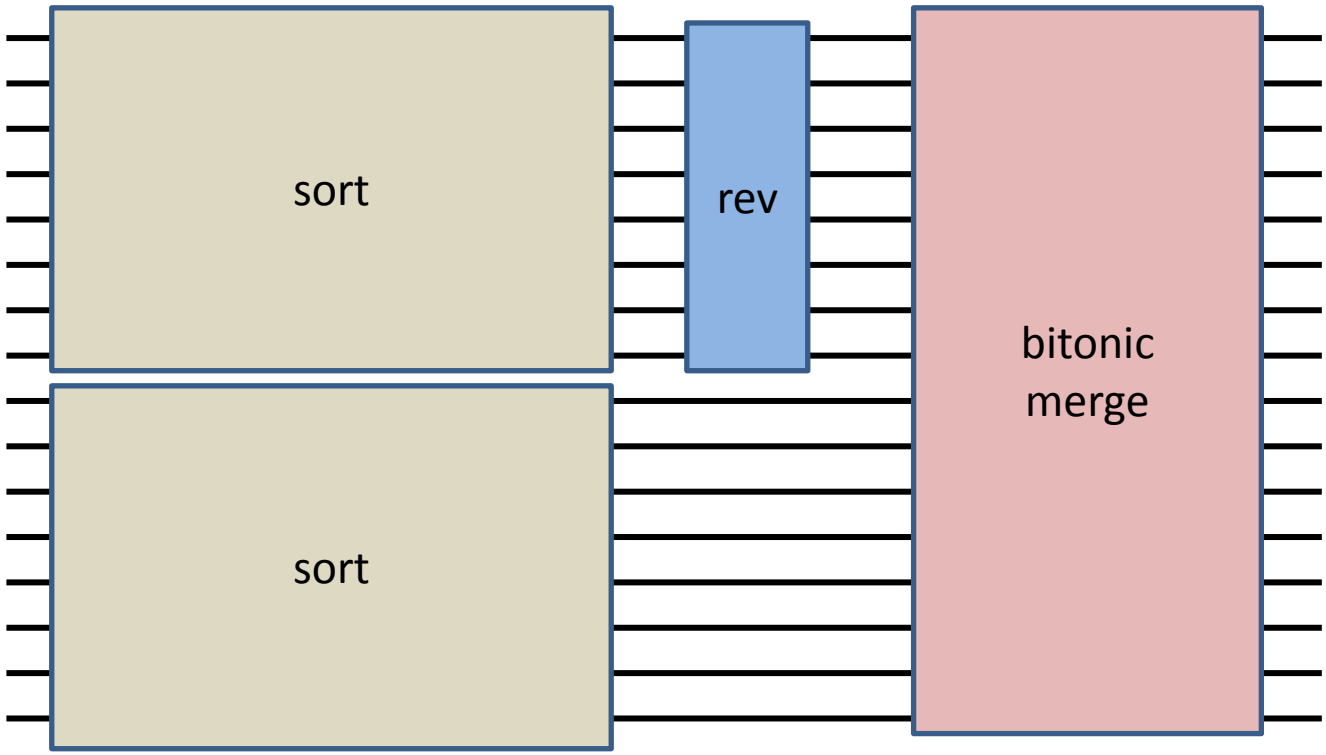
bitonic_sort (merger)



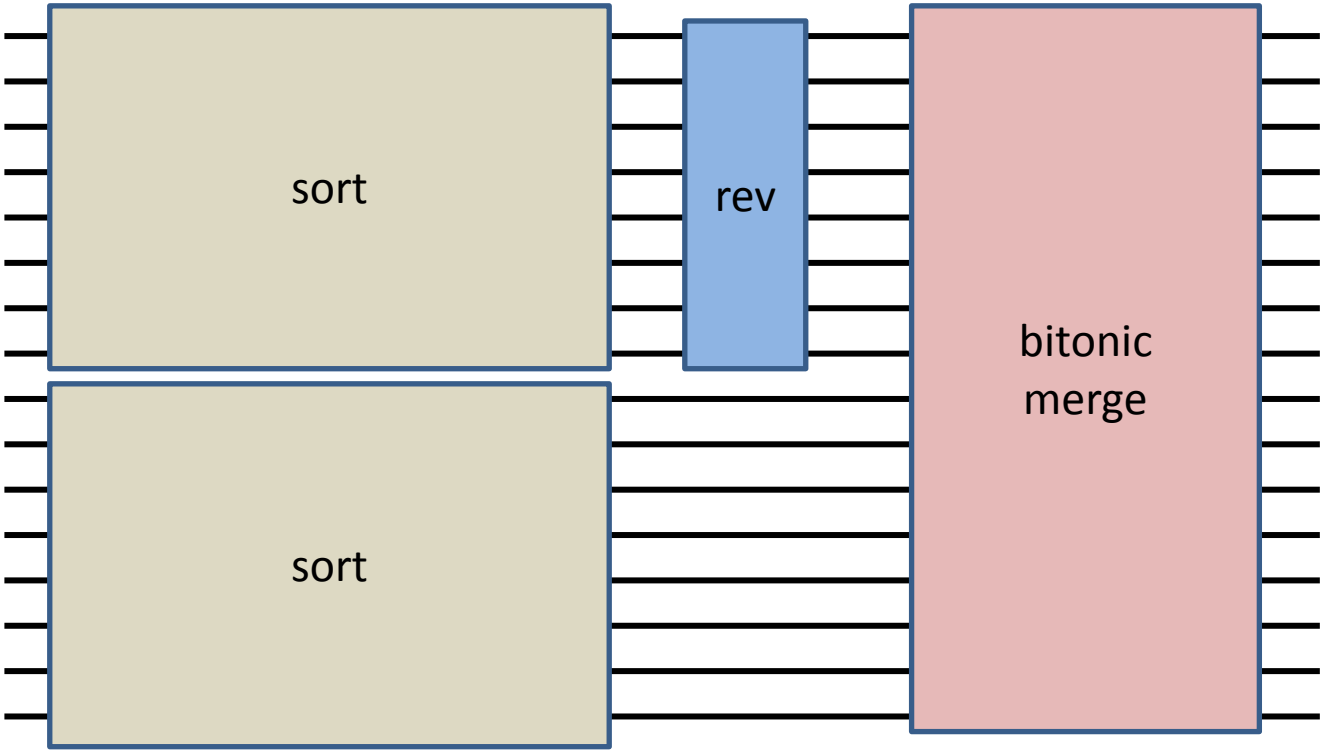
bitonic sort

```
function batcher_sort(a) =
if (#a == 1) then a
else
    let b = {batcher_sort(x) : x in bottop(a)};
    in bitonic_sort(b[0]++reverse(b[1]));
```

bitonic sort



bitonic sort



For some fun with sorting networks in Obsidian, see
http://www.cse.chalmers.se/~joels/writing/expressive.pdf

parentheses matching

For each index, return the index of the matching parenthesis

```
function parentheses_match(string) =
let
    depth = plus_scan({if c==`( then 1 else -1 : c in string});
    depth = {d + (if c==`( then 1 else 0): c in string; d in depth};
    rnk = permute([0:#string], rank(depth));
    ret = interleave(odd_elts(rnk), even_elts(rnk))
in permute(ret, rnk);
```

one scan, a map, a zipWith, two permutes and an interleave,
also rank and odd_elts and even_elts

parentheses matching

For each index, return the in

```
permute([7,8,9],[2,1,0]);
permute([7,8,9],[1,2,0]);

it = [9, 8, 7] : [int]

it = [9, 7, 8] : [int]
```

```
function parentheses_match(
let
   depth = plus_scan({if c==`( then 1 else -1 : c in string});
   depth = {d + (if c==`( then 1 else 0): c in string; d in depth};
   rnk = permute([0:#string], rank(depth));
   ret = interleave(odd_elts(rnk), even_elts(rnk))
in permute(ret, rnk);
```

one scan, a map, a zipWith, two permutes and an interleave, also rank and odd_elts and even_elts

parentheses matching

For each index, return the in

```
rank([6,8,9,7]);

it = [0, 2, 3, 1] : [int]

rank([6,8,9,7,9]);

it = [0, 2, 3, 1, 4] : [int]
```

```
function parentheses_match(
let
   depth = plus_scan({if c==`(
   depth = {d + (if c==`( then 1 e
   rnk = permute([0:#string], rank(depth));
   ret = interleave(odd_elts(rnk), even_elts(rnk))
in permute(ret, rnk);
```

one scan, a map, a zipWith, two permutes and an interleave, also rank and odd_elts and even_elts

parentheses matching

For each index, return the index of the matching pa

A ”step through” of this function is provided at end of these slides

```
function parentheses_match(string) =
let
   depth = plus_scan({if c==`( then 1 else -1 : c in string});
   depth = {d + (if c==`( then 1 else 0): c in string; d in depth};
   rnk = permute([0:#string], rank(depth));
   ret = interleave(odd_elts(rnk), even_elts(rnk))
in permute(ret, rnk);
```

one scan, a map, a zipWith, two permutes and an interleave, also rank and odd_elts and even_elts

Break?

What does Nested mean??

```
{plus_scan(a) : a in [[2,3], [8,3,9], [7]]};
```

```
it = [[0, 2], [0, 8, 11], [0]] : [[int]]
```

What does Nested mean??

sequence of sequences
apply to each of a PARALLEL
function

```
{plus_scan(a) : a in [[2,3], [8,3,9], [7]]};
```

```
it = [[0, 2], [0, 8, 11], [0]] : [[int]]
```

What does Nested mean??

sequence of sequences
apply to each of a PARALLEL
function

```
{plus_scan(a) : a in [[2,3], [8,3,9], [7]]};
```

```
it = [[0, 2], [0, 8, 11], [0]] : [[int]]
```

Implemented using Blelloch's Flattening Transformation, which converts nested parallelism into flat. Brilliant idea, challenging to make work in fancier languages (see DPH and good work on Manticore (ML))

What does Nested mean?? Another example

```
function svxv (sv, v) =
sum ({x * v[i] : (x, i) in sv});
```

```
function smxv (sm, v) =
{ svxv(row, v) : row in sm }
```

Nested parallelism

Arbitrarily nested parallel loops + fork-join

Assumes no synchronization among parallel
tasks except at join points => a task can only sync with its parent (sometimes called fully strict)

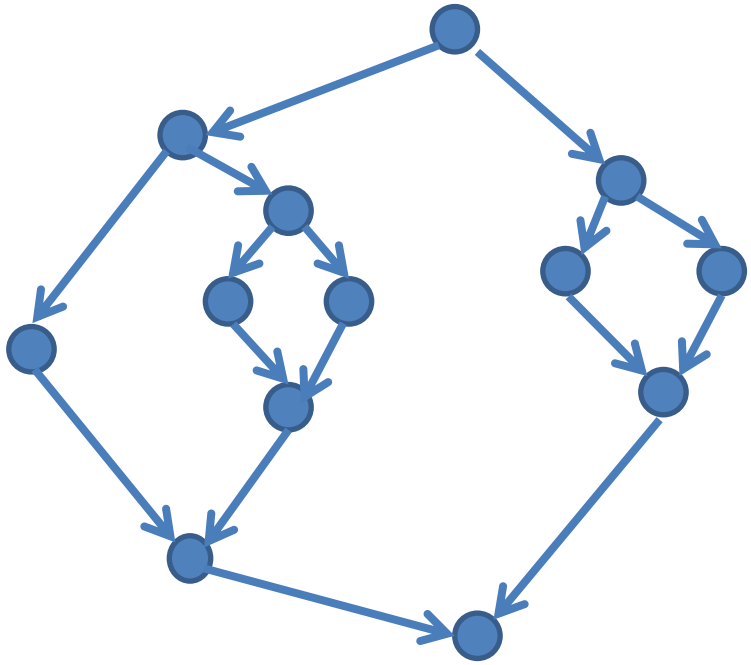
Deterministic (in absence of race conditions)

Advantages (according to Blelloch):

- Good schedulers are known
- Easy to understand, debug, and analyze

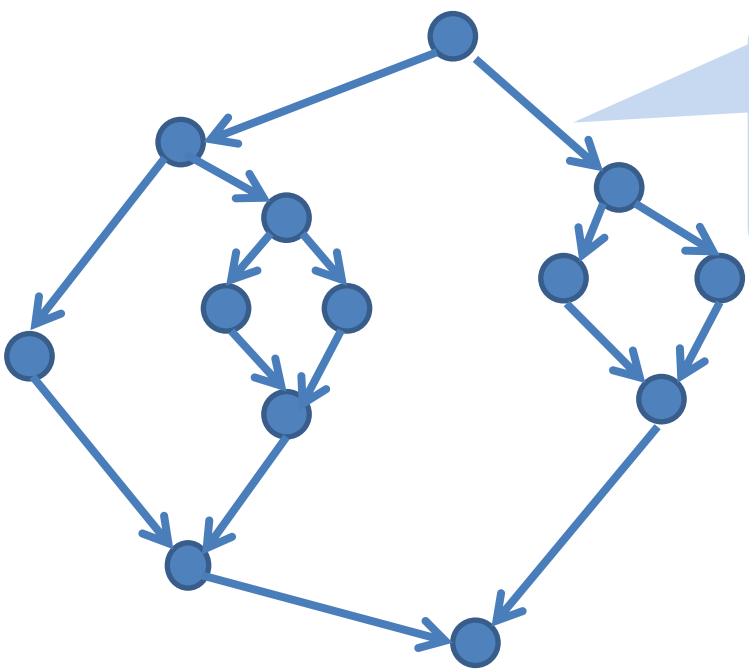
Nested Parallelism

Dependence graph is series-parallel



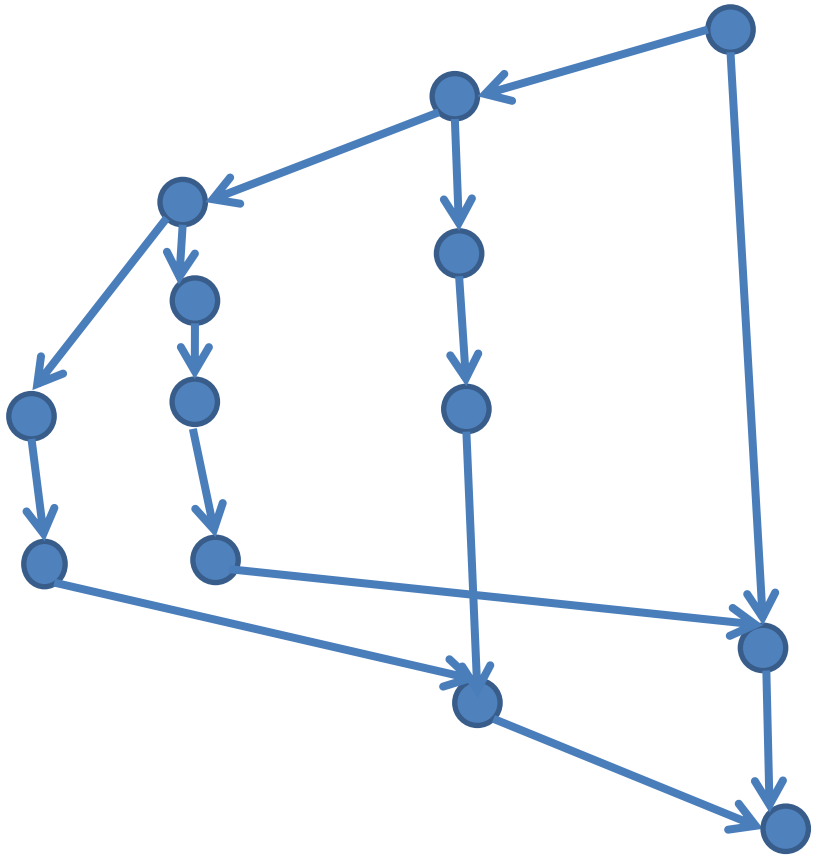
Nested Parallelism

Dependence graph is series-parallel

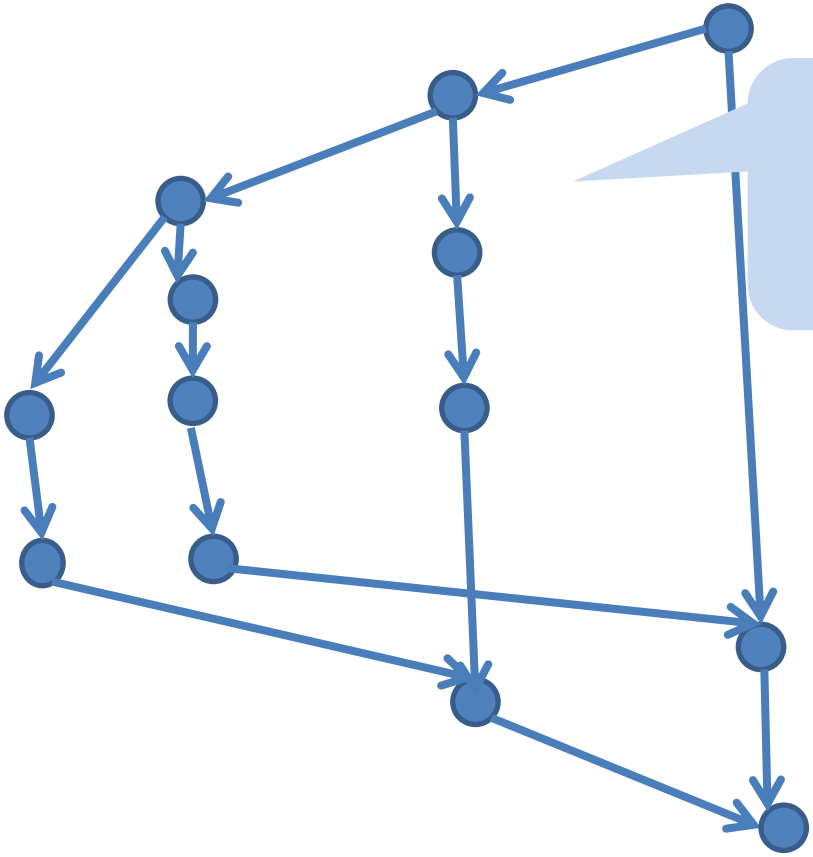


Task can only synchronise with its parent

But not



But not



Here, a task can only synchronise with an ancestor (strict (but not fully strict))

Back to examples

this prescan is actually flat

```
function scan_op(op,identity,a) =
if #a == 1 then [identity]
else
  let e = even_elts(a);
      o = odd_elts(a);
      s = scan_op(op,identity,{op(e,o): e in e; o in o})
  in interleave(s,{op(s,e): s in s; e in e});
```

Back to examples
Batcher's bitonic merge IS NESTED

```
function bitonic_sort(a) =
if (#a == 1) then a
else
    let
            bot = subseq(a,0,#a/2);
            top = subseq(a,#a/2,#a);
            mins = {min(bot,top):bot;top};
            maxs = {max(bot,top):bot;top};
    in flatten({bitonic_sort(x) : x in [mins,maxs]});
```

and so is the sort

Back to examples
Batcher's bitonic merge IS NESTED

nestedness is good for D&C
and for irregular computations

```
function bitonic_sort(a) =
if (#a == 1) then a
else
    let
            bot = subseq(a,0,#a/2);
            top = subseq(a,#a/2,#a);
            mins = {min(bot,top):bot;top};
            maxs = {max(bot,top):bot;top};
    in flatten({bitonic_sort(x) : x in [mins,maxs]});
```

and so is the sort

Back to examples
parentheses matching is FLAT

For each index, return the index of the matching parenthesis

```
function parentheses_match(string) =
let
    depth = plus_scan({if c==`( then 1 else -1 : c in string});
    depth = {d + (if c==`( then 1 else 0): c in string; d in depth};
    rnk = permute([0:#string], rank(depth));
    ret = interleave(odd_elts(rnk), even_elts(rnk))
in permute(ret, rnk);
```

What about a cost model?

Blelloch empasises

1) work : total number of operations

   represents total cost (integral of needed resources over time = running time on one processor)

2) depth or span: longest chain of sequential (functional) dependencies

   best possible running time on an unlimited number of processors

claims:

1) easier to think about algorithms based on work and depth than to use running time on machine with P processors (e.g. PRAM)
2) work and depth predict running time on various different machines

   (at least in the abstract)

What about a cost model?

Blelloch empasises

1) work : total number of operations

e = running time on

cost model is language based rather than machine based

2) de dependencies

best possible running time on an unlim of processors

claims:

1) easier to think about algorithms based on work and depth than to use running time on machine with P processors (e.g. PRAM)
2) work and depth predict running time on various different machines
   (at least in the abstract)

Part 1: simple language based performance model

Call-by-value λ-calculus

$$\lambda x.\, e \Downarrow \lambda x.\, e \qquad \text{(LAM)}$$

$$\frac{e_1 \Downarrow \lambda x.\, e \quad e_2 \Downarrow v \quad e[v/x] \Downarrow v'}{e_1\, e_2 \Downarrow v'} \qquad \text{(APP)}$$

slide from Blelloch's ICFP10 invited talk

The Parallel λ-calculus: cost model

$$e \Downarrow v; w, d$$

Reads: expression *e* evaluates to *v* with work *w* and span *d*.

- **Work** (W): sequential work
- **Span** (D): parallel depth

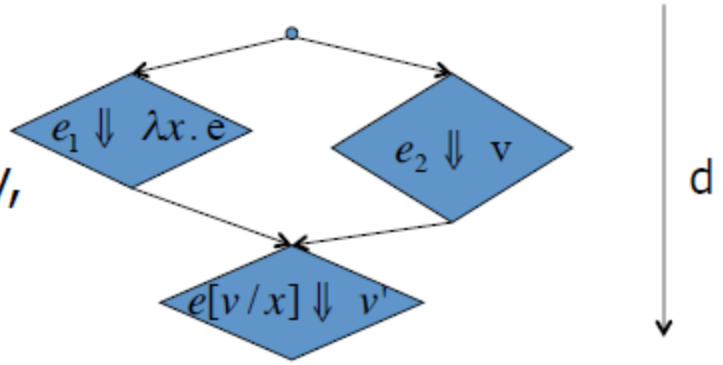
slide from Blelloch's ICFP10 invited talk

The Parallel λ-calculus: cost model

$$\lambda x.\,\mathrm{e} \Downarrow \lambda x.\,\mathrm{e};\ 1,1 \qquad \text{(LAM)}$$

$$\frac{e_1 \Downarrow \lambda x.\,e;\ w_1, d_1 \quad e_2 \Downarrow v;\ w_2, d_2 \quad e[v/x] \Downarrow v';\ w_3, d_3}{e_1\ e_2 \Downarrow v';\ 1 + w_1 + w_2 + w_3,\ 1 + \max(d_1, d_2) + d_3} \qquad \text{(APP)}$$

Work adds
Span adds sequentially,
and max in parallel



slide from Blelloch's ICFP10 invited talk

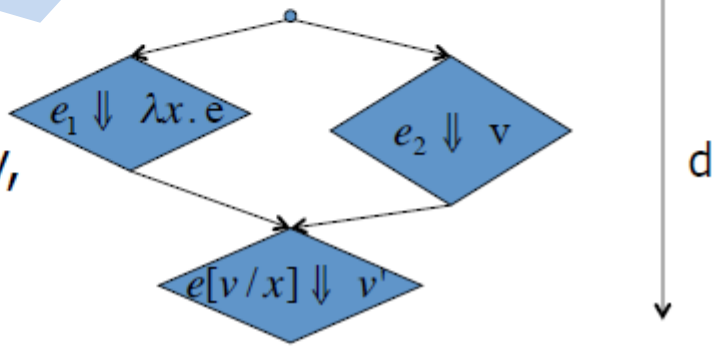
The Parallel λ-calculus: cost model

$\lambda x.e \Downarrow \lambda x.e$ (LAM)

$$\frac{e_1 \Downarrow \quad}{e_1}$$

Oops. The colour is wrong. Should be pink (not blue)

Work adds
Span adds sequentially,
and max in parallel



slide from Blelloch's ICFP10 invited talk

The Parallel λ-calculus cost model

$$\lambda x.\,\mathrm{e} \Downarrow \lambda x.\,\mathrm{e};\ 1,1 \qquad \text{(LAM)}$$

$$\frac{e_1 \Downarrow \lambda x.\ e;\ w_1,d_1 \quad e_2 \Downarrow v;\ w_2,d_2 \quad e[v/x] \Downarrow v';\ w_3,d_3}{e_1\ e_2 \Downarrow v';\ 1+w_1+w_2+w_3,\ 1+\max(d_1,d_2)+d_3} \qquad \text{(APP)}$$

$$c \Downarrow c;\ 1,1 \qquad \text{(CONST)}$$

$$\frac{e_1 \Downarrow c;\ w_1,d_1 \quad e_2 \Downarrow v;\ w_2,d_2 \quad \delta(c,v) \Downarrow v'}{e_1\ e_2 \Downarrow v';\ 1+w_1+w_2,\ 1+\max(d_1,d_2)} \qquad \text{(APPC)}$$

$$c_n = 0,\cdots,n,+,+_0,\cdots,+_n,<,<_0,\cdots,<_n,\times,\times_0,\cdots,\times_n,\cdots \qquad \text{(constants)}$$

slide from Blelloch's ICFP10 invited talk

Adding Functional Arrays: NESL

$$\{e_1 : x \text{ in } e_2 \mid e_3\}$$

$$\frac{e'[v_i/x] \Downarrow v_i';\ w_i, d_i \quad i \in \{1 \ldots n\}}{\{e' : x \text{ in } [v_1 \ldots v_n]\} \Downarrow [v_1' \ldots v_n'];\ 1 + \sum_{i=1}^{n} w_i,\ 1 + \max_{i=1}^{|v|} d_i}$$

Primitives:

```
<- : 'a seq * (int,'a) seq -> 'a seq
```

```
• [g,c,a,p] <- [(0,d),(2,f),(0,i)]
     [i,c,f,p]
```

[ICFP95]

```
elt, index, length
```

slide from Blelloch's ICFP10 invited talk

Adding Functional Arrays: NESL

$$\{e_1 : x \text{ in } e_2 \mid e_3\}$$

$$e'[v_i / x] \Downarrow v_i'; w_i, d_i \quad i \in \{1 \ldots n\}$$

Brings us back to the paper at the beginning (which is from ICFP 96)

- `[g,c,a,p] <- [(0,d),(2,f),(`
  `[i,c,f,p]`

`elt, index, length`

[ICFP95]

slide from Blelloch's ICFP10 invited talk

Adding Functional Arrays: NESL

$$\{e_1 : x \text{ in } e_2 \mid e_3\}$$

Blelloch:
programming based cost models could change the way people think about costs and open door for other kinds of abstract costs
doing it in terms of machines.... "that's so last century"

```
<- : 'a seq * (int,'a) seq -> 'a seq
• [g,c,a,p] <- [(0,d),(2,f),(0,i)]
     [i,c,f,p]

elt, index, length
```

[ICFP95]

slide from Blelloch's ICFP10 invited talk

Aside: Research needed!

Blelloch has pointed the way

ICFP96

Spoonhower et al (JFP'11)
https://www.cs.cmu.edu/afs/cs.cmu.edu/Web/People/blelloch/papers/SBHG11.pdf

Cache and IO efficient functional algs (with Harper, POPL 13)
http://www.cs.cmu.edu/~rwh/papers/iolambda/short.pdf

Cost models that programmers can really use need to be developed

Back to ICFP96 paper

The Second Half:
Provable Implementation Bounds

Theorem [FPCA95]:If $e \Downarrow v; w, d$ then v can be calculated from e on a CREW PRAM with p processors in $O\left(\frac{w}{p} + d \log p\right)$ time.

Can't really do better than: $\max\left(\frac{w}{p}, d\right)$

If w/p > d log p then "work dominates"

We refer to w/p as the parallelism.

slide from Blelloch's ICFP10 invited talk

Ba

Brent's Lemma: might expect O(w/p + d)
log p term is because of how flattening is done (flatten the arrays and use segmented scans)
So scan is essential in the implementation too, not just a tool for the programmer

Provable I ... tion Bounds

Theorem [FPCA95]: ... v; w,d then v can be calculated from e ... a CREW PRAM with p processors in $O\left(\frac{w}{p} + d\log p\right)$ time.

Can't really do better than: $\max\left(\frac{w}{p}, d\right)$

If w/p > d log p then "work dominates"

We refer to w/p as the parallelism.

slide from Blelloch's ICFP10 invited talk

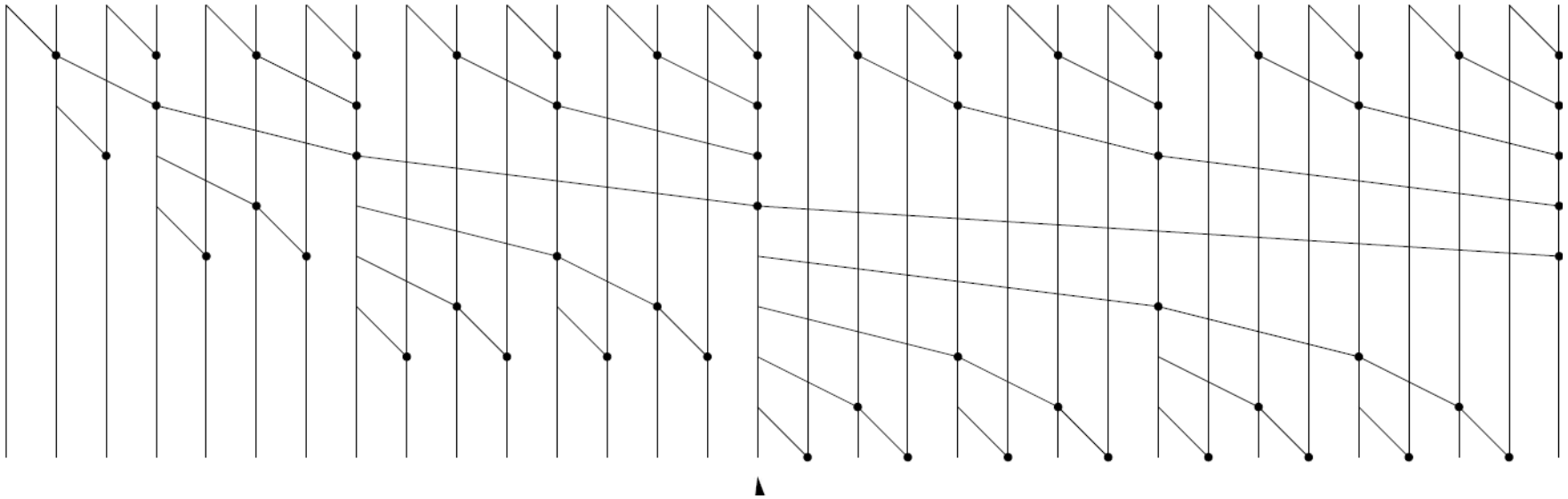
Background info: Brent's lemma

If a computation can be performed in *t* steps with *q* operations on a parallel computer (formally, a PRAM) with an unbounded number of processors, then the computation can be performed in *t + (q-t)/p* steps with *p* processors

http://maths-people.anu.edu.au/~brent/pd/rpb022.pdf

(paper from '74)

Back to our scan



oblivious or data independent computation

$N = 2^n$ inputs, work of dot is 1
work = ?
depth = ?

and bitonic sort? (see Blelloch's ICFP10 invited talk for quicksort etc.)

From the NESL quick reference

Basic Sequence Functions

Basic Operations	Description	Work	Depth
#a	Length of a	O(1)	O(1)
a[i]	ith element of a	O(1)	O(1)
dist(a,n)	Create sequence of length n with a in each element.	O(n)	O(1)
zip(a,b)	Elementwise zip two sequences together into a sequence of pairs.	O(n)	O(1)
[s:e]	Create sequence of integers from s to e (not inclusive of e)	O(e-s)	O(1)
[s:e:d]	Same as [s:e] but with a stride d.	O((e-s)/d)	O(1)

Scans

plus_scan(a)	Execute a scan on a using the + operator	O(n)	O(log n)
min_scan(a)	Execute a scan on a using the minimum operator	O(n)	O(log n)
max_scan(a)	Execute a scan on a using the maximum operator	O(n)	O(log n)
or_scan(a)	Execute a scan on a using the or operator	O(n)	O(log n)
and_scan(a)	Execute a scan on a using the and operator	O(n)	O(log n)

Data Parallel Haskell (DPH) intentions

NESL was a seminal breakthrough but, fifteen years later it remains largely un-exploited. Our goal is to adopt the key insights of NESL, embody them in a modern, widely-used functional programming language, namely Haskell, and implement them in a state-of-the-art Haskell compiler (GHC). The resulting system, Data Parallel Haskell, will make nested data parallelism available to real users.

Doing so is not straightforward. NESL a first-order language, has very few data types, was focused entirely on nested data parallelism, and its implementation is an interpreter. Haskell is a higher-order language with an extremely rich type system; it already includes several other sorts of parallel execution; and its implementation is a compiler.

http://www.cse.unsw.edu.au/~chak/papers/fsttcs2008.pdf

PREPRINT: To be presented at ICFP '12, September 9–15, 2012, Copenhagen, Denmark.

Nested Data-Parallelism on the GPU

Lars Bergstrom
University of Chicago
larsberg@cs.uchicago.edu

John Reppy
University of Chicago
jhr@cs.uchicago.edu

Abstract

Graphics processing units (GPUs) provide both memory bandwidth and arithmetic performance far greater than that available on CPUs but, because of their *Single-Instruction-Multiple-Data* (SIMD) architecture, they are hard to program. Most of the programs ported to GPUs thus far use traditional data-level parallelism, performing only operations that operate uniformly over vectors.

NESL is a first-order functional language that was designed to allow programmers to write irregular-parallel programs — such as parallel divide-and-conquer algorithms — for wide-vector parallel computers. This paper presents our port of the NESL implementation to work on GPUs and provides empirical evidence that nested data-parallelism (NDP) on GPUs significantly outperforms CPU-based implementations and matches or beats newer GPU languages that support only flat parallelism. While our performance does not match that of hand-tuned CUDA programs, we argue that the notational conciseness of NESL is worth the loss in performance. This work provides the first language implementation that directly supports NDP on a GPU.

uniform problem subdivisions and non-uniform memory access, such as divide-and-conquer algorithms.

Most GPU programming is done with the CUDA [NVI11b] and OpenCL [Khr11] languages, which provide the illusion of C-style general-purpose programming, but which actually impose restrictions. There have been a number of efforts to support GPU programming from higher-level languages, usually by embedding a data-parallel DSL into the host language, but these efforts have been limited to regular parallelism [CBS11, MM10, CKL+11].

The current best practice for irregular parallelism on a GPU is for skilled programmers to laboriously hand code applications. The literature is rife with implementations of specific irregular-parallel algorithms for GPUs [BP11, DR11, MLBP12, MGG12]. These efforts typically require many programmer-months of effort to even meet the performance of the original optimized sequential C program.

GPUs have some common characteristics with the wide-vector supercomputers of the 1980's, which similarly provided high-performance SIMD computations. NESL is a first-order functional language developed by Guy Blelloch in the early 1990's that was

NESL on GPU!

First NDP language on GPU

PREPRINT: To be presented at ICFP '12, September 9–15, 2012, Copenhagen, Denmark.

Nested Data-Parallelism on the GPU

Lars Bergstrom
University of Chicago
larsberg@cs.uchicago.edu

John Reppy
University of Chicago
jhr@cs.uchicago.edu

Abstract

Graphics processing units (GPUs) provide both memory bandwidth and arithmetic performance far greater than that available on CPUs but, because of their *Single-Instruction-Multiple-Data* (SIMD) architecture, they are hard to program. Most of the programs ported to GPUs thus far use traditional data-level parallelism, performing only operations that operate uniformly over vectors.

NESL is a first-order functional language that was designed to allow programmers to write irregular-parallel programs — such as parallel divide-and-conquer algorithms — for wide-vector parallel computers. This paper presents our port of the NESL implementation to work on GPUs and provides empirical evidence that nested data-parallelism (NDP) on GPUs significantly outperforms CPU-based implementations and matches or beats newer GPU languages that support only flat parallelism. While our performance does not match that of hand-tuned CUDA programs, we argue that the notational conciseness of NESL is worth the loss in performance. This work provides the first language implementation that directly supports NDP on a GPU.

uniform problem subdivisions and non-uniform memory access, such as divide-and-conquer algorithms.

Most GPU programming is done with the CUDA [NVI11b] and OpenCL [Khr11] languages, which provide the illusion of C-style general-purpose programming, but which actually impose restrictions. There have been a number of efforts to support GPU programming from higher-level languages, usually by embedding a data-parallel DSL into the host language, but these efforts have been limited to regular parallelism [CBS11, MM10, CKL+11].

The current best practice for irregular parallelism on a GPU is for skilled programmers to laboriously hand code applications. The literature is rife with implementations of specific irregular-parallel algorithms for GPUs [BP11, DR11, MLBP12, MGG12]. These efforts typically require many programmer-months of effort to even meet the performance of the original optimized sequential C program.

GPUs have some common characteristics with the wide-vector supercomputers of the 1980's, which similarly provided high-performance SIMD computations. NESL is a first-order functional language developed by Guy Blelloch in the early 1990's that was

End

Next lecture (Thursday)

DPH (nested)

Repa (flat)

parentheses matching

For each index, return the index of the matching parenthesis

```
function parentheses_match(string) =
let
    depth = plus_scan({if c==`( then 1 else -1 : c in string});
    depth = {d + (if c==`( then 1 else 0): c in string; d in depth};
    rnk = permute([0:#string], rank(depth));
    ret = interleave(odd_elts(rnk), even_elts(rnk))
in permute(ret, rnk);
```

(	)	(	(	)	(	)	)	(	(	(	)	)	)
1	-1	1	1	-1	1	-1	-1	1	1	1	-1	-1	-1

(	)	(	(	)	(	)	)	(	(	(	)	)	)
1	-1	1	1	-1	1	-1	-1	1	1	1	-1	-1	-1
0	1	0	1	2	1	2	1	0	1	2	3	2	1

prescan
(+)

(	)	(	(	)	(	)	)	(	(	(	)	)	)
1	-1	1	1	-1	1	-1	-1	1	1	1	-1	-1	-1
0	1	0	1	2	1	2	1	0	1	2	3	2	1
1	1	1	2	2	2	2	1	1	2	3	3	2	1

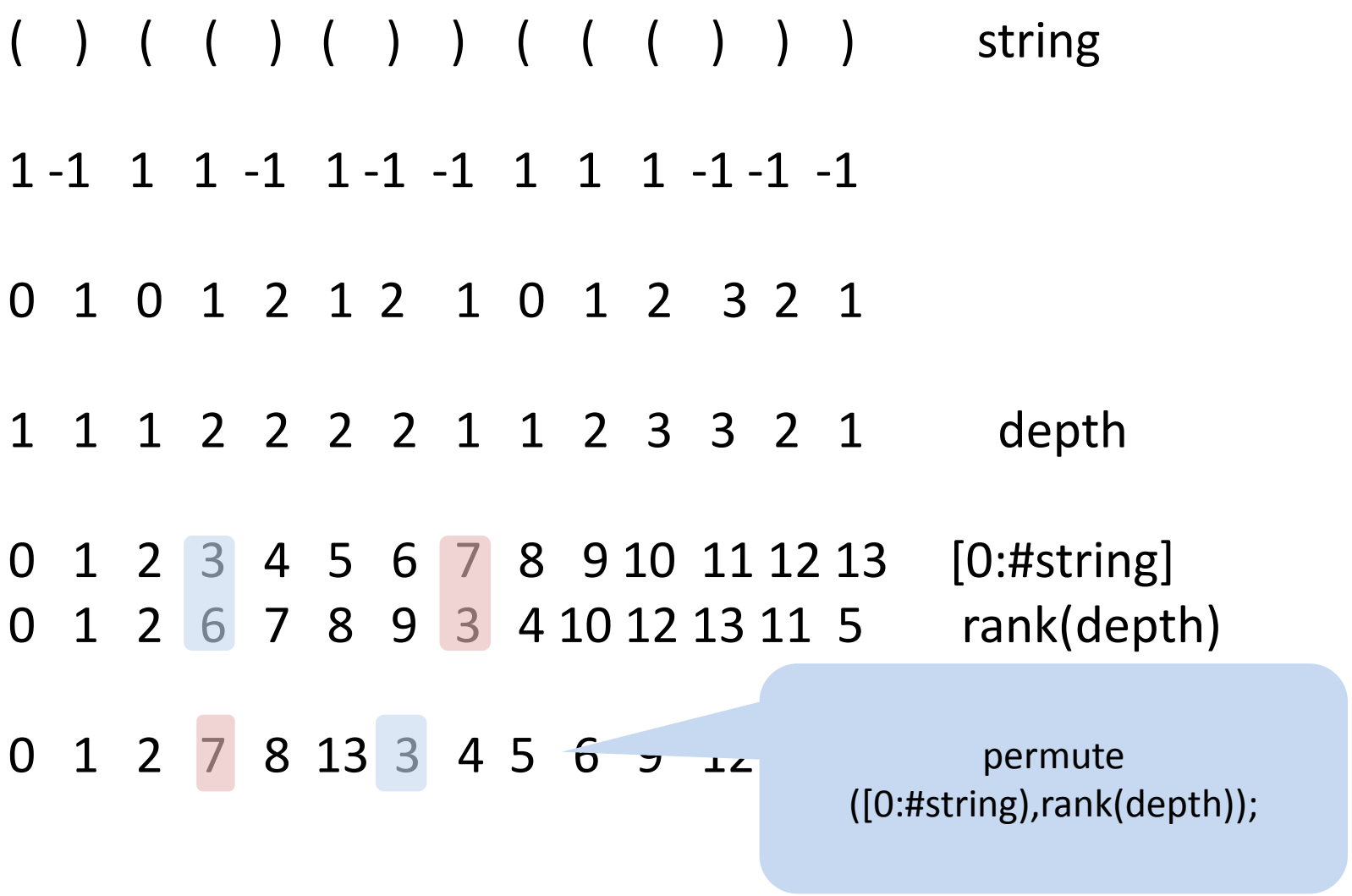
+1 if (
+0 if)

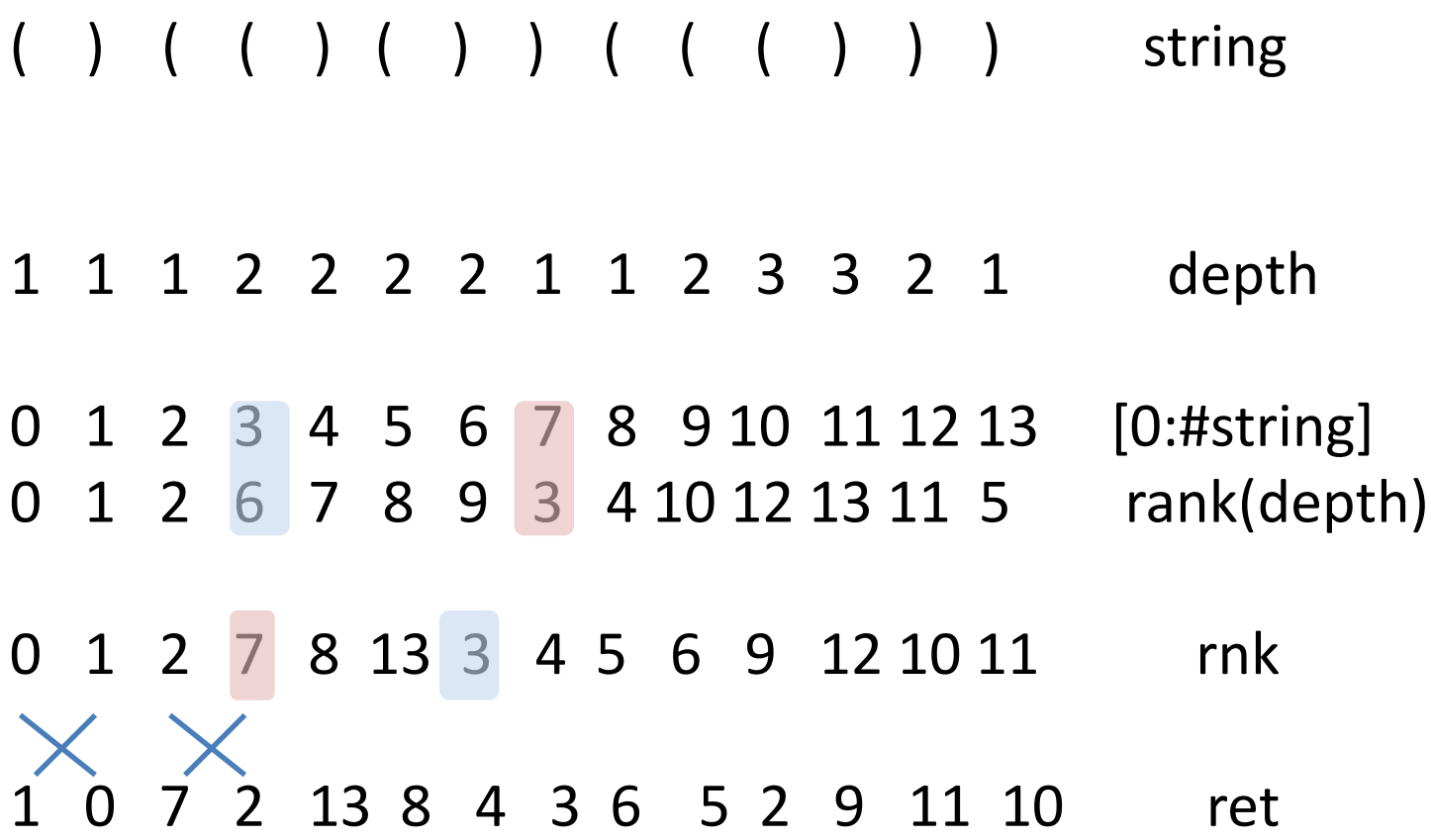
(	)	(	(	)	(	)	)	(	(	(	)	)	)	
1	-1	1	1	-1	1	-1	-1	1	1	1	-1	-1	-1	
0	1	0	1	2	1	2	1	0	1	2	3	2	1	
1	1	1	2	2	2	2	1	1	2	3	3	2	1	depth

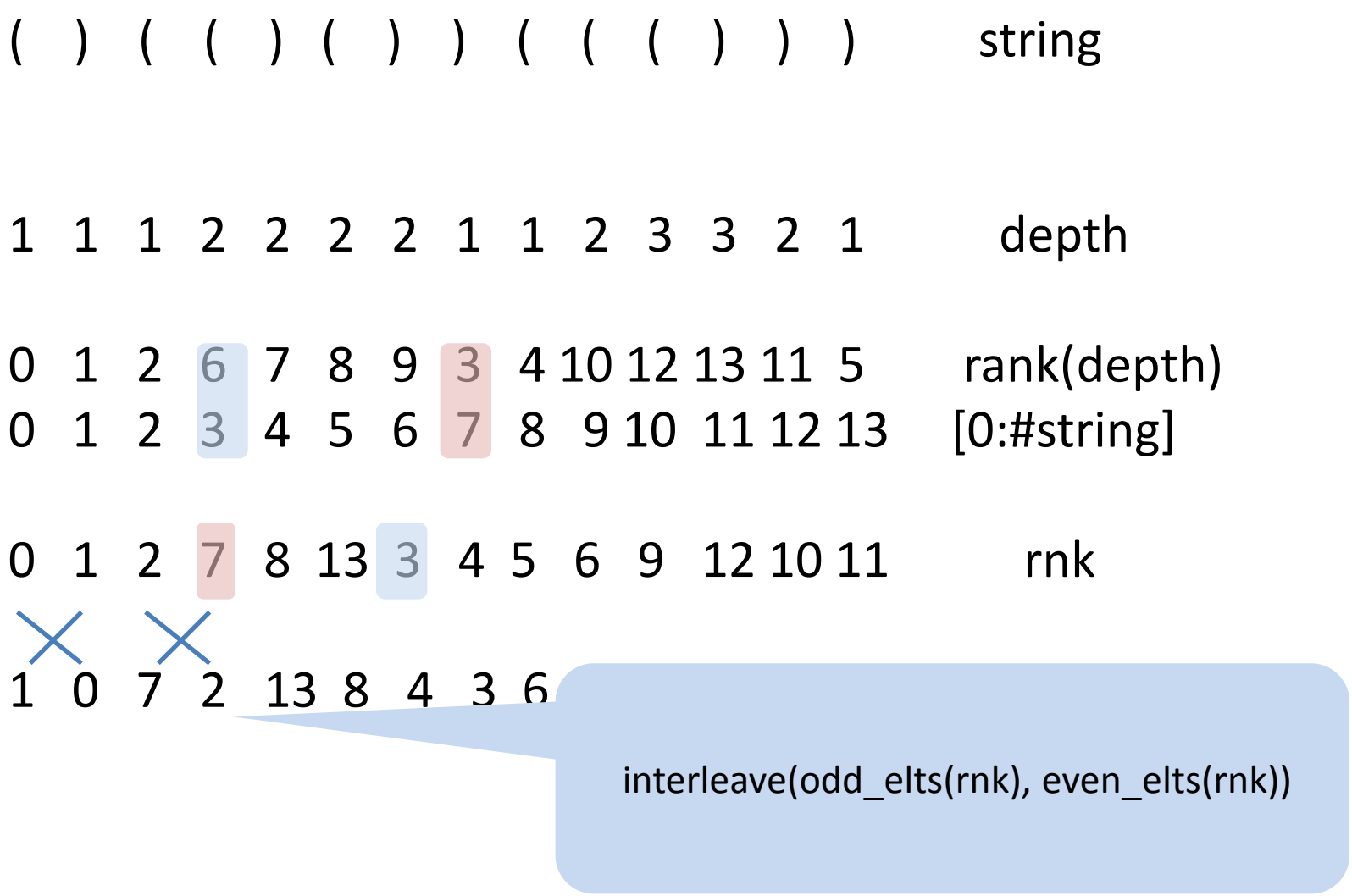
+1 if (
+0 if)

(	)	(	(	)	(	)	)	(	(	(	)	)	)	string
1	-1	1	1	-1	1	-1	-1	1	1	1	-1	-1	-1	
0	1	0	1	2	1	2	1	0	1	2	3	2	1	
1	1	1	2	2	2	2	1	1	2	3	3	2	1	depth
0	1	2	6	7	8	9	3	4	10	12	13	11	5	rank(depth)

(	)	(	(	)	(	)	)	(	(	(	)	)	)	string
1	-1	1	1	-1	1	-1	-1	1	1	1	-1	-1	-1	
0	1	0	1	2	1	2	1	0	1	2	3	2	1	
1	1	1	2	2	2	2	1	1	2	3	3	2	1	depth
0	1	2	3	4	5	6	7	8	9	10	11	12	13	[0:#string]
0	1	2	6	7	8	9	3	4	10	12	13	11	5	rank(depth)
0	1	2	7	8	13	3	4	5	6	9	12	10	11	rnk

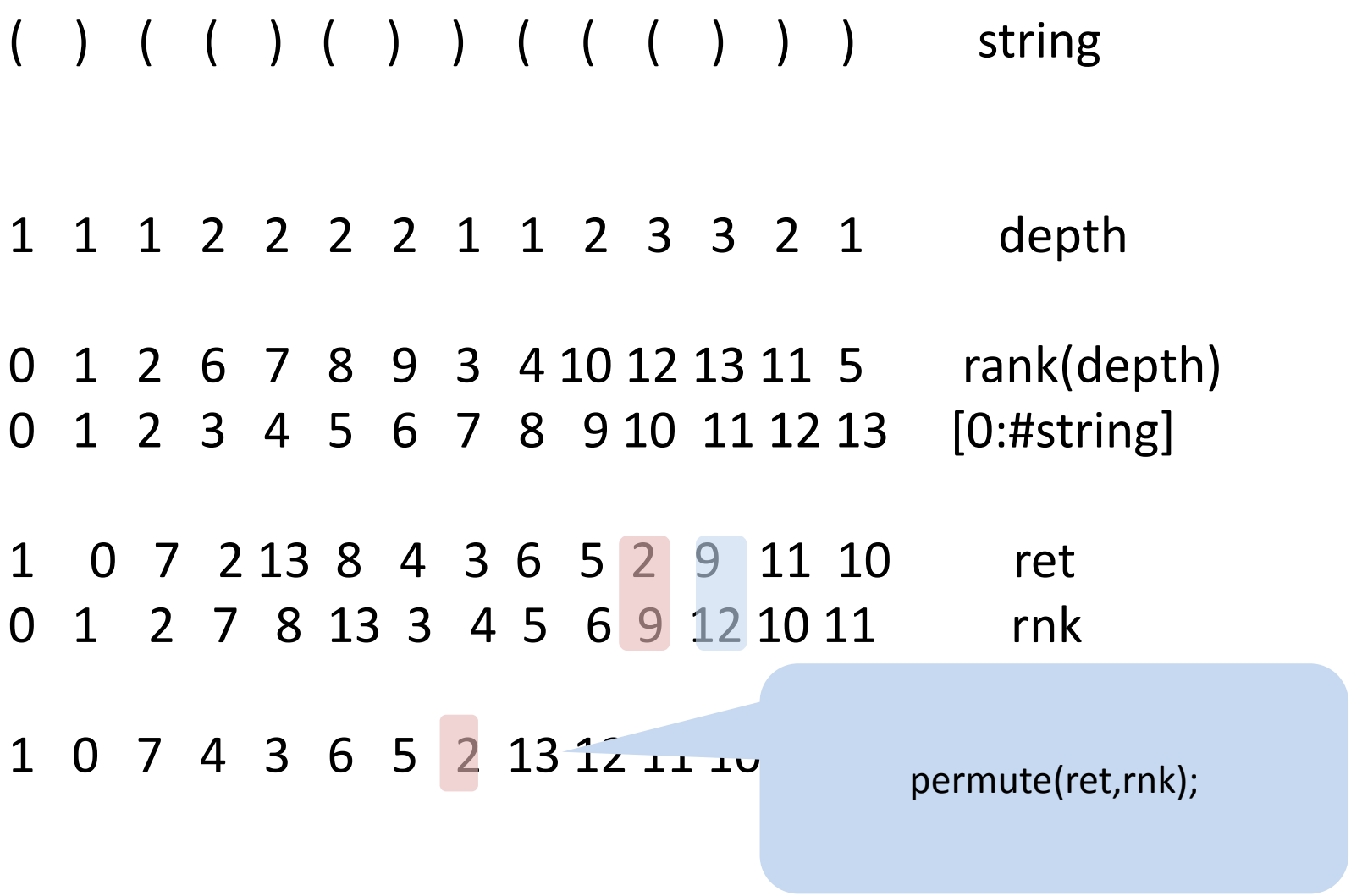

( ) (( ) ( )) ((( )))
string
1 -1 1 1 -1 1 -1 -1 1 1 1 -1 -1 -1
0 1 0 1 2 1 2 1 0 1 2 3 2 1
1 1 1 2 2 2 2 1 1 2 3 3 2 1
depth
0 1 2 3 4 5 6 7 8 9 10 11 12 13
[0:#string]
0 1 2 6 7 8 9 3 4 10 12 13 11 5
rank(depth)
0 1 2 7 8 13 3 4 5 6
permute
([0:#string),rank(depth));

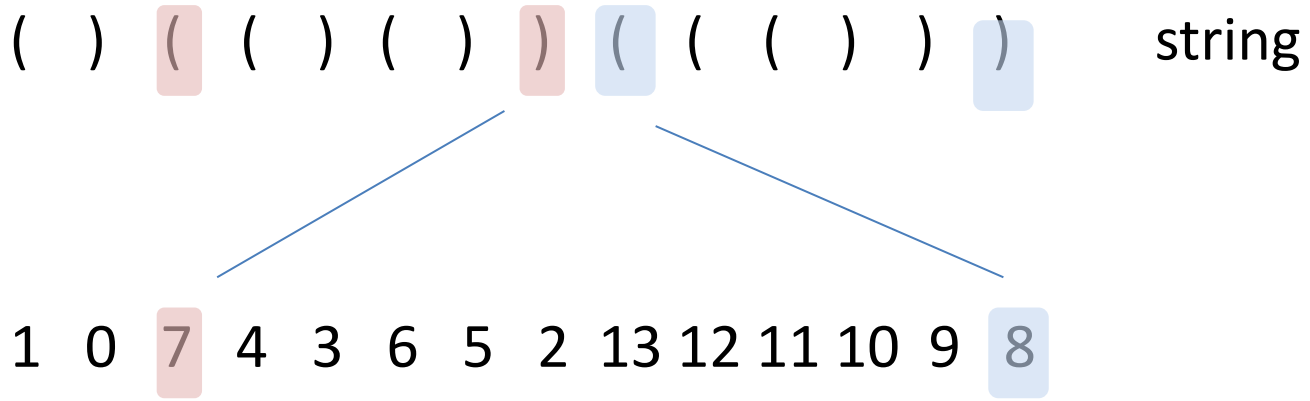

( ) (( ) ( )) ((( ))) string
1 1 1 2 2 2 2 1 1 2 3 3 2 1 depth
0 1 2 3 4 5 6 7 8 9 10 11 12 13 [0:#string]
0 1 2 6 7 8 9 3 4 10 12 13 11 5 rank(depth)
0 1 2 7 8 13 3 4 5 6 9 12 10 11 rnk
1 0 7 2 13 8 4 3 6 5 2 9 11 10 ret


( ) (( ) ( )) ((( ))) string
1 1 1 2 2 2 2 1 1 2 3 3 2 1 depth
0 1 2 6 7 8 9 3 4 10 12 13 11 5 rank(depth)
0 1 2 3 4 5 6 7 8 9 10 11 12 13 [0:#string]
0 1 2 7 8 13 3 4 5 6 9 12 10 11 rnk
1 0 7 2 13 8 4 3 6
interleave(odd_elts(rnk), even_elts(rnk))

(	)	(	(	)	(	)	)	(	(	(	)	)	)	string
1	1	1	2	2	2	2	1	1	2	3	3	2	1	depth
0	1	2	6	7	8	9	3	4	10	12	13	11	5	rank(depth)
0	1	2	3	4	5	6	7	8	9	10	11	12	13	[0:#string]
1	0	7	2	13	8	4	3	6	5	2	9	11	10	ret
0	1	2	7	8	13	3	4	5	6	9	12	10	11	rnk

1	0	7	4	3	6	5	2	13	12	11	10	9	8


( ) (( ) ( )) ((( ))) string
1 1 1 2 2 2 2 1 1 2 3 3 2 1 depth
0 1 2 6 7 8 9 3 4 10 12 13 11 5 rank(depth)
0 1 2 3 4 5 6 7 8 9 10 11 12 13 [0:#string]
1 0 7 2 13 8 4 3 6 5 2 9 11 10 ret
0 1 2 7 8 13 3 4 5 6 9 12 10 11 rnk
1 0 7 4 3 6 5 2 13
permute(ret,rnk);


( ) (( ) ( )) ((( ))) string
1 0 7 4 3 6 5 2 13 12 11 10 9 8