Parallel Parsing: How Hard Can It Be?

Jean-Philippe Bernardy Koen Claessen

CHALMERS | GÖTEBORG UNIVERSITY

Parallel Functional Programming Course, May 6, 2013

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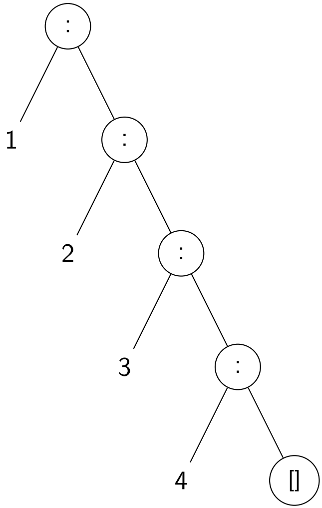
FP workhorse: lists



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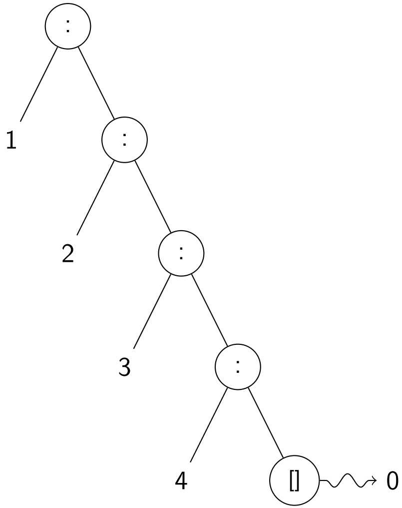

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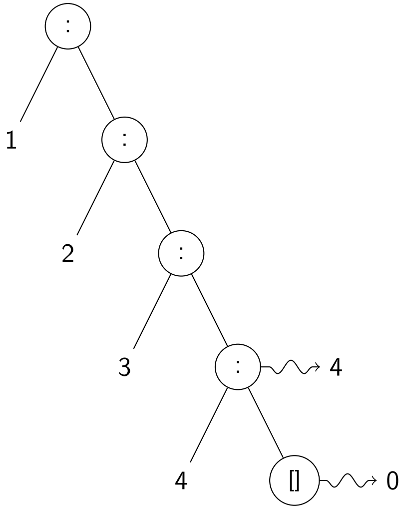

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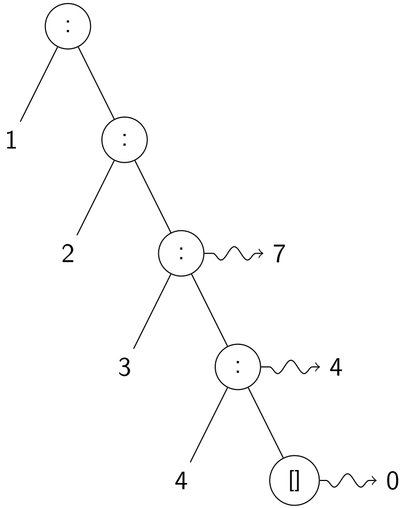

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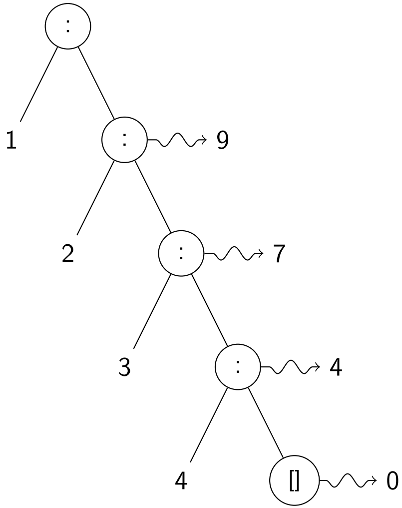

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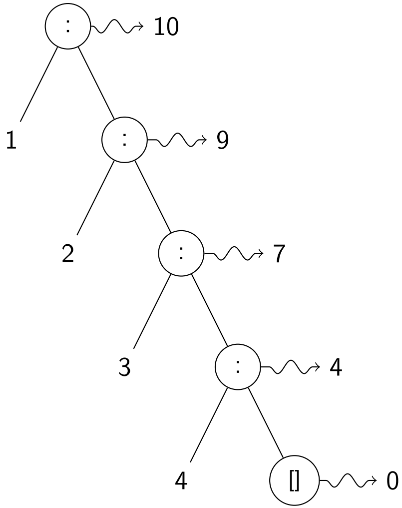

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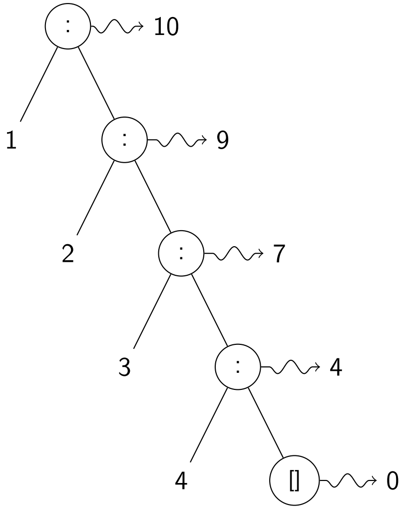

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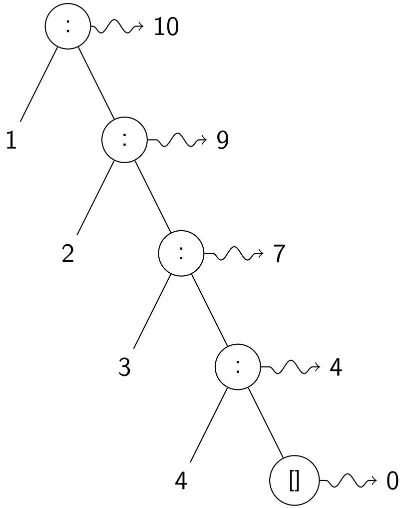
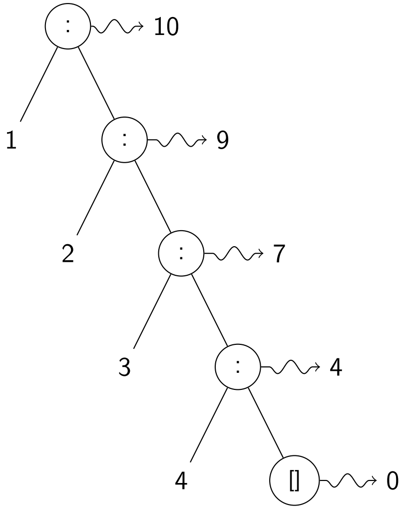


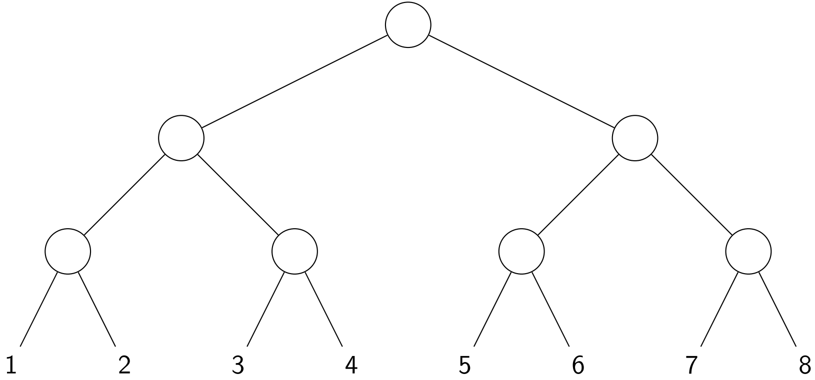
- Built-in sequentiality

FP workhorse: lists



- Built-in sequentiality
- Bad!

Exploiting parallelism: sum over a tree



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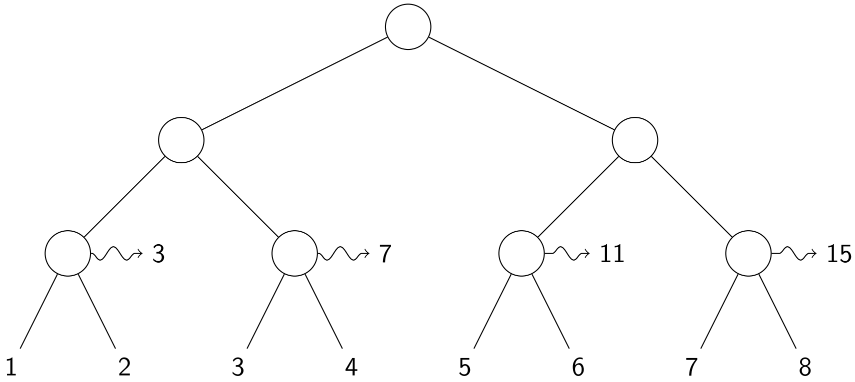

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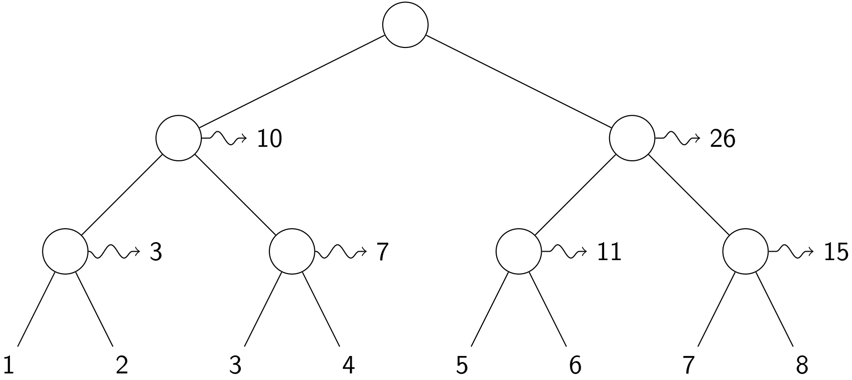

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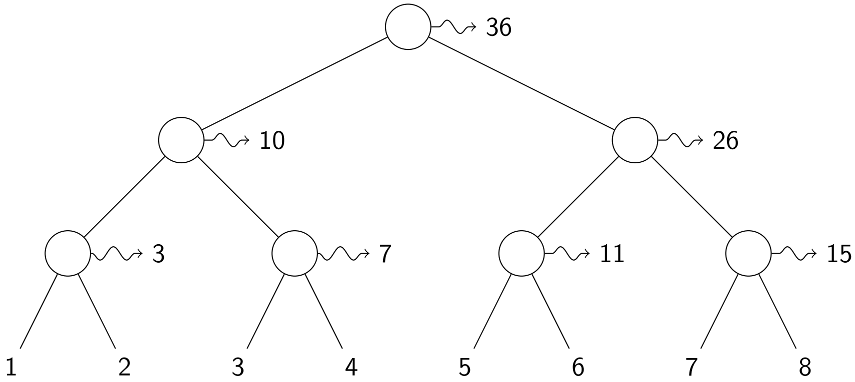

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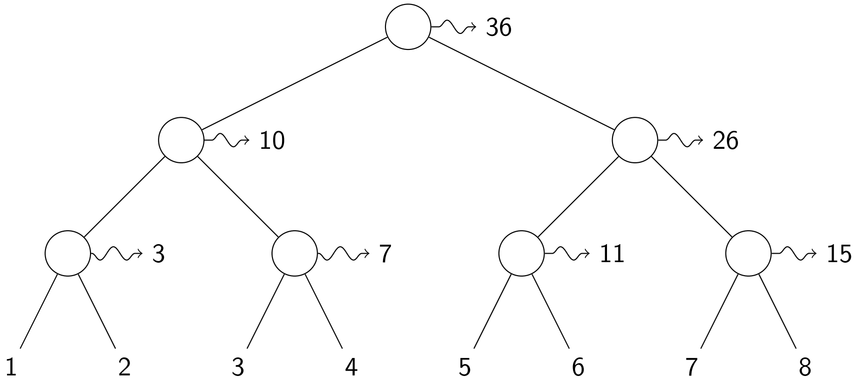

- Picture a little computer at each node.
- The program "flows down" and the data "flows up".
- Computers of the future will have such a fractal structure.

On a mission to reinvent programming, using:

- Trees
- Divide and Conquer

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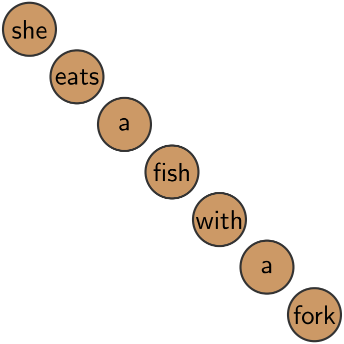
- Guy Steele, invited talk at ICFP2009 (and other venues)
- MapReduce

Chart parsing



$\mathcal{G}$ (CNF)

S	$\rightarrow$	NP VP
VP	$\rightarrow$	VP PP
VP	$\rightarrow$	VP NP
VP	$\rightarrow$	eats
PP	$\rightarrow$	P NP
NP	$\rightarrow$	Det N
NP	$\rightarrow$	she
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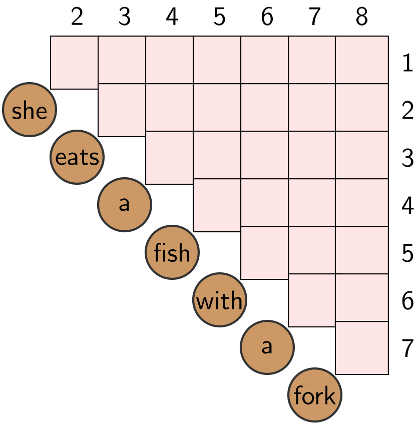

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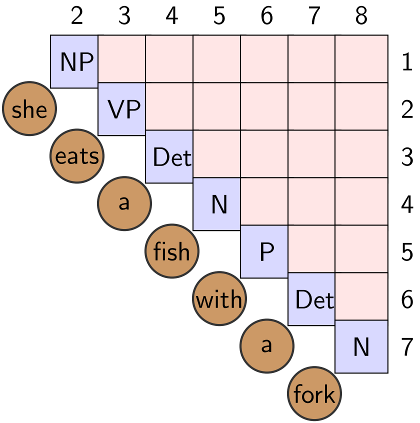
R_{ij} = all non-terminals generating the input substring $w_{i..j}$

Chart parsing



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$$R_{i,i+1} = \{A \mid A \rightarrow w_i \in \mathcal{G}\}$$

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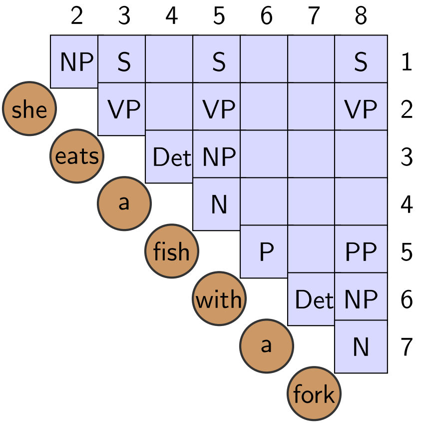

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$$R_{i,i+1} = \{A \mid \boxed{A \rightarrow w_i} \in \mathcal{G}\}$$

$$R_{ij} = \{A \mid k \in [i+1..j-1], B \in R_{ik}, C \in R_{kj}, \boxed{A \rightarrow BC} \in \mathcal{G}\}$$

Parsing Specification

Structure on sets of non-terminals :

$$x + y = x \cup y$$

$$x \cdot y = \{N \mid A \in x, B \in y, \ N \to AB \ \in \mathcal{G}\}$$

Parsing Specification

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Structure on matrices:

$$(A + B)_{ij} = A_{ij} + B_{ij}$$
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Structure on matrices:

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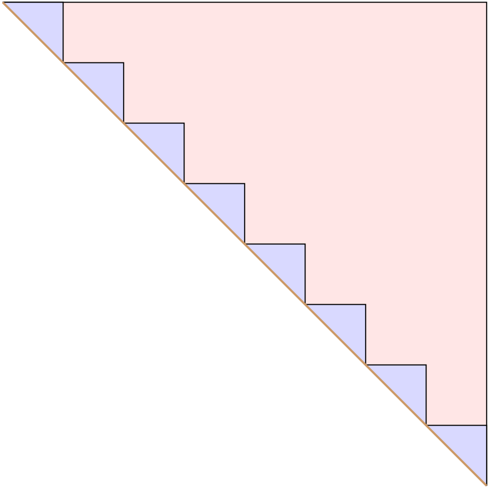
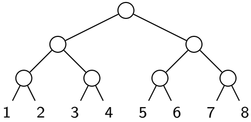
Find R, such that

$$R = I(w) + R \cdot R \tag{1}$$

Chart parsing as Divide and Conquer





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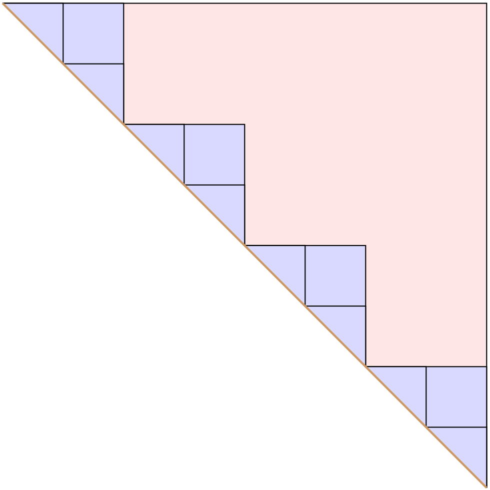

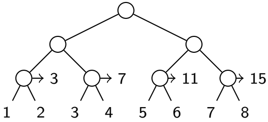

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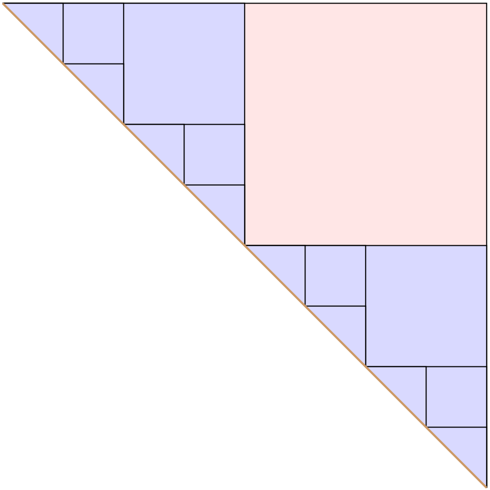

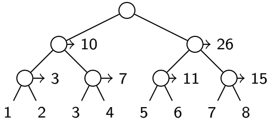

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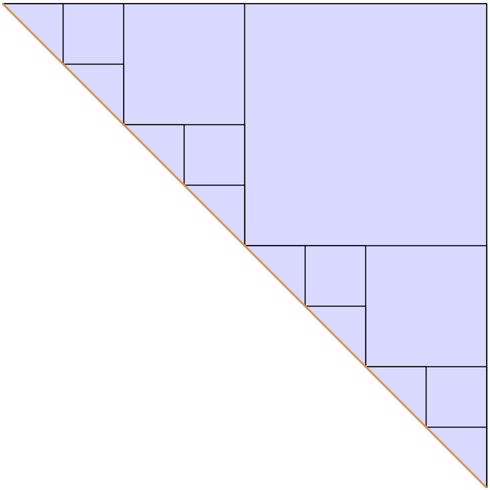
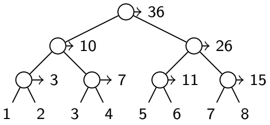
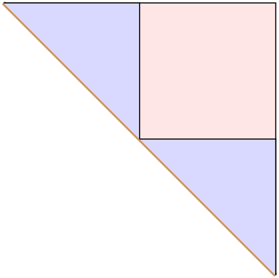




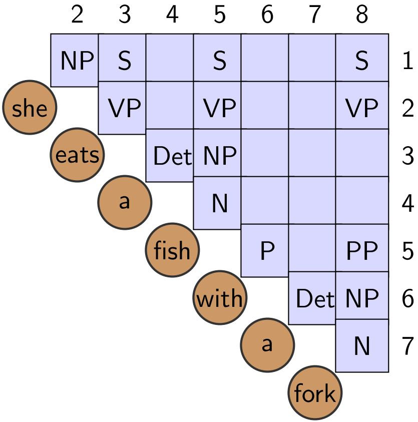
Efficiency

- space usage quadratic in the size of input string?!
- runtime cubic in the size of input string?!



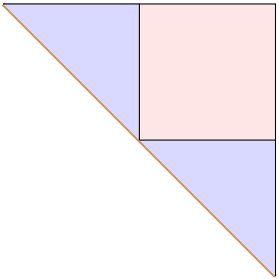
The combination operator takes cubic time?!
Failure to parallelize?!

A sparse matrix



$$\#A \leq \left\lceil \alpha \sum_{(i,j)\in \mathrm{dom}(A)} \frac{1}{(j-i)^2} \right\rceil$$

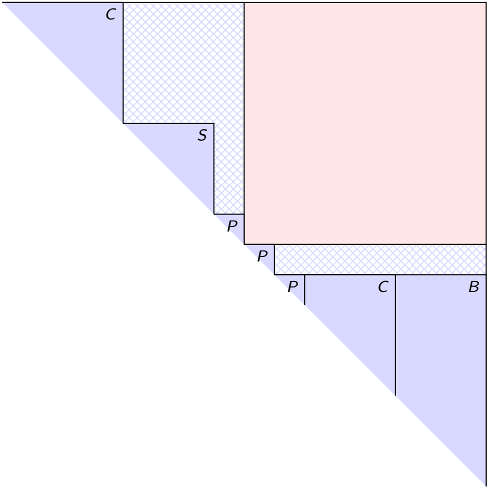
Cheap combination



The square to fill is sparse

- To fill it should be quick
- Good space usage
- Good time-usage
- Parallelizable

How much work?



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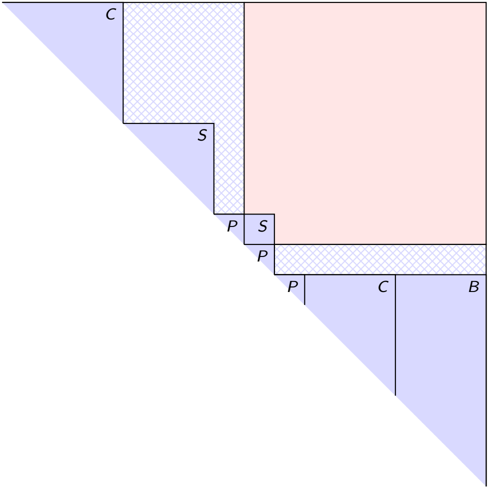

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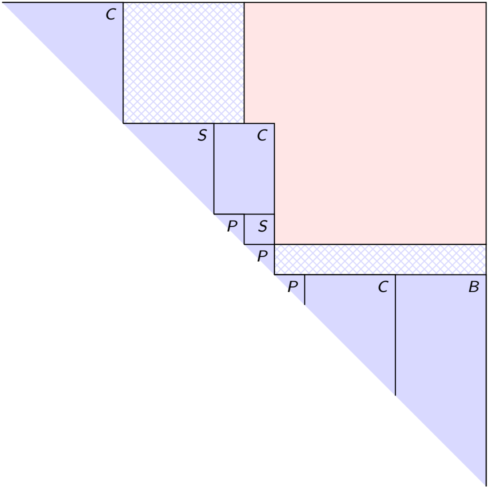

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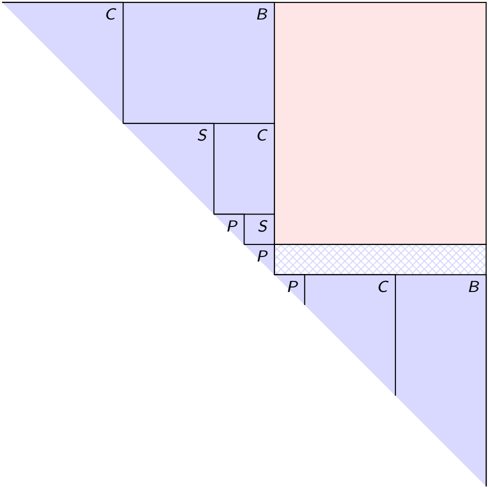

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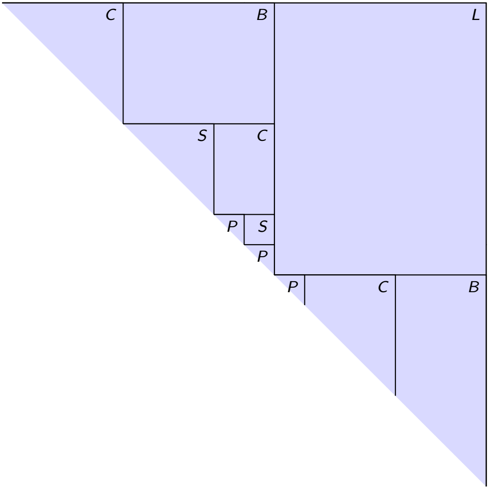

Deriving Efficitent Transitive Closure Algorithm

Problem: find R such that $R = R \cdot R + W$.

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$$W = \begin{bmatrix} A & X \\ 0 & B \end{bmatrix} \qquad\qquad R = \begin{bmatrix} A' & X' \\ 0 & B' \end{bmatrix}$$

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$$\begin{bmatrix} A' & X' \\ 0 & B' \end{bmatrix} = \begin{bmatrix} A' & X' \\ 0 & B' \end{bmatrix} \cdot \begin{bmatrix} A' & X' \\ 0 & B' \end{bmatrix} + \begin{bmatrix} A & X \\ 0 & B \end{bmatrix}$$

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$$\begin{aligned} A' &= A'A' + A \\ X' &= A'X' + X'B' + X \\ B' &= B'B' + B \end{aligned}$$

Deriving Efficient Chart Concatenation

Problem: find Y such that $Y = AY + YB + X = V(A, X, B)$.

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$$\begin{aligned} \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = {} & \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix} \cdot \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \\ & + \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \cdot \begin{bmatrix} B_{11} & B_{12} \\ 0 & B_{22} \end{bmatrix} + \begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{bmatrix} \end{aligned}$$

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$$\begin{aligned}
Y_{11} &= A_{11}Y_{11} + A_{12}Y_{21} + Y_{11}B_{11} + 0 + X_{11} \\
Y_{12} &= A_{11}Y_{12} + A_{12}Y_{22} + Y_{11}B_{12} + Y_{12}B_{22} + X_{12} \\
Y_{21} &= 0 + A_{22}Y_{21} + Y_{21}B_{11} + 0 + X_{21} \\
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\end{aligned}$$

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Y_{21} & = & 0 & + A_{22}Y_{21} & + Y_{21}B_{11} & + 0 & + X_{21} \\
Y_{22} & = & 0 & + A_{22}Y_{22} & + Y_{21}B_{12} & + Y_{22}B_{22} & + X_{22}
\end{array}$$

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Y_{21} & = & A_{22}Y_{21} & + X_{21} + 0 & + Y_{21}B_{11} \\
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$$
\begin{array}{rclclcll}
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Y_{12} & = & A_{11}Y_{12} & + & X_{12} & + & A_{12}Y_{22} + Y_{11}B_{12} & + \;\; Y_{12}B_{22} \\
Y_{21} & = & A_{22}Y_{21} & + & X_{21} & + & 0 & + \;\; Y_{21}B_{11} \\
Y_{22} & = & A_{22}Y_{22} & + & X_{22} & + & Y_{21}B_{12} & + \;\; Y_{22}B_{22}
\end{array}
$$

$$
\begin{array}{rll}
Y_{11} & = V(A_{11}, & X_{11} + A_{12}Y_{21} \qquad\qquad\quad , \; B_{11}) \\
Y_{12} & = V(A_{11}, & X_{12} + A_{12}Y_{22} + Y_{11}B_{12}, \; B_{22}) \\
Y_{21} & = V(A_{22}, & X_{21} \qquad\qquad\qquad\qquad\quad\;\; , \; B_{11}) \\
Y_{22} & = V(A_{22}, & X_{22} + Y_{21}B_{12} \qquad\qquad\quad , \; B_{22})
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$$\begin{aligned}
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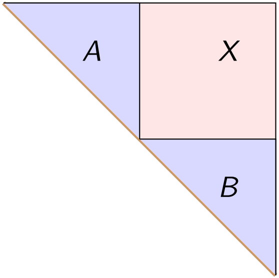
Y_{12} &= V(A_{11},\ X_{12} + A_{12} Y_{22} + Y_{11} B_{12} &&,\ B_{22}) \\
Y_{21} &= V(A_{22},\ X_{21} &&,\ B_{11}) \\
Y_{22} &= V(A_{22},\ X_{22} + Y_{21} B_{12} &&,\ B_{22})
\end{aligned}$$

No circular dependencies! Done!

Valiant's algorithm for transitive closure

$$Y = V(A, X, B)$$



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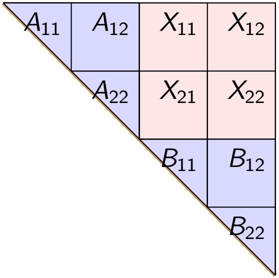

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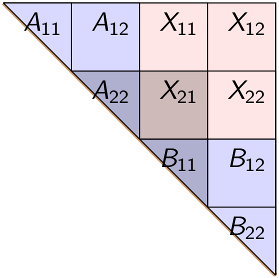

Valiant's algorithm for transitive closure

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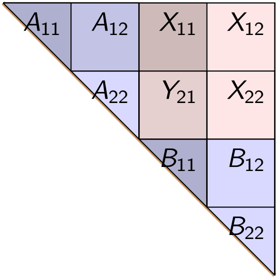

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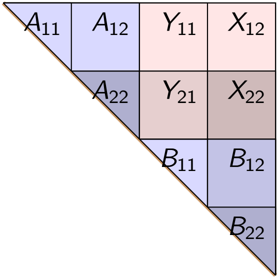

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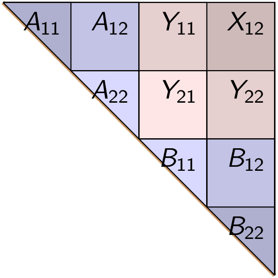

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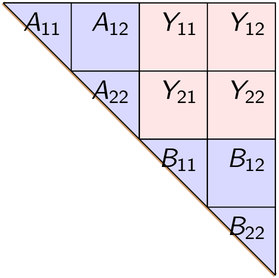


Haskell Implementation: Sparse Matrix Structure

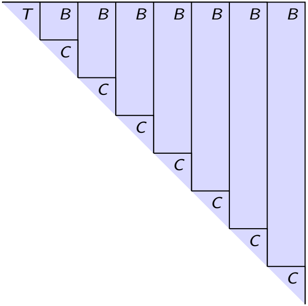
$$
\begin{array}{l}
\textbf{import}\ \mathit{Prelude}\ (\mathit{Eq}\ (..)) \\
\textbf{class}\ \mathit{RingLike}\ a\ \textbf{where} \\
\quad \mathit{zero} :: a \\
\quad (+) :: a \to a \to a \\
\quad (\cdot) :: a \to a \to a \\
\textbf{data}\ M\ a = Q\ (M\ a)\ (M\ a)\ (M\ a)\ (M\ a) \mid Z \mid \mathit{One}\ a \\
q\ Z\ Z\ Z\ Z = Z \\
q\ a\ \ b\ \ c\ \ d = Q\ a\ b\ c\ d \\
\mathit{one}\ x = \textbf{if}\ x \equiv \mathit{zero}\ \textbf{then}\ Z\ \textbf{else}\ \mathit{One}\ x
\end{array}
$$

Haskell Implementation: algorithm

$$
\begin{array}{llll}
\multicolumn{4}{l}{\textbf{instance}\ (Eq\ a, RingLike\ a) \Rightarrow RingLike\ (M\ a)\ \textbf{where} \quad \text{-- } \ldots} \\
\multicolumn{4}{l}{v :: (Eq\ a, RingLike\ a) \Rightarrow M\ a \to M\ a \to M\ a \to M\ a} \\
v\ a & Z & b = Z & \\
v\ Z & (One\ x) & Z = One\ x & \\
v\ (Q\ a_{11}\ a_{12}\ Z\ a_{22}) & (Q\ x_{11}\ x_{12}\ x_{21}\ x_{22}) & (Q\ b_{11}\ b_{12}\ Z\ b_{22}) &
\end{array}
$$

$$
\begin{aligned}
&= q\ y_{11}\ y_{12}\ y_{21}\ y_{22} \\
&\quad \textbf{where}\ y_{21} = v\ a_{22}\ \ x_{21} \qquad\qquad\qquad\qquad\qquad\quad b_{11} \\
&\qquad\qquad y_{11} = v\ a_{11}\ (x_{11} + a_{12} \cdot y_{21} \qquad\qquad\quad)\ b_{11} \\
&\qquad\qquad y_{22} = v\ a_{22}\ (x_{22} + \qquad\qquad\quad y_{21} \cdot b_{12})\ b_{22} \\
&\qquad\qquad y_{12} = v\ a_{11}\ (x_{12} + a_{12} \cdot y_{22} + y_{11} \cdot b_{12})\ b_{22}
\end{aligned}
$$

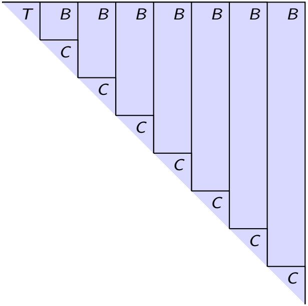
Recursion in the grammar



$B \rightarrow \mathit{TitlePage}$

$B \rightarrow BC$

Recursion in the grammar



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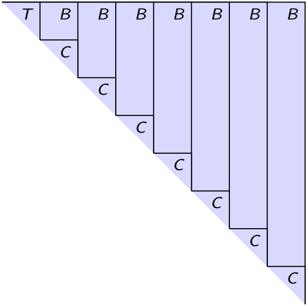

$B \to TitlePage$

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Bad!

- The combination has a lot of work to do (at least linear)

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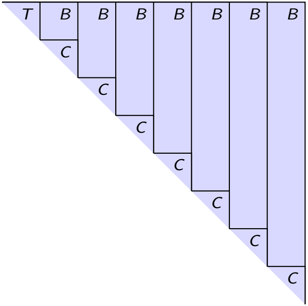

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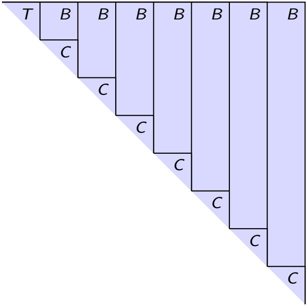
Recursion in the grammar



$B \rightarrow TitlePage$

$B \rightarrow BC$

Bad!

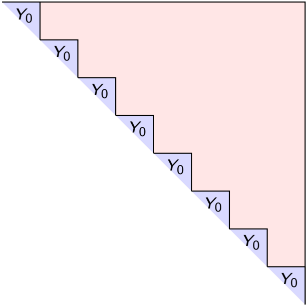
- The combination has a lot of work to do (at least linear)
- AST is a list

Solution:

$B' \rightarrow TitlePage\ B$

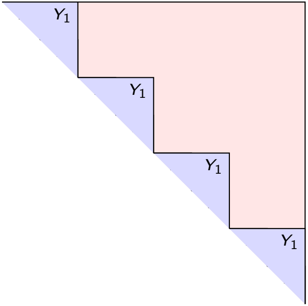
$B \rightarrow C*$

Binary encoding of lists: idea



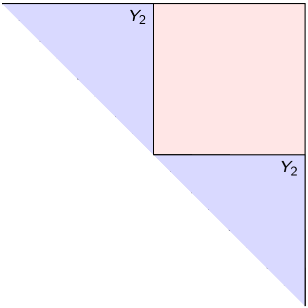
$L \to Y*$

Binary encoding of lists: idea



$L \to Y*$

Binary encoding of lists: idea



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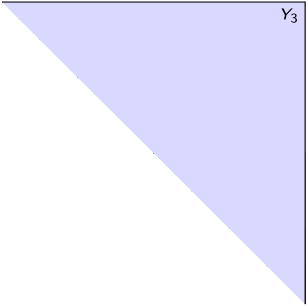
Binary encoding of lists: idea



$$L \to Y*$$

Conclusion

- Valiant (75) does parsing using matrix multiply; yields the most efficient known CF recognition algorithm: $O(n^{2.3727})$. [1]
- The *very same* algorithm yields parsing on usual inputs in $O(n \log^3 n)$, and $O(n \log^4 n)$ incremental/parallel complexity.
- The implementation is purely functional and fits on a slide! Why aren't all CS students taught this?!
- Implemented in BNFC: push-button technology.
- It *is* fast:
  - Experiments indicate the theory is pessimitic by a factor of $\log n$.
  - Incremental parsing of a 8000-line C program in less than 1 millisecond.

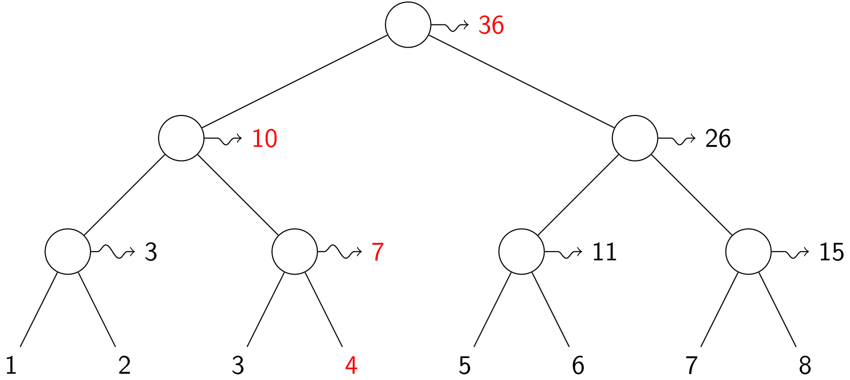
Full details: `http://cse.chalmers.se/~bernardy/PP.pdf`

[1] Complexity of the Coppersmith-Winograd algorithm

Take home message

- Use balanced trees; divide and conqueer; associative operations.
- Technically: sequence homomorphisms
- To be able to find associative operators: enlarge the search space

BONUS: Incremental computation (1)



BONUS: Incremental computation (2)