# Local search algorithms

## Chapter 4, Sections 1–2

# Outline

- ♢ Hill-climbing
- ♢ Simulated annealing
- ♢ Genetic algorithms (briefly)
- ♢ Local search in continuous spaces (very briefly)

# Iterative improvement algorithms

In many optimization problems, the **path** is irrelevant;
the goal state itself is the solution

Then the state space can be the set of "complete" configurations
- e.g., for 8-queens, a configuration can be any board with 8 queens
- e.g., for TSP, a configuration can be any complete tour

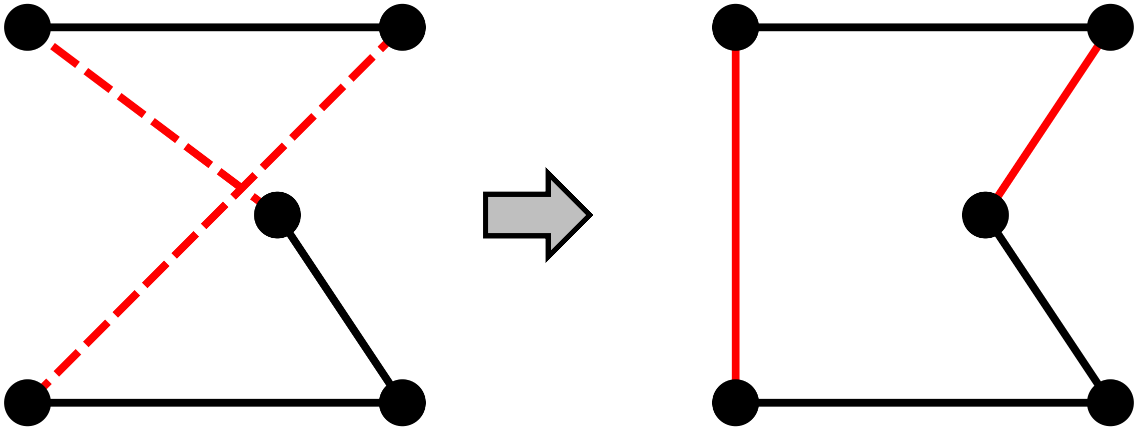
In such cases, we can use iterative improvement algorithms;
we keep a single "current" state, and try to improve it
- e.g., for 8-queens, we gradually move some queen to a better place
- e.g., for TSP, we start with any tour and gradually improve it

The goal would be to find an **optimal** configuration
- e.g., for 8-queens, an optimal config. is where no queen is threatened
- e.g., for TSP, an optimal configuration is the shortest route

This takes constant space, and is suitable for online as well as offline search

# Example: Travelling Salesperson Problem

Start with any complete tour, and perform pairwise exchanges

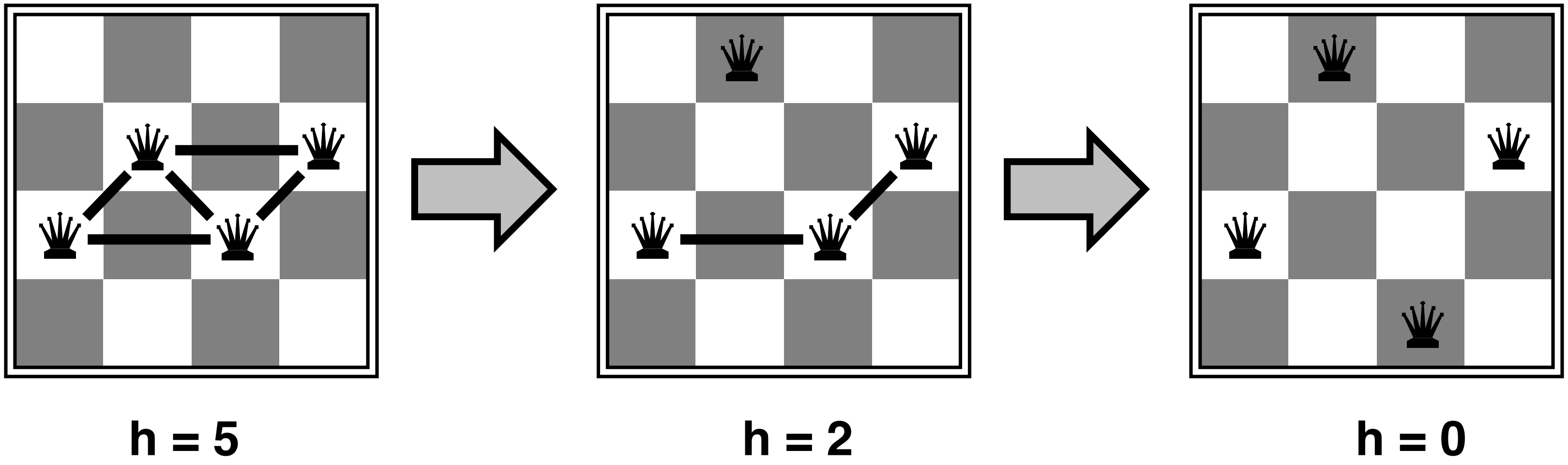


Variants of this approach get within 1% of optimal very quickly
with thousands of cities

# Example: $n$-queens

Put $n$ queens on an $n \times n$ board, with no two queens on the same column

Move a queen to reduce the number of conflicts;
repeat until we cannot move any queen anymore
– then we are at a local maximum, hopefully it is global too



This almost always solves $n$-queens problems almost instantaneously for very large $n$ (e.g., $n = 1$ **million**)

# Hill-climbing (or gradient ascent/descent)

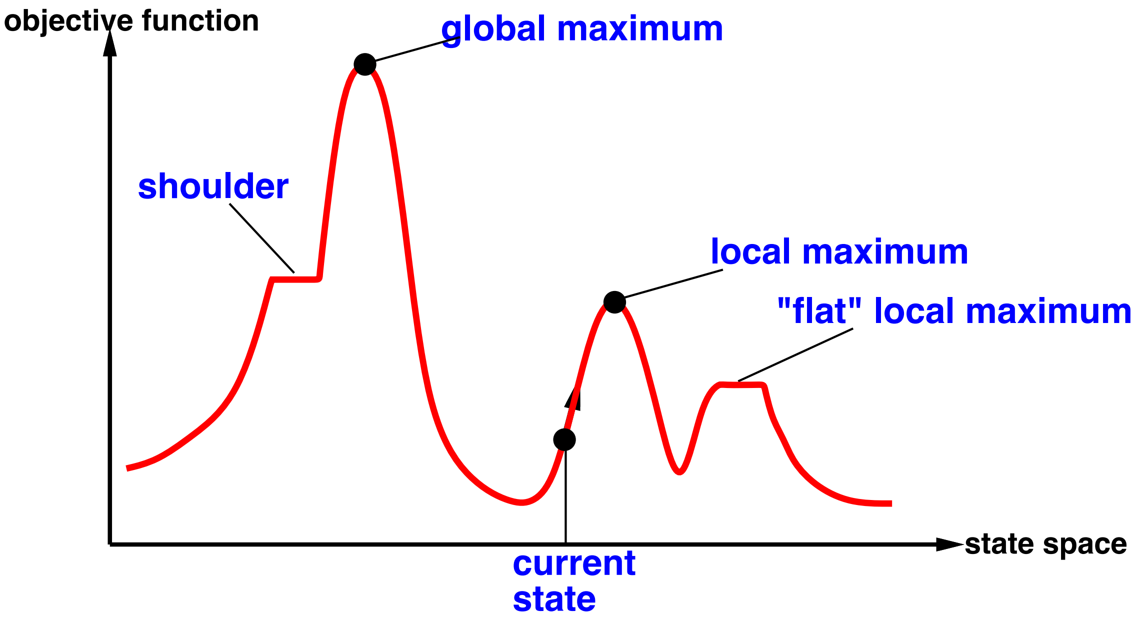
"Like climbing Everest in thick fog with amnesia"

```
function HILL-CLIMBING( problem) returns a state that is a local maximum
    inputs: problem, a problem
    local variables: current, a node
                     neighbor, a node

    current $\leftarrow$ MAKE-NODE(INITIAL-STATE[problem])
    loop do
        neighbor $\leftarrow$ a highest-valued successor of current
        if VALUE[neighbor] $\leq$ VALUE[current] then return STATE[current]
        current $\leftarrow$ neighbor
    end
```

# Hill-climbing contd.

It is useful to consider the state space landscape:



Random-restart hill climbing overcomes local maxima
- trivially complete, given enough time

Random sideways moves
☺ escapes from shoulders ☹ loops on flat maxima

# Simulated annealing

Idea: Escape local maxima by allowing some "bad" moves
**but gradually decrease their size and frequency**

```
function SIMULATED-ANNEALING(problem, schedule) returns a solution state
    inputs: problem, a problem
            schedule, a mapping from time to "temperature"

    current ← MAKE-NODE(INITIAL-STATE[problem])
    for t ← 1 to ∞ do
        T ← schedule[t]
        if T = 0 then return current
        next ← a randomly selected successor of current
        ΔE ← VALUE[next] – VALUE[current]
        if ΔE > 0 then current ← next
        else current ← next only with probability $e^{\Delta E/T}$
```

Note: The *schedule* should **decrease** the temperature $T$
so that it gradually goes to 0

# Local beam search

Idea: keep $k$ states instead of 1; choose top $k$ of all their successors

This is **not** the same as $k$ searches run in parallel!

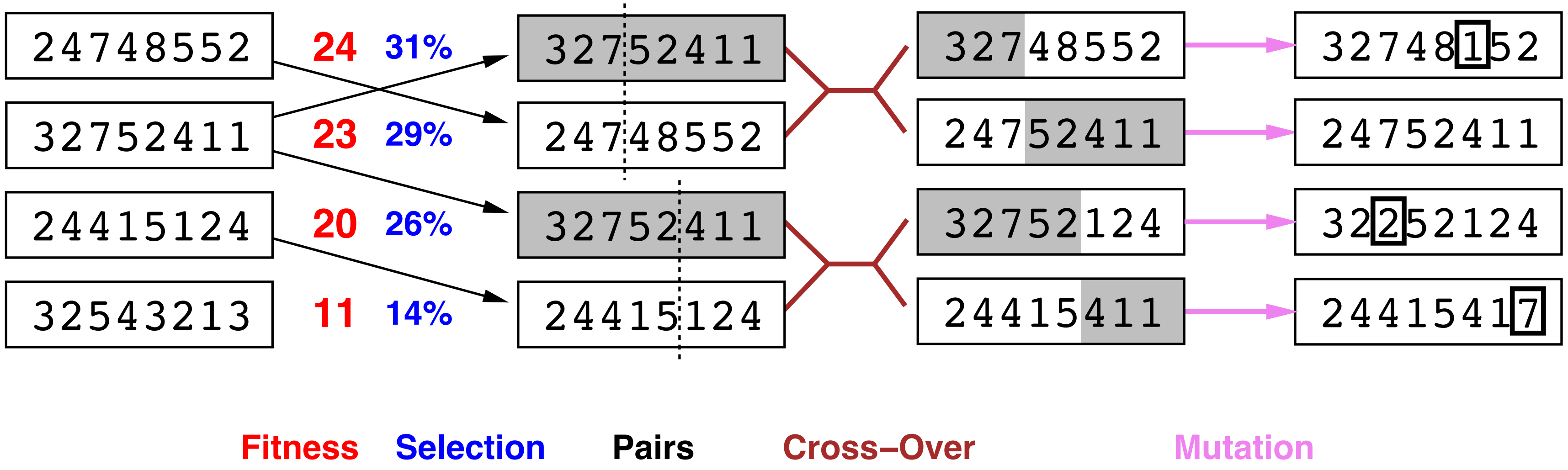
Problem: quite often, all $k$ states end up on same local hill

Idea: choose $k$ successors randomly, biased towards good ones
("Stochastic local beam search")

# Genetic algorithms (briefly)

Idea:

- a variant of stochastic local beam search
- generate successors from **pairs** of states
- the states have to be encoded as strings



24748552
32752411
24415124
32543213

24 31%
23 29%
20 26%
11 14%

32752411
24748552
32752411
24415124

32748552
24752411
32752124
24415411

32748152
24752411
32252124
24415417

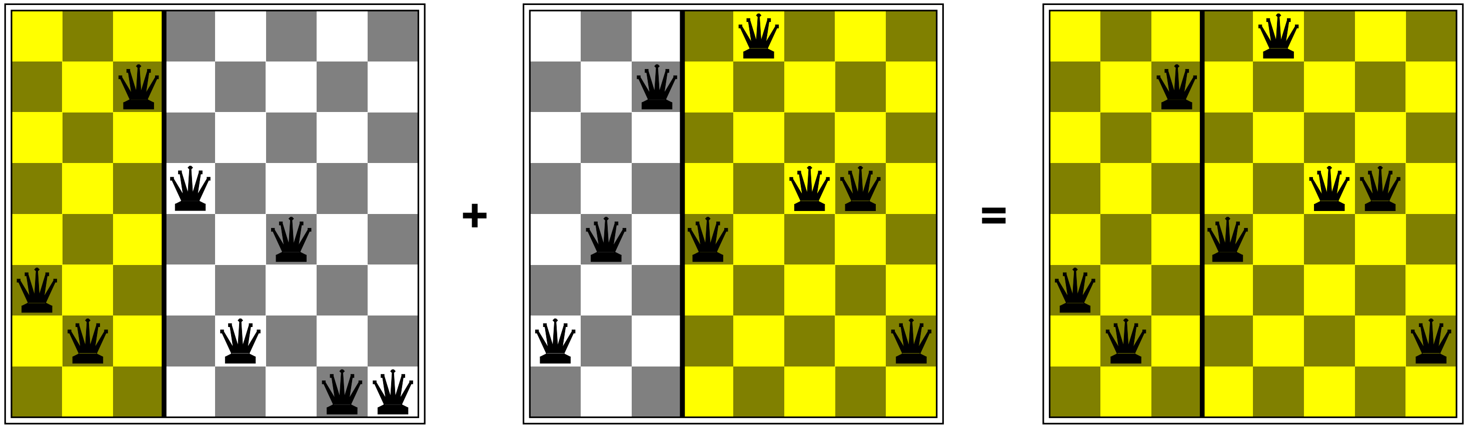
**Fitness** **Selection** **Pairs** **Cross–Over** **Mutation**

Note: $24 / (24{+}23{+}20{+}11) \; = \; 31\%$

# Genetic algorithms contd.

GAs require that the states are encoded as strings

The ‘crossover helps **iff substrings are meaningful components**



3 2 7 5 2 4 1 1 + 2 4 7 4 8 5 5 2 = 3 2 7 4 8 5 5 2

# Continuous state spaces (very briefly)

Suppose we want to site three airports in Romania:

– 6-D state space is defined by $(x_1, y_2)$, $(x_2, y_2)$, $(x_3, y_3)$

– objective function $f(x_1, y_2, x_2, y_2, x_3, y_3) =$
   the sum of squared distances from each city to nearest airport

Discretization methods turn continuous space into discrete space,
e.g., empirical gradient considers $\pm\delta$ change in each coordinate

Gradient methods compute

$$\nabla f = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial y_1}, \frac{\partial f}{\partial x_2}, \frac{\partial f}{\partial y_2}, \frac{\partial f}{\partial x_3}, \frac{\partial f}{\partial y_3}\right)$$

to increase/reduce $f$, e.g., by $\mathbf{x} \leftarrow \mathbf{x} + \alpha \nabla f(\mathbf{x})$

Sometimes we can solve for $\nabla f(\mathbf{x}) = 0$ exactly (e.g., with one city).
Newton–Raphson (1664, 1690) iterates $\mathbf{x} \leftarrow \mathbf{x} - \mathbf{H}_f^{-1}(\mathbf{x}) \nabla f(\mathbf{x})$
to solve $\nabla f(\mathbf{x}) = 0$, where $\mathbf{H}_{ij} = \partial^2 f / \partial x_i \partial x_j$