Model Checking Concurrent Programs A Taster

Wolfgang Ahrendt

Department of Computer Science and Engineering Chalmers University of Technology and University of Gothenburg

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Level of System (Implementation) Description



- Finitely many states (finite datatypes)
- Automated proofs are (in principle) possible
- Simplification, unfaithful modeling inevitable

- Concrete level
 - Infinite datatypes
 - (pointer chains, dynamic arrays, streams)
 - Complex datatypes and control structures, general programs
 - Realistic programming model (e.g., Java)
 - Automated proofs (in general) impossible!

Expressiveness of Specification

Simple

- Simple or general properties
- Finitely many case distinctions
- Approximation, low precision
- Automated proofs are (in principle) possible

Complex

- Full behavioural specification
- Quantification over infinite domains
- High precision, tight modeling
- Automated proofs (in general) impossible!

Main Approaches

Abstract programs, Simple properties	Abstract programs, Complex properties
Concrete programs,	Concrete programs,
Simple properties	Complex properties

Main Approaches



Main Approaches



Proof Automation

"Automated" Proof ("batch-mode")

- No interaction during verification necessary
- Proof may fail or result inconclusive Tuning of tool parameters necessary
- Formal specification still "by hand"
- "Semi-Automated" Proof ("interactive")
 - Interaction may be required during proof
 - Need certain knowledge of tool internals Intermediate inspection can help
 - Proof is checked by tool



Model Checking

System Model

System Property

[]! (criticalSectP && criticalSectQ)



Hardware verification

- Good match between limitations of technology and application
- Intel, Motorola, AMD, ...
- Software verification
 - Specialized software: control systems, protocols
 - Typically no checking of executable source code, but of abstractions
 - Bell Labs, Ericsson, Microsoft

PROMELA is an acronym

Process meta-language

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PROMELA is a language for modeling concurrent systems
multi-threaded

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- multi-threaded
- synchronisation and message passing

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- ▶ few control structures, pure (no side-effects) expressions

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Process meta-language

 $\operatorname{PROMELA}$ is a language for modeling concurrent systems

- multi-threaded
- synchronisation and message passing
- few control structures, pure (no side-effects) expressions
- data structures with finite and fixed bounds

What is **PROMELA** Not?

PROMELA is not a programming language

Very small language, not intended to program real systems

- No pointers
- No methods/procedures
- No libraries
- No GUI, no standard input
- No floating point types
- Fair scheduling policy (during verification)
- No data encapsulation
- Non-deterministic

Guarded Commands: Selection

```
active proctype P() {
   byte a = 5, b = 5;
   byte max, branch;
   if
      :: a >= b -> max = a; branch = 1
      :: a <= b -> max = b; branch = 2
   fi
```

}

Guarded Commands: Selection

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active proctype P() {
   byte a = 5, b = 5;
   byte max, branch;
   if
      :: a >= b -> max = a; branch = 1
      :: a <= b -> max = b; branch = 2
   fi
}
```

Observations

- Guards may "overlap" (more than one can be true at the same time)
- Any alternative whose guard is true is randomly selected
- ► When no guard true: process blocks until one becomes true

Guarded Commands: Repetition

```
active proctype P() { /* computes gcd */
int a = 15, b = 20;
do
    :: a > b -> a = a - b
    :: b > a -> b = b - a
    :: a == b -> break
    od
}
```

Guarded Commands: Repetition

```
active proctype P() { /* computes gcd */
int a = 15, b = 20;
do
    :: a > b -> a = a - b
    :: b > a -> b = b - a
    :: a == b -> break
    od
}
```

Observations

,

- Any alternative whose guard is true is randomly selected
- Only way to exit loop is via break or goto
- ► When no guard true: loop blocks until one becomes true

Sources of Non-Determinism

- 1. Non-deterministic choice of alternatives with overlapping guards
- 2. Scheduling of concurrent processes

$\label{eq:promela} Promela \ Computations$

```
1 active [2] proctype P() {
2    byte n;
3    n = 1;
4    n = 2
5 }
```

PROMELA Computations

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Notation

- Program pointer (line #) for each process in upper compartment
- Value of all variables in lower compartment

PROMELA Computations

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Notation

- Program pointer (line #) for each process in upper compartment
- Value of all variables in lower compartment

Computations are either infinite or terminating or blocking

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Interleaving

Can represent possible interleavings in a DAG



Usage Scenario of PROMELA

1. Model the essential features of a system in $\ensuremath{\operatorname{PROMELA}}$

- abstract away from complex (numerical) computations
 - make usage of non-deterministic choice of outcome
- replace unbounded data structures with finite approximations
- assume fair process scheduler

2. Select properties that the PROMELA model must satisfy

- Generic Properties
 - Mutal exclusion for access to critical resources
 - Absence of deadlock
 - Absence of starvation
- System-specific properties
 - Event sequences (e.g., system responsiveness)

Model Checker (MC) is designed to prove the user wrong.

MC does *not* try to prove correctness properties. It tries the opposite.

MC tuned to find counter example to correctness property.

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MC tuned to find counter example to correctness property.

Why can an MC also prove correctness properties?

MC's search for counter examples is exhaustive.

 \Rightarrow Finding no counter example proves stated correctness properties.

What does 'exhaustive search' mean here?



resolving non-determinism in all possible ways

What does 'exhaustive search' mean here?

exhaustive search

resolving non-determinism in all possible ways

For model checking **PROMELA** code, two kinds of non-determinism to be resolved:

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exhaustive search

resolving non-determinism in all possible ways

For model checking $\operatorname{PROMELA}$ code,

two kinds of non-determinism to be resolved:

- explicit, local: if/do statements
 - :: guardX -> ...
 - :: guardY -> ...
What does 'exhaustive search' mean here?



resolving non-determinism in all possible ways

For model checking $\operatorname{PROMELA}$ code,

two kinds of non-determinism to be resolved:

explicit, local:

 \mathbf{if}/\mathbf{do} statements

- :: guardX -> ...
- :: guardY -> ...

implicit, global:

scheduling of concurrent processes

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- simulating a model (randomly/interactively)
- generating a verifier

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exhaustively checks PROMELA model against correctness properties

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- in case the check is negative: generates a failing run of the model

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main functionality of SPIN:

- simulating a model (randomly/interactively/guided)
- generating a verifier

verifier generated by $\ensuremath{\mathrm{SPIN}}$ is a C program performing model checking:

- exhaustively checks PROMELA model against correctness properties
- in case the check is negative: generates a failing run of the model, to be simulated by SPIN







Given PROMELA model M, and correctness properties C_1, \ldots, C_n .

• Be R_M the set of all possible runs of M.

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- ▶ If *M* is not correct, then each $r \in (R_M \setminus (R_{M,C_1} \cap ... \cap R_{M,C_n}))$ is a counter example.

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But how to state Correctness Properties?

Stating Correctness Properties







stating properties within model using

assertion statements



stating properties within model using

- assertion statements
- meta labels
 - end labels
 - accept labels
 - progress labels



stating properties within model using

- assertion statements
- meta labels
 - end labels
 - accept labels
 - progress labels

stating properties outside model using

- never claims
- temporal logic formulas

Definition (Assertion Statements)

Assertion statements in PROMELA are statements of the form assert(*expr*) were *expr* is any PROMELA expression.

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```
...
stmt1;
assert(max == a);
stmt2;
...
```

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Typically, *expr* is of type bool.

assert(expr) can appear wherever a statement is expected.

```
...
stmt1;
assert(max == a);
stmt2;
...
if
if
:: b1 -> stmt3;
assert(x < y)
:: b2 -> stmt4
...
```

quoting from file max.pml:

```
/* after choosing a,b from {1,2,3} */
if
    :: a >= b -> max = b
    :: a <= b -> max = a
fi;
assert( max == (a>b -> a : b) )
```

Generate Verifier in C



Command Line Execution

Generate Verifier in C

> spin -a max.pml

SPIN generates Verifier in C, called pan.c

(plus helper files)

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Compile To Executable Verifier



Command Line Execution

compile to executable verifier

Compile To Executable Verifier



Command Line Execution compile to executable verifier > gcc -o pan pan.c

C compiler generates executable verifier pan

Compile To Executable Verifier





C compiler generates executable verifier pan

pan: historically "protocol analyzer", now "process analyzer"

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Run Verifier (= Model Check)



Command Line Execution

run verifier pan

>./pan or > pan

Run Verifier (= Model Check)



Command Line Execution

run verifier pan

- >./pan or > pan
 - prints "errors: 0"

Run Verifier (= Model Check)



Command Line Execution

run verifier pan

>./pan or > pan

▶ prints "errors: 0" ⇒ Correctness Property verified!
Run Verifier (= Model Check)



Command Line Execution

run verifier pan

- >./pan or > pan
 - prints "errors: 0", or
 - prints "errors: n" (n > 0)

Run Verifier (= Model Check)



Command Line Execution

run verifier pan

- >./pan or > pan
 - prints "errors: 0", or
 - ▶ prints "errors: n" $(n > 0) \Rightarrow$ counter example found!

Run Verifier (= Model Check)



Command Line Execution

run verifier pan

- >./pan or > pan
 - prints "errors: 0", or
 - ▶ prints "errors: n" (n > 0) ⇒ counter example found! records failing run in max.pml.trail

Model Checking: A Taster

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Guided Simulation

To examine failing run: employ simulation mode, "guided" by trail file.



Command Line Execution

inject a fault, re-run verification, and then:

> spin - t - p - l max.pml

Output of Guided Simulation

can look like:

Output of Guided Simulation

can look like:

assignments in the run

Output of Guided Simulation

can look like:

assignments in the run values of variables whenever updated

What did we do so far?

following whole cycle (most primitive example, assertions only)



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following whole cycle (most primitive example, assertions only)



Variables declared outside of the processes are global to all processes.

Variables declared inside a process are local to that processes. byte n;

```
proctype P(byte id; byte incr) {
    byte t;
    ...
}
n is global
t is local
```

pragmatics of modeling with global data:

shared memory of concurrent systems often modeled by global variables of numeric (or array) type

status of shared resources (printer, traffic light, ...) often modeled by global variables of Boolean or enumeration type (bool/mtype).

communication mediums of distributed systems often modeled by global variables of channel type (chan).

```
byte n = 0;
active proctype P() {
  n = 1;
  printf("Process_P,_n__=_%d\n", n)
}
```

```
byte n = 0;
active proctype P() {
  n = 1;
  printf("Process_P,_n_n_=,%d\n", n)
}
active proctype Q() {
  n = 2;
  printf("Process_Q,_n_=,%d\n", n)
}
```

```
byte n = 0;
active proctype P() {
  n = 1;
  printf("Process_P,__n__=_%d\n", n)
}
active proctype Q() {
  n = 2;
  printf("Process_Q,_n__=_%d\n", n)
}
```

how many outputs possible?

```
byte n = 0;
active proctype P() {
  n = 1;
  printf("Process_P,_n_n_=_%d\n", n)
}
active proctype Q() {
  n = 2;
  printf("Process_Q,_n_=_%d\n", n)
}
```

how many outputs possible?

different processes can interfere on global data

- 1. interleave0.pml SPIN simulation, SPINSPIDER automata + transition system
- interleave1.pml SPIN simulation, adding assertion, fine-grained execution model, model checking
- 3. interleave5.pml SPIN simulation, SPIN model checking, trail inspection

Show Mutual Exclusion

```
int critical = 0;
active proctype P() {
  do :: printf("Punon-criticaluactions\n");
         P_{in}CS = true;
         !Q in CS:
         /* begin critical section */
         critical++:
         printf("P_{||}uses_{||}shared_{||}recourses \n");
         assert(critical < 2);
         critical--:
         /* end critical section */
         P_in_CS = false
  od
}
active proctype Q() {
  ...correspondingly...
3
```

Verify Mutual Exclusion of this

SPIN (./pan -E) shows no assertion is violated \Rightarrow mutual exclusion is verified

Verify Mutual Exclusion of this

SPIN (./pan -E) shows no assertion is violated \Rightarrow mutual exclusion is verified

still SPIN (without -E) reports (invalid end state) \Rightarrow deadlock

Invalid End State:

- A process does not finish at its end
- Two or more inter-dependent processes do not finish at the end Real deadlock

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Find Deadlock with SPIN:

- Verify to produce a failing run trail
- Simulate to see how the processes get to the interlock
- Fix the code

Atomicity against Deadlocks

solution:

checking and setting the flag in one atomic step

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checking and setting the flag in one atomic step

```
atomic {
    !Q_in_CS;
    P_in_CS = true
}
```

Creates a channel, which is stored in *name*

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Messages communicated via the channel are *n*-tuples \in *type*₁ $\times ... \times$ *type*_n

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Can buffer up to *capacity* messages, if *capacity* $\geq 1 \Rightarrow$ *"buffered channel"*

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Messages communicated via the channel are *n*-tuples \in *type*₁ $\times ... \times$ *type*_n

Can buffer up to *capacity* messages, if *capacity* \geq 1 \Rightarrow *"buffered channel"*

The channel has *no* buffer, if *capacity* = 0 ⇒ *"rendezvous channel"*

chan ch = [2] of { mtype, byte, bool }

Creates a channel, which is stored in ch

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Messages communicated via ch are 3-tuples \in mtype \times byte \times bool Given, e.g., mtype {red, yellow, green}, an example message can be: green, 20, false

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Creates a channel, which is stored in ch

Messages communicated via ch are 3-tuples \in mtype \times byte \times bool Given, e.g., mtype {red, yellow, green}, an example message can be: green, 20, false

ch is a *buffered channel*, buffering up to 2 messages

Sending and Receiving

send statement has the form:

name ! $expr_1, \dots, expr_n$

- name: channel variable
- expr₁, ..., expr_n: sequence of expressions, where number and types match message type
- ▶ sends values of $expr_1$, ..., $expr_n$ as one message
- example: ch ! green, 20, false

receive statement has the form:

name ? var₁, ... , var_n

- name: channel variable
- var₁, ..., var_n: sequence of variables, where number and types match message type
- assigns values of message to var₁, ..., var_n
- example: ch ? color, time, flash

Rendezvous Channels

```
chan ch = [0] of { byte, byte };
```

```
/* global to make visible in SpinSpider */
byte hour, minute;
```

```
active proctype Sender() {
    printf("ready\n");
    ch ! 11, 45;
    printf("Sent\n")
}
```

```
active proctype Receiver() {
    printf("steady\n");
    ch ? hour, minute;
    printf("Received\n")
}
```

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    printf("steady\n");
    ch ? hour, minute;
    printf("Received\n")
}
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```
Which interleavings can occur?
```

Rendezvous Channels

```
chan ch = [0] of { byte, byte };
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/* global to make visible in SpinSpider */
byte hour, minute;
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active proctype Sender() {
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}
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active proctype Receiver() {
    printf("steady\n");
    ch ? hour, minute;
    printf("Received\n")
}
```

Which interleavings can occur? \Rightarrow ask SPINSPIDER
through JSPIN: SPINSPIDER on ReadySteady.pml

Rendezvous are Synchronous

On a rendezvous channel:

transfer of message from sender to receiver is synchronous, i.e., one single operation

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transfer of message from sender to receiver is synchronous, i.e., one single operation



```
chan request = [0] of { mtype };
chan reply = [0] of { mtype };
mtype = { nice, rude };
active proctype Server() {
 mtype msg;
 do :: request ? msg; reply ! msg
  od
}
active proctype NiceClient() {
 mtype msg;
  request ! nice; reply ? msg;
}
active proctype RudeClient() {
 mtype msg;
  request ! rude; reply ? msg
}
```

```
chan request = [0] of { mtype };
chan reply = [0] of { mtype };
mtype = { nice, rude };
active proctype Server() {
 mtype msg;
 do :: request ? msg; reply ! msg
  od
}
active proctype NiceClient() {
  mtype msg;
  request ! nice; reply ? msg;
  assert(msg == nice)
}
active proctype RudeClient() {
 mtype msg;
  request ! rude; reply ? msg
}
```

```
chan request = [0] of { mtype };
chan reply = [0] of { mtype };
mtype = { nice, rude };
active proctype Server() {
 mtype msg;
 do :: request ? msg; reply ! msg
  od
}
active proctype NiceClient() {
  mtype msg;
  request ! nice; reply ? msg;
  assert(msg == nice)
                                  Is the assertion valid?
}
active proctype RudeClient() {
 mtype msg;
  request ! rude; reply ? msg
}
```

```
chan request = [0] of { mtype };
chan reply = [0] of { mtype };
mtype = { nice, rude };
active proctype Server() {
 mtype msg;
 do :: request ? msg; reply ! msg
 od
}
active proctype NiceClient() {
  mtype msg;
  request ! nice; reply ? msg;
  assert(msg == nice)
                                  Is the assertion valid? Ask SPIN.
}
active proctype RudeClient() {
 mtype msg;
  request ! rude; reply ? msg
}
```

```
active [2] proctype Server() {
 mtype msg;
 do :: request ? msg; reply ! msg
  od
}
active proctype NiceClient() {
  mtype msg;
  request ! nice; reply ? msg;
}
active proctype RudeClient() {
 mtype msg;
  request ! rude; reply ? msg
}
```

```
active [2] proctype Server() {
 mtype msg;
 do :: request ? msg; reply ! msg
  od
}
active proctype NiceClient() {
  mtype msg;
  request ! nice; reply ? msg;
  assert(msg == nice)
}
active proctype RudeClient() {
  mtype msg;
  request ! rude; reply ? msg
}
```

```
active [2] proctype Server() {
 mtype msg;
 do :: request ? msg; reply ! msg
  od
}
active proctype NiceClient() {
  mtype msg;
  request ! nice; reply ? msg;
                                   And here?
  assert(msg == nice)
}
active proctype RudeClient() {
  mtype msg;
  request ! rude; reply ? msg
}
```

```
active [2] proctype Server() {
 mtype msg;
 do :: request ? msg; reply ! msg
  od
}
active proctype NiceClient() {
  mtype msg;
  request ! nice; reply ? msg;
  assert(msg == nice)
                                   And here? Analyse with SPIN.
}
active proctype RudeClient() {
  mtype msg;
  request ! rude; reply ? msg
}
```

One way to fix the protocol:

clients declare local reply channel + send it to server

Sending Channels via Channels

```
mtype = { nice, rude };
chan request = [0] of { mtype, chan };
active [2] proctype Server() {
 mtype msg; chan ch;
 do :: request ? msg, ch;
        ch ! msg
 od
}
active proctype NiceClient() {
  chan reply = [0] of { mtype }; mtype msg;
  request ! nice, reply; reply ? msg;
  assert( msg == nice )
}
active proctype RudeClient() {
  chan reply = [0] of { mtype }; mtype msg;
  request ! rude, reply; reply ? msg
}
```

Sending Channels via Channels

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mtype = { nice, rude };
chan request = [0] of { mtype, chan };
active [2] proctype Server() {
 mtype msg; chan ch;
 do :: request ? msg, ch;
        ch ! msg
 od
}
active proctype NiceClient() {
  chan reply = [0] of { mtype }; mtype msg;
  request ! nice, reply; reply ? msg;
  assert( msg == nice )
}
active proctype RudeClient() {
  chan reply = [0] of { mtype }; mtype msg;
  request ! rude, reply; reply ? msg
}
      verify with SPIN
```

Recapitulation: Formalisation













Formalisation: Syntax, Semantics, Proving









Transition systems (aka Kripke Structures)





Transition systems (aka Kripke Structures)



- Each state s_i has its own propositional interpretation I_i
 - Convention: list values of variables in ascending lexicographic order
- Computations, or runs, are infinite paths through states
 - Intuitively 'finite' runs modelled by looping on final states
- In general, infinitely many different runs possible
- How to express (for example) that p changes its value infinitely often in each run?



(Linear) Temporal Logic

An extension of propositional logic that allows to specify properties of all runs

(Linear) Temporal Logic—Syntax

An extension of propositional logic that allows to specify properties of all runs

Syntax

Based on propositional signature and syntax

Extension with three connectives:

Always If ϕ is a formula then so is $\Box \phi$

Eventually If ϕ is a formula then so is $\Diamond \phi$

Until If ϕ and ψ are formulas then so is $\phi \mathcal{U}\psi$





Given a finite alphabet (vocabulary) Σ A word $w \in \Sigma^*$ is a finite sequence

$$w = a_o \cdots a_n$$

with $a_i \in \Sigma, i \in \{0, \dots, n\}$ $\mathcal{L} \subseteq \Sigma^*$ is called a language Given a finite alphabet (vocabulary) Σ An ω -word $w \in \Sigma^{\omega}$ is an infinite sequence

 $w = a_o \cdots a_k \cdots$

with $a_i \in \Sigma, i \in \mathbb{N}$ $\mathcal{L}^{\omega} \subseteq \Sigma^{\omega}$ is called an ω -language

Büchi Automaton

Definition (Büchi Automaton)

A (non-deterministic) Büchi automaton over an alphabet Σ consists of a

- finite, non-empty set of locations Q
- ▶ a non-empty set of initial/start locations $I \subseteq Q$
- ▶ a set of accepting locations $F = \{F_1, \ldots, F_n\} \subseteq Q$
- a transition relation $\delta \subseteq Q \times \Sigma \times Q$

Example

$$\Sigma = \{a, b\}, Q = \{q_1, q_2, q_3\}, I = \{q_1\}, F = \{q_2\}$$



Büchi Automaton—Executions and Accepted Words

Definition (Execution)

Let $\mathcal{B} = (Q, I, F, \delta)$ be a Büchi automaton over alphabet Σ . An execution of \mathcal{B} is a pair (w, v), with

•
$$w = a_o \cdots a_k \cdots \in \Sigma^{\omega}$$

•
$$v = q_o \cdots q_k \cdots \in Q^\omega$$

where $q_0 \in I$, and $(q_i, a_i, q_{i+1}) \in \delta$, for all $i \in \mathbb{N}$

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Definition (Accepted Word)

A Büchi automaton \mathcal{B} accepts a word $w \in \Sigma^{\omega}$, if there exists an execution (w, v) of \mathcal{B} where some accepting location $f \in F$ appears infinitely often in v

Let $\mathcal{B} = (Q, I, F, \delta)$ be a Büchi automaton, then

 $\mathcal{L}^{\omega}(\mathcal{B}) = \{ w \in \Sigma^{\omega} | w \in \Sigma^{\omega} \text{ is an accepted word of } \mathcal{B} \}$

denotes the ω -language recognised by \mathcal{B} .
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An ω -language for which an accepting Büchi automaton exists is called ω -regular language.

Example, ω -Regular Expression

Which language is accepted by the following Büchi automaton?



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Solution: $(a + b)^* (ab)^\omega$	$[NB: (ab)^\omega = a(ba)^\omega]$
-----------------------------------	------------------------------------

Example, ω -Regular Expression

Which language is accepted by the following Büchi automaton?



Solution: $(a + b)^* (ab)^{\omega}$ [NB: $(ab)^{\omega} = a(ba)^{\omega}$]

 $\omega\text{-}\mathrm{regular}$ expressions like standard regular expression

ab a then b

a + b a or b

- a* arbitrarily, but finitely often a
- **new:** a^{ω} infinitely often a

Formal Verification: Model Checking



Check whether a formula is valid in all runs of a transition system Given a transition system \mathcal{T} (e.g., derived from a PROMELA program) Verification task: is the LTL formula ϕ satisfied in all runs of \mathcal{T} , i.e.,

 $\mathcal{T} \models \phi$?

$$\mathcal{T} \models \phi$$
 ?

1. Represent transition system \mathcal{T} as Büchi automaton $\mathcal{B}_{\mathcal{T}}$ such that $\mathcal{B}_{\mathcal{T}}$ accepts exactly those words corresponding to runs through \mathcal{T}

$$\mathcal{T} \models \phi$$
 ?

- 1. Represent transition system \mathcal{T} as Büchi automaton $\mathcal{B}_{\mathcal{T}}$ such that $\mathcal{B}_{\mathcal{T}}$ accepts exactly those words corresponding to runs through \mathcal{T}
- 2. Construct Büchi automaton $\mathcal{B}_{\neg\phi}$ for negation of formula ϕ

$$\mathcal{T} \models \phi$$
 ?

- 1. Represent transition system \mathcal{T} as Büchi automaton $\mathcal{B}_{\mathcal{T}}$ such that $\mathcal{B}_{\mathcal{T}}$ accepts exactly those words corresponding to runs through \mathcal{T}
- Construct Büchi automaton B_{¬φ} for negation of formula φ
 If

$$\mathcal{L}^\omega(\mathcal{B}_\mathcal{T})\cap\mathcal{L}^\omega(\mathcal{B}_{\neg\phi})=\emptyset$$

then ϕ holds.

$$\mathcal{T} \models \phi$$
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lf

$$\mathcal{L}^\omega(\mathcal{B}_\mathcal{T})\cap\mathcal{L}^\omega(\mathcal{B}_{\neg\phi})
eq\emptyset$$

then each element of the set is a counterexample for ϕ .

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To check $\mathcal{L}^{\omega}(\mathcal{B}_{\mathcal{T}}) \cap \mathcal{L}^{\omega}(\mathcal{B}_{\neg\phi})$ construct intersection automaton and search for cycle through accepting state

Representing a Model as a Büchi Automaton

First Step: Represent transition system \mathcal{T} as Büchi automaton $\mathcal{B}_{\mathcal{T}}$ accepting exactly those words representing a run of \mathcal{T}

Example

```
active proctype P () {
do
    :: atomic {
      !wQ; wP = true
    };
    Pcs = true;
    atomic {
      Pcs = false;
      wP = false
    }
od }
```

First location skipped and second made ${\bf atomic}$ just to keep automaton small; similar code for process Q

Model Checking: A Taster

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Model Checking: A Taster

Representing a Model as a Büchi Automaton

First Step: Represent transition system \mathcal{T} as Büchi automaton $\mathcal{B}_{\mathcal{T}}$ accepting exactly those words representing a run of \mathcal{T}

Example



The property we want to check is $\phi = \Box \neg Pcs$ (which does not hold)

Büchi Automaton $B_{\neg\phi}$ for $\neg\phi$

Second Step:

Construct Büchi Automaton corresponding to negated LTL formula

 $\mathcal{T} \models \phi$ holds iff there is no accepting run of \mathcal{T} for $\neg \phi$

Simplify $\neg \phi = \neg \Box \neg Pcs = \Diamond Pcs$

Büchi Automaton $B_{\neg\phi}$ for $\neg\phi$

Second Step:

Construct Büchi Automaton corresponding to negated LTL formula

 $\mathcal{T} \models \phi \text{ holds iff there is no accepting run of } \mathcal{T} \text{ for } \neg \phi$ Simplify $\neg \phi = \neg \Box \neg Pcs = \Diamond Pcs$

Büchi Automaton $\mathcal{B}_{\neg\phi}$

$$\mathcal{P} = \{ wP, wQ, Pcs, Qcs \}, \ \Sigma = 2^{\mathcal{P}}$$



$$\Sigma_{\textit{Pcs}} = \{ \textit{I} | \textit{I} \in \Sigma, \textit{Pcs} \in \textit{I} \}, \quad \Sigma_{\textit{Pcs}}^{c} = \Sigma - \Sigma_{\textit{Pcs}}$$

Third Step: $\mathcal{L}^{\omega}(\mathcal{B}_{\mathcal{T}}) \cap \mathcal{L}^{\omega}(\mathcal{B}_{\neg \phi}) = \emptyset$?

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 ?



$$\begin{array}{ll} \mathsf{Third Step:} \quad \mathcal{L}^\omega(\mathcal{B}_\mathcal{T}) \cap \mathcal{L}^\omega(\mathcal{B}_{\neg \phi}) \neq \emptyset \end{array}$$

Counterexample



$$\begin{array}{ll} \mathsf{Third} \ \mathsf{Step:} \quad \mathcal{L}^\omega(\mathcal{B}_\mathcal{T}) \cap \mathcal{L}^\omega(\mathcal{B}_{\neg \phi}) \neq \emptyset \end{array}$$

Counterexample Construction of intersection automaton



Applying Temporal Logic to Critical Section Problem

```
We want to verify [](critical<=1) as a correctness property of:
int critical = 0;
```

```
active proctype P() {
  do :: printf("P_non-critical_actions\n");
         atomic {
           !Q_in_CS;
           P_{in}CS = true
         }
         critical++:
         printf("P_{||}uses_{||}shared_{||}recourses \n");
         critical--:
         P_in_CS = false
  od
}
active proctype Q() {
```

```
\dots correspondingly \dots
```

Model Checking a Safety Property with $\rm _{JSPIN}$

edit 'LTL fomula' field of $\rm _JSPIN$

- 1. load PROMELA file in JSPIN (not necessarily containing ltl ...)
- 2. enter [] (critical <= 1) in LTL text field of JSPIN
- 3. select Translate to create a 'never claim', corresponding to the negation of the formula
- 4. ensure Safety is selected
- 5. select Verify
- 6. (if necessary) select Stop to terminate too long verification

Demo: csGhostLTL.pml

Theory behind SPIN

1. Represent the interleaving of all processes as a single automaton (only one process advances in each step), called \mathcal{M}

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then ϕ holds in \mathcal{M} , otherwise we have a counterexample

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 To check L^ω(M) ∩ L^ω(NC_{¬φ}) construct intersection automaton (both automata advance in each step) and search for accepting run

Temporal Model Checking without Ghost Variables

```
We want to verify mutual exclusion without using ghost variables
bool inCriticalP = false, inCriticalQ = false;
active proctype P() {
  do :: atomic {
           !inCriticalQ;
           inCriticalP = true
        }
        /* critical activity */
cs:
        inCriticalP = false
  od
}
/* similar for process Q with same label cs: */
ltl m { []!(P@cs && Q@cs) }
```

Demo: noGhost.pml

- SPIN targets software, instead of hardware verification ("Software Engineering using Formal Methods")
- 2001 ACM Software Systems Award (other winning software systems include: Unix, TCP/IP, WWW, Tcl/Tk, Java)
- used for safety critical applications
- distributed freely as research tool, well-documented, actively maintained, large user-base in academia and in industry
- ▶ annual SPIN user workshops series held since 1995
- **•** based on standard theory of (ω -)automata and linear temporal logic

In order to

- ► learn more about Software Model Checking (SPIN)
- learn about Deductive Verification (KeY) of
 - a real-world language, here Java (without abstraction)
 - w.r.t. more complex, problem specific properties

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you are welcome to my course:

Software Engineering using Formal Methods