Software Engineering using Formal Methods

Proof Obligations

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This Part

making the connection between

JML

and

Dynamic Logic / KeY

- generating,
- understanding,
- and proving

DL proof obligations from JML specifications

Tutorial Example

we follow 'KeY Quicktour for JML' (cited below as [KQJ])

paper + sources:
see 'KeY Quicktour' on course page, under 'Links, Papers, and Software'

scenario: simple `PayCard`

Inspecting JML Specification

inspect `quicktour/jml/paycard/PayCard.java`

follow [KQJ, 2.2]

New JML Feature I: Nested Specification Cases

method charge() has nested specification case: X

```
@ public normal_behavior
@ requires amount > 0;
@ {|
@   requires amount + balance < limit && isValid()==true;
@   ensures \result == true;
@   ensures balance == amount + \old(balance);
@   assignable balance;
@
@   also
@
@   requires amount + balance >= limit;
@   ensures \result == false;
@   ensures unsuccessfulOperations
@           == \old(unsuccessfulOperations) + 1;
@   assignable unsuccessfulOperations;
@ |}
```

Nested Specification Cases

nested specification cases allow to factor out common preconditions

```
@ public normal_behavior
@ requires R;
@ {|
@   requires R1;
@   ensures E1;
@   assignable A1;
@
@   also
@
@   requires R2;
@   ensures E2;
@   assignable A2;
@ |}
```

expands to ... (next page)

Nested Specification Cases

(previous page) ... expands to

```
@ public normal_behavior
@ requires R;
@ requires R1;
@ ensures E1;
@ assignable A1;
@
@ also
@
@ public normal_behavior
@ requires R;
@ requires R2;
@ ensures E2;
@ assignable A2;
```

Nested Specification Cases

```
@ public normal_behavior
@ requires amount > 0;
@ {|
@   requires amount + balance < limit && isValid()==true;
@   ensures \result == true;
@   ensures balance == amount + \old(balance);
@   assignable balance;
@
@   also
@
@   requires amount + balance >= limit;
@   ensures \result == false;
@   ensures unsuccessfulOperations
@           == \old(unsuccessfulOperations) + 1;
@   assignable unsuccessfulOperations;
@ |}
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expands to ... (next page)

Nested Specification Cases

(previous page) ... expands to

```
@ public normal_behavior
@ requires amount > 0;
@ requires amount + balance < limit && isValid()==true;
@ ensures \result == true;
@ ensures balance == amount + \old(balance);
@ assignable balance;
@
@ also
@
@ public normal_behavior
@ requires amount > 0;
@ requires amount + balance >= limit;
@ ensures \result == false;
@ ensures unsuccessfulOperations
@         == \old(unsuccessfulOperations) + 1;
@ assignable unsuccessfulOperations;
```

Recall: `pure` vs. `assignable \nothing`

method `charge()` has exceptional behavior case:

```
@ public exceptional_behavior
@   requires amount <= 0;
@   assignable \nothing;
```

`assignable \nothing` prohibits side effects

difference to **`pure`**:

- **`pure`** is method-global, also prohibits non-termination & exceptions
- **`assignable`** clause is local to specification case
- **`pure`** not usable in this particular context

Generating Proof Obligations (POs)

generate **EnsuresPost** PO for normal behavior of `charge()`

follow [KQJ, 3.1+3.2]

summary:

- start KeY prover
- in `quicktour/jml`, open `paycard`
- select `paycard` > `PayCard` > `charge` and **EnsuresPost**
- inspect **Assumed Invariants**

  assuming less invariants:

  - is fully sound
  - can compromise provability

  sometimes invariants of *other* classes also needed (select class+inv.)
- select contract which **modifies** `balance`

  (in JML: **`modifies`** synonymous for **`assignable`**)
- **Current Goal** pane displays proof obligation as DL sequent

Generating Proof Obligations

for loading more proof obligations:
re-open **Proof Obligation Browser** under **Tools** menu (or **Ctrl-B**)

generate **EnsuresPost** PO for normal behavior of `isValid()`

generate **EnsuresPost** PO for exceptional behavior of `charge()`

generate **PreservesOwnInv** PO for `charge()`

expressing that `charge()` preserves all invariants (of its own class)

follow [KQJ, 4.3.1+4.3.2]

Translating JML to POs in DL

in the following:

principles of translating JML to proof obligations in DL

- issues in translating arithmetic expressions
- translating **`this`**
- identifying the method's implementation
- translating boolean JML expressions to first-order logic formulas
- translating preconditions
- translating class invariants
- translating postconditions
- storing **`\old`** fields prior to method invocation
- storing actual parameters prior to method invocation
- expressing that 'exceptions are (not) thrown'
- *putting everything together*

Translating JML to POs in DL

WARNING:

following presentation is

- incomplete
- not fully precise
- simplifying
- omitting details/complications
- deviating from exact implementation in KeY

aim of the following:

enable you to read/understand proof obligations

(notational remark: stick to ASCII syntax of KeY logic in this lecture)

Issues on Translating Arithmetic Expressions

often:

- KeY replaces arithmetic JAVA operators by generalized operators, generic towards various integer semantics (JAVA, Math).
  example: "`+`" becomes "`javaAddInt`"
- KeY inserts casts like `(jint)`,
  needed for type hierarchy among primitive types.
  example: "`0`" becomes "`(jint)(0)`"

(no need to memorize this)

Translating `this`

both

- explicit
- implicit

`this` reference translated to `self`

e.g., given class

```
public class MyClass {
  ...
  private int f;
  ...
}
```

- `f` translated to `self.f`
- `this.f` translated to `self.f`

Identifying the Method's Implementation

JAVA's dynamic dispatch selects a method's implementation *at runtime*

for a method call `m(`*args*`)`,
KeY models selection of implementation from `package.Class` by
`m(`*args*`)@package.Class`

example:

`charge(x)@paycard.PayCard`
executes class `paycard.PayCard`'s implementation of method call
`charge(x)`

Translating Boolean JML Expressions

first-order logic treated fundamentally different in JML and KeY logic

JML

- formulas no separate syntactic category
- instead:
  JAVA's **`boolean`** expressions extended with first-order concepts
  (i.p. quantifiers)

KeY logic

- formulas and expressions completely separate
- truth constants **`true`**, **`false`** are formulas,
  `boolean` constants `TRUE`, `FALSE` are expressions
- atomic formulas take expressions as arguments; e.g.:
  - `x - y < 5`
  - `b = TRUE`

$\mathcal{F}$ Translates `boolean` JML Expressions to Formulas

$\mathcal{F}$(`v`)	=	`v = TRUE`
$\mathcal{F}$(`f`)	=	$\mathcal{T}$(`f`) `= TRUE`
$\mathcal{F}$(`m()`)	=	$\mathcal{T}$(`m`)`() = TRUE`
$\mathcal{F}$(`!b_0`)	=	`!`$\mathcal{F}$(`b_0`)
$\mathcal{F}$(`b_0 && b_1`)	=	$\mathcal{F}$(`b_0`) `&` $\mathcal{F}$(`b_1`)
$\mathcal{F}$(`b_0 || b_1`)	=	$\mathcal{F}$(`b_0`) `|` $\mathcal{F}$(`b_1`)
$\mathcal{F}$(`b_0 ==> b_1`)	=	$\mathcal{F}$(`b_0`) `->` $\mathcal{F}$(`b_1`)
$\mathcal{F}$(`b_0 <==> b_1`)	=	$\mathcal{F}$(`b_0`) `<->` $\mathcal{F}$(`b_1`)
$\mathcal{F}$(`e_0 == e_1`)	=	$\mathcal{E}$(`e_0`) `=` $\mathcal{E}$(`e_1`)
$\mathcal{F}$(`e_0 != e_1`)	=	`!`$\mathcal{E}$(`e_0`) `=` $\mathcal{E}$(`e_1`)
$\mathcal{F}$(`e_0 >= e_1`)	=	$\mathcal{E}$(`e_0`) `>=` $\mathcal{E}$(`e_1`)

`v`/`f`/`m()` **`boolean`** variables/fields/pure methods

`b_0`, `b_1` **`boolean`** JML expressions

`e_0`, `e_1` JAVA expressions

$\mathcal{T}$ may add '`self.`' or '`@ClassName`' (see pp. 16, 17)

$\mathcal{E}$ may add casts, transform operators (see p. 15)

$\mathcal{F}$ Translates `boolean` JML Expressions to Formulas

$\mathcal{F}($`(\forall T x; e_0)`$)$ = `\forall T x;`
`!x = null ->` $\mathcal{F}($`e_0`$)$

$\mathcal{F}($`(\exists T x; e_0)`$)$ = `\exists T x;`
`!x = null &` $\mathcal{F}($`e_0`$)$

$\mathcal{F}($`(\forall T x; e_0; e_1)`$)$ = `\forall T x;`
`!x = null &` $\mathcal{F}($`e_0`$)$
`->` $\mathcal{F}($`e_1`$)$

$\mathcal{F}($`(\exists T x; e_0; e_1)`$)$ = `\exists T x;`
`!x = null`
`&` $\mathcal{F}($`e_0`$)$ `&` $\mathcal{F}($`e_1`$)$

Translating Preconditions

if selected contract *Contr* has preconditions

```
@ requires b_1;
@ ...
@ requires b_n;
```

they are translated to

$$\mathcal{PRE}(\mathit{Contr}) = \mathcal{F}(\texttt{b\_1})\ \texttt{\&}\ \ldots\ \texttt{\&}\ \mathcal{F}(\texttt{b\_n})$$

Translating Class Invariants

the invariant

```
class C {
   ...
   //@ invariant inv_i;
   ...
}
```

is translated to

$$\mathcal{INV}(\texttt{inv\_i})$$

$$=$$

$$\texttt{\textbackslash forall C o; ((o.<created> = TRUE \& !o = null) ->} \\ \{\texttt{self:=o}\}\mathcal{F}(\texttt{inv\_i}))$$

Translating Postconditions

if selected contract *Contr* has postconditions

```
@ ensures b_1;
@ ...
@ ensures b_n;
```

they are translated to

$$\mathcal{POST}(\mathit{Contr}) \\ = \\ \mathcal{F}(\texttt{b\_1})\ \texttt{\&}\ \ldots\ \texttt{\&}\ \mathcal{F}(\texttt{b\_n})$$

special treatment of expressions in post-condition: see next slide

Translating Expressions in Postconditions

below, we assume the following assignable clause

```
@ assignable <assignable_fields>;
```

translating expressions in postconditions (interesting cases only):

$$\mathcal{E}(\texttt{\textbackslash result}) = \texttt{result}$$

$$\mathcal{E}(\texttt{\textbackslash old(e)}) = \mathcal{E}_{old}(\texttt{e})$$

$\mathcal{E}_{old}$ defined like $\mathcal{E}$, with the exception of:

$$\begin{array}{lcl} \mathcal{E}_{old}(\texttt{e.f}) & = & \texttt{fAtPre}(\mathcal{E}_{old}(\texttt{e})) \\ \mathcal{E}_{old}(\texttt{f}) & = & \texttt{fAtPre(self)} \end{array}$$

for f $\in$ *<assignable_fields>*

'`fAtPre`' intuitively refers to field '`f`' *in the pre-state*
But the logic does not know. Must be expressed in formula (next slide).

Storing Pre-State of a Field

given an **assignable** field f of class C

```
class C {
  ...
  private T f;
  ...
}
```

translation of postcondition replaces f in **\old**(...) by fAtPre (p. 24)
left to do: store pre-state values of f in fAtPre

$$\mathcal{STORE}(\texttt{f})$$
$$=$$
$$\texttt{\textbf{\textbackslash for} C o; fAtPre(o) := o.f}$$

note: not a formula, but a quantified update
(more proper explanation next lecture)

Storing Pre-State of All Assignable Fields

if selected contract *Contr* has assignable clause:

```
@ assignable f_1, ..., f_n;
```

then pre-state of *all* assignable fields can be stored by *one* parallel update:

$$\mathcal{STORE}(\mathit{Contr}) \\ = \\ \{\ \mathcal{STORE}(\texttt{f\_1})\ ||\ \ldots\ \ ||\ \mathcal{STORE}(\texttt{f\_n})\ \}$$

Expressing Normal Termination

how can you express in DL:
method call `m()` will not throw an exception
(if method body from class `C` in package `p` is executed)

```
\<{ exc = null;
    try {
      m()@p.C;
    } catch (Throwable e) {
      exc = e;
    }
  }\> exc = null
```

note difference:

- Java assignments
- equation, i.e., formula

Expressing Exceptional Termination

how can you express in DL:
method call `m()` will throw an exception
(if method body from class `C` in package `p` is executed)

```
\<{ exc = null;
    try {
      m()@p.C;
    } catch (Throwable e) {
      exc = e;
    }
  }\> !exc = null & <exc has right type>
```

PO for Normal Behavior Contract

PO for a normal behavior contract *Contr* for `void` method `m()`, with chosen assumed invariants `inv_1`, ..., `inv_n`

```
==>
      INV(inv_1)
    & ...
    & INV(inv_n)
    & PRE(Contr)
 -> STORE(Contr)
      \<{ exc = null;
          try {
            m()@p.C;
          } catch (Throwable e) {
            exc = e;
          }
        }\> exc = null & POST(Contr)
```

PO for Normal Behavior Allowing Non-Termination

PO for a normal behavior contract *Contr* for method `m()`,
where *Contr* has clause **`diverges true;`**

```
==>
      𝓘𝓝𝓥(inv_1)
    & ...
    & 𝓘𝓝𝓥(inv_n)
    & 𝓟𝓡𝓔(Contr)
 -> 𝓢𝓣𝓞𝓡𝓔(Contr)
      \[{ exc = null;
          try {
            m()@p.C;
          } catch (Throwable e) {
            exc = e;
          }
        }\] exc = null  & 𝓟𝓞𝓢𝓣(Contr)
```

PO for Normal Behavior of Non-Void Method

PO for a normal behavior contract *Contr* for **non-void** method `m()`,

```
==>
      INV(inv_1)
    & ...
    & INV(inv_n)
    & PRE(Contr)
 -> STORE(Contr)
      \<{ exc = null;
          try {
            result = m()@p.C;
          } catch (Throwable e) {
            exc = e;
          }
        }\> exc = null & POST(Contr)
```

recall: $\mathcal{POST}$(*Contr*) translates `\result` to `result` (p. 24)

PO for Preserving Invariants

assume method `m()` has contracts $Contr_1$, ..., $Contr_j$

PO stating that:

Invariants `inv_1`, ..., `inv_n` are preserved
in all cases covered by a contracts.

```
==>
     INV(inv_1) & ... & INV(inv_n)
   & ( PRE(Contr_1) | ...  | PRE(Contr_1)  )
 -> \[{ exc = null;
        try {
          m()@p.C;
        } catch (Throwable e) {
          exc = e;
        }
     }\] INV(inv_1) & ... & INV(inv_n)
```

Examples

don't fit on slide: execute quicktour with KeY instead

Literature for this Lecture

Essential

KeY Quicktour see course page, under 'Links, Papers, and Software'