

# Parallel Functional Programming

## Lecture 8

### Data Parallelism II

Mary Sheeran

(with thanks to Ben Lippmeier for  
borrowed slides)

<http://www.cse.chalmers.se/edu/course/pfp>

# Data parallelism

Perform *same* computation on a collection of *differing* data values

examples: HPF (High Performance Fortran)  
CUDA

Both support only **flat data parallelism**

Flat : each of the individual computations on (array) elements is sequential

those computations don't need to communicate

parallel computations don't spark further parallel computations

# Regular, Shape-polymorphic, Parallel Arrays in Haskell

Gabriele Keller<sup>†</sup>   Manuel M. T. Chakravarty<sup>†</sup>   Roman Leshchinskiy<sup>†</sup>  
Simon Peyton Jones<sup>‡</sup>   Ben Lippmeier<sup>†</sup>

<sup>†</sup>Computer Science and Engineering, University of New South Wales  
{keller,chak,rl,benl}@cse.unsw.edu.au

<sup>‡</sup>Microsoft Research Ltd, Cambridge  
simonpj@microsoft.com

API for purely functional, collective operations over dense, rectangular, multi-dimensional arrays supporting shape polymorphism

ICFP 2010

# Ideas

Purely functional array interface using collective (whole array) operations like map, fold and permutations can

- combine efficiency and clarity
- focus attention on structure of algorithm, away from low level details

Influenced by work on algorithmic skeletons based on Bird  
Meertens formalism

Provides shape polymorphism not in a standalone specialist compiler like SAC, but using the Haskell type system

# terminology

## **Regular arrays**

dense, rectangular, most elements non-zero

## **shape polymorphic**

functions work over arrays of arbitrary dimension

# terminology

## Regular arrays

dense, rectan

## shape polym

functions wo

note: the arrays are purely functional and immutable

All elements of an array are demanded at once -> parallelism

P processing elements, n array elements =>  $n/P$  consecutive elements on each proc. element

# version

I use Repa 2.1.1.5 (which works with the GHC that you get with the current Haskell platform)

If you have GHC 7.4 installed, you can use a later Repa, which has more array types (and doubtless better performance)

# example

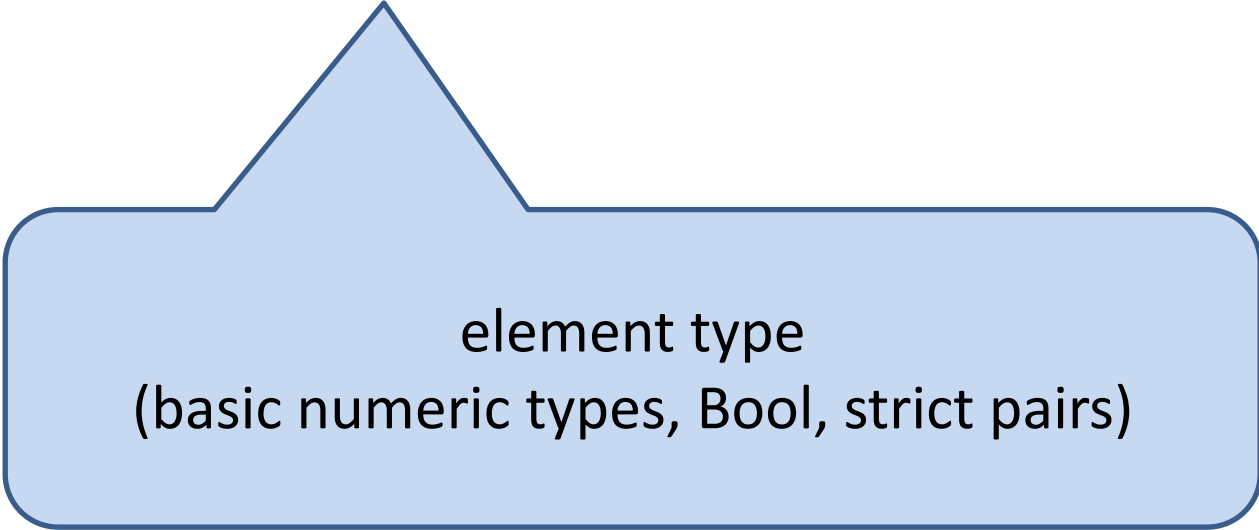
```
import Data.Array.Repa as A
```

```
transpose2D :: Elt e => Array DIM2 e -> Array DIM2 e
```



# example

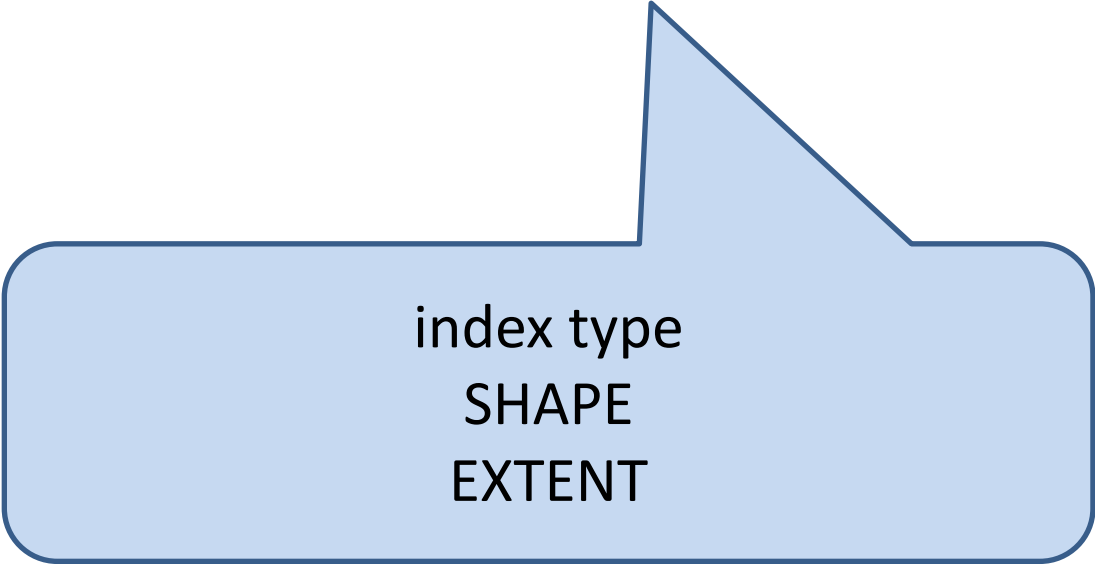
```
transpose2D :: Elt e => Array DIM2 e -> Array DIM2 e
```



element type  
(basic numeric types, Bool, strict pairs)

# example

`transpose2D :: Elt e => Array DIM2 e -> Array DIM2 e`



index type  
SHAPE  
EXTENT

# example

`transpose2D :: Elt e => Array DIM2 e -> Array DIM2 e`

DIM0 = Z (scalar)  
DIM1 = DIM0 :: Int  
DIM2 = DIM1 :: Int

# snoc lists

Haskell lists are cons lists

`1:2:3:[]` is the same as `[1,2,3]`

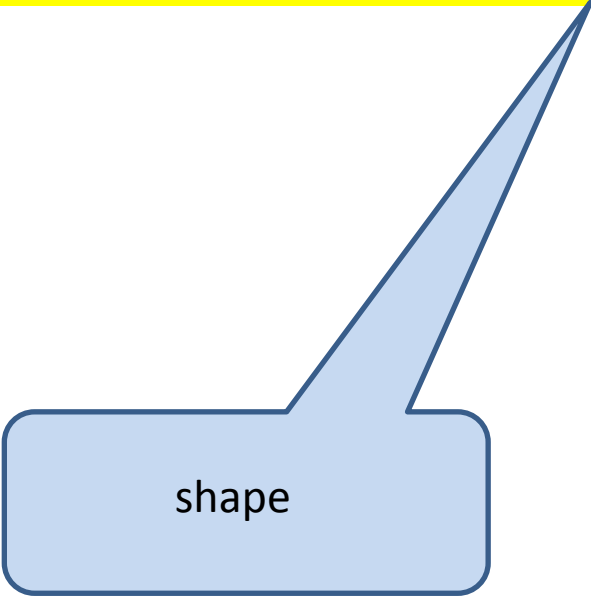
Repa uses snoc lists at type level for shape types  
and at value level for shapes

`DIM2 = Z :: Int :: Int` is a shape type

`Z :: i :: j` read as `(i,j)` is an index into a two dim. array

# examples

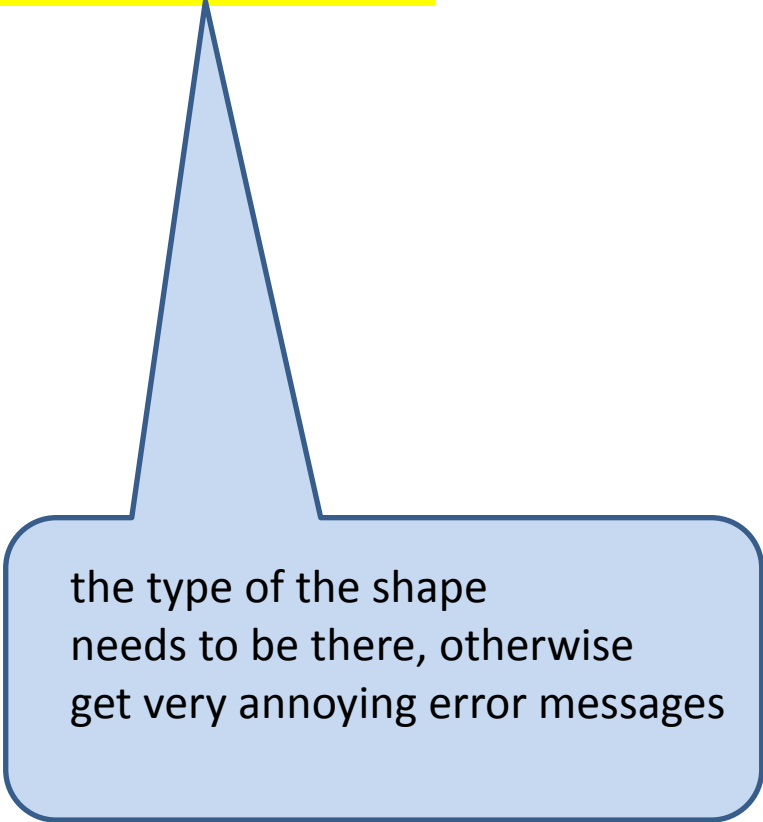
```
*Main> let y = fromList ((Z :: 2 :: 3 :: 3) :: DIM3) [1..18]
```



shape

# examples

```
*Main> let y = fromList ((Z :: 2 :: 3 :: 3) :: DIM3) [1..18]
```



the type of the shape  
needs to be there, otherwise  
get very annoying error messages

# examples

```
*Main> let y = fromList ((Z :: 2 :: 3 :: 3) :: DIM3) [1..18]
```

```
*Main> y
```

```
Array (Z :: 2 :: 3 :: 3) [1.0,2.0,3.0,4.0,5.0,6.0,7.0,8.0,9.0,10.0,11.0,12.0,13.0,14.0,15.0,16.0,17.0,18.0]
```

```
*Main> extent y
```

```
((Z :: 2) :: 3) :: 3
```

# examples

```
*Main> let y = fromList ((Z :: 2 :: 3 :: 3) :: DIM3) [1..18]
```

```
*Main> y ! (Z :: 0 :: 0 :: 0)
```

```
1.0
```

```
*Main> y ! (Z :: 1 :: 1 :: 1)
```

```
14.0
```



# examples

```
*Main> let y = fromList ((Z :: 2 :: 3 :: 3) :: DIM3) [1..18]
```

```
*Main> y ! (Z :: 0 :: 0 :: 20)
```

```
*** Exception: .\Data\Vector\Generic.hs:237 (!): index out of bounds (20,18)
```



bounds checking is done at RUN TME

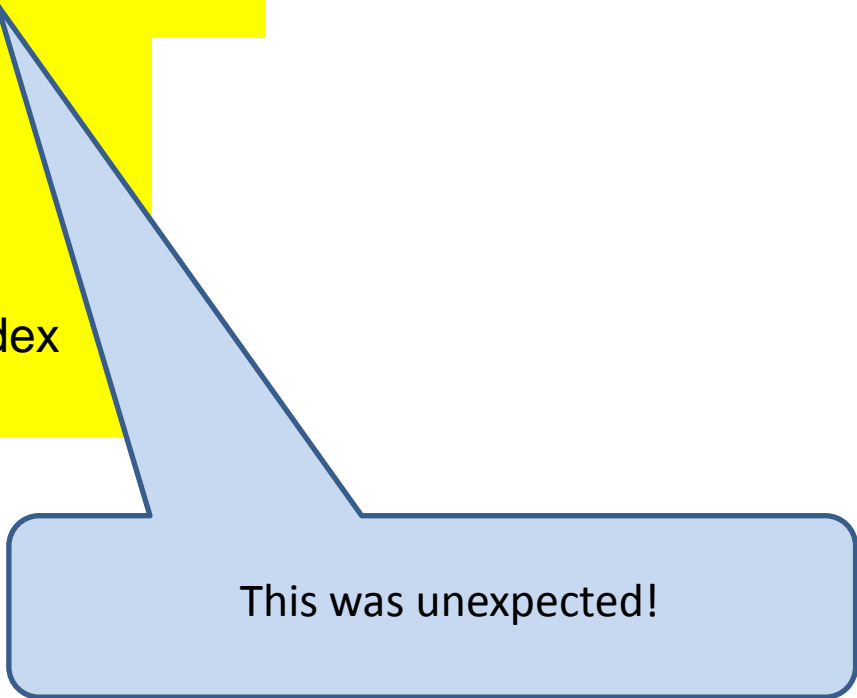
# examples

```
*Main> let y = fromList ((Z :: 2 :: 3 :: 3) :: DIM3)
[1..18]
```

```
*Main> y ! (Z :: 0 :: 0 :: 9)
10.0
```

```
*Main> y ! (Z :: 0 :: 0 :: 17)
18.0
```

```
*Main> y ! (Z :: 0 :: 17 :: 0)
*** Exception:
.\Data\Vector\Generic.hs:237 (!): index
out of bounds (51,18)
```



This was unexpected!

```
*Main> let z = fromList (Z :: 2 :: 3 :: DIM2) [1..6]
*Main> transpose2D z
Array (Z :: 3 :: 2) [1.0,4.0,2.0,5.0,3.0,6.0]
```

1	2	3
4	5	6

1	4
2	5
3	6

transpose2D :: Elt e => Array DIM2 e -> Array DIM2 e  
transpose2D a = backpermute (swap s) swap a

where

s = extent a

swap (Z :: i :: j) = Z :: j :: i

```
transpose2D :: Elt e => Array DIM2 e -> Array DIM2 e  
transpose2D a = backpermute (swap s) swap a
```

where

```
s = extent a  
swap (Z :: i :: j) = Z :: j :: i
```

s is the shape (or extent) of the array a

transpose2D :: Elt e => Array DIM2 e -> Array DIM2 e  
transpose2D a = backpermute (swap s) swap a

where

s = extent a

swap (Z :: i :: j) = Z :: j :: i

swap i and j  
= swap rows and columns

an index space transformation

transpose2D :: Elt e => Array DIM2 e -> Array DIM2 e  
transpose2D a = backpermute (swap s) swap a

where

s = extent a

swap (Z :: i :: j) = Z :: j :: i

swap i and j  
= swap rows and columns

an index space transformation

1	2	3
4	5	6

1	4
2	5
3	6

```
backpermute :: (Shape shin, Shape shout, Elt a) =>  
             shout -> (shout -> shin) -> Array shin a -> Array shout a
```

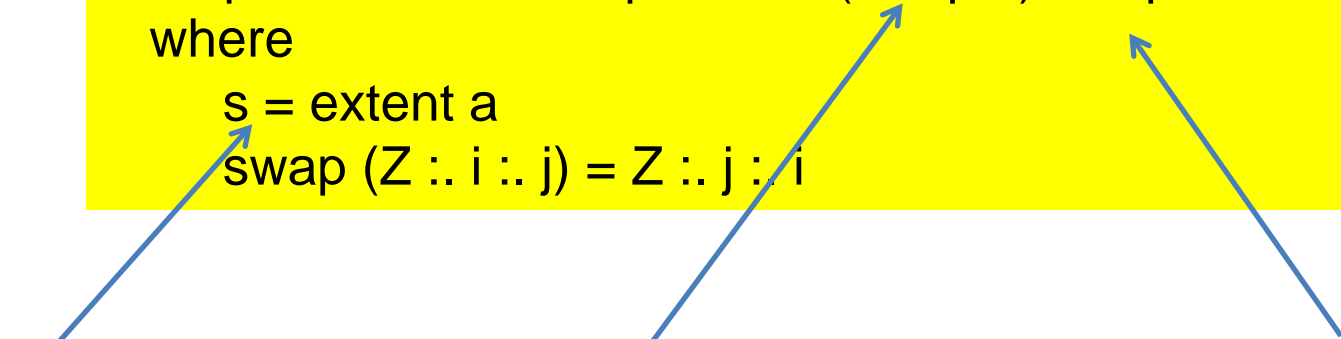
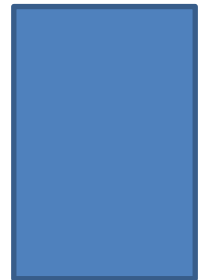
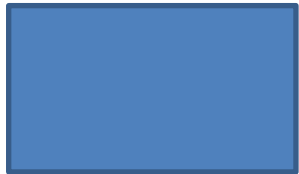


transpose2D :: Elt e => Array DIM2 e -> Array DIM2 e  
transpose2D a = backpermute (swap s) swap a

where

s = extent a

swap (Z :: i :: j) = Z :: j :: i



# more general transpose (on inner two dimensions)

```
transpose :: (Shape sh, Elt e) => Array ((sh :: Int) :: Int) e -> Array ((sh :: Int) :: Int) e
```

more general transpose  
(on inner two dimensions)  
is provided

```
transpose :: (Shape sh, Elt e) => Array ((sh :: Int) :: Int) e -> Array ((sh :: Int) :: Int) e
```

This type says an array with at least 2 dimensions.  
The function is **shape polymorphic**

more general transpose  
(on inner two dimensions)  
is provided

```
transpose :: (Shape sh, Elt e) => Array ((sh :: Int) :: Int) e -> Array ((sh :: Int) :: Int) e
```

Functions with at-least constraints become a parallel map over the unspecified dimensions (called rank generalisation)

Important way to express parallel patterns

```
*Main> let w = fromList (Z :: 2 :: 3 :: 3 :: DIM3) [1..(18 ::Int)]
```

```
*Main> A.transpose w
```

```
Array (Z :: 2 :: 3 :: 3) [1,4,7,2,5,8,3,6,9, 10,13,16,11,14,17,12,15,18]
```

A.sum :: (Shape sh, Elt a, Num a) => Array (sh :: Int) a -> Array sh a



reduces shape by one dimension

```
*Main> w
```

```
Array (Z :: 2 :: 3 :: 3) [1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18]
```

```
*Main> A.sum w
```

```
Array (Z :: 2 :: 3)      [6, 15, 24, 33, 42, 51]
```

```
*Main> w
Array (Z :: 2 :: 3 :: 3) [1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18]
*Main> A.sum w
Array (Z :: 2 :: 3)      [6, 15, 24, 33, 42, 51]
```

note that for 1D arrays, sum is implemented as a parallel tree reduction as (+) is known to be associative.  
Generic folds etc. are sequential (in this version of Repa)  
For > 1D arrays, both are sequential, but many of them happen at once because of rank generalisation



# backpermute can change the shape

```
selEven :: (Shape sh, Elt e) => Array (sh:.Int) e -> Array (sh:.Int) e
{-# INLINE selEven #-}
selEven !arr = force $ backpermute new_shape expand arr
  where
    (ns :.n) = extent arr
    new_shape = ns :.((n+1) `div` 2)
    expand (is :.i) = is :.(i * 2)
```

# backpermute can change the shape

```
selEven :: (Shape sh, Elt e) => Array (sh:.Int) e -> Array (sh:.Int) e
{-# INLINE selEven #-}
selEven !arr = force $ backpermute new_shape expand arr
  where
    (ns :.n) = extent arr
    new_shape = ns :.((n+1) `div` 2)
    expand (is :.i) = is :.(i * 2)
```

Note how the new shape depends only on the old shape and not on the data in the array  
(My def. differs slightly from that in the paper.)

# backpermute can change the shape

```
selEven :: (Shape sh, Elt e) => Array (sh:.Int) e -> Array (sh:.Int) e  
{-# INLINE selEven #-}
```

```
selEven !arr = force $ backpermute new_shape expand arr  
where
```

```
  (ns :.n) = extent arr  
  new_shape = ns :.((n+1) `div` 2)  
  expand (is :.i) = is :.(i * 2)
```

```
selOdd :: (Shape sh, Elt e) => Array (sh:.Int) e -> Array (sh:.Int) e  
{-# INLINE selOdd #-}
```

```
selOdd !arr = force $ backpermute new_extent expand arr  
where
```

```
  (ns :.n) = extent arr  
  new_extent = ns :.(n `div` 2)  
  expand (is :.i) = is :.(i * 2 + 1)
```

```
*Main> let w = fromList (Z :: 2 :: 3 :: 3 :: DIM3) [1..(18 ::Int)]
*Main> selEven w
Array (Z :: 2 :: 3 :: 2) [1,3,4,6,7,9,10,12,13,15,16,18]
*Main> selOdd w
Array (Z :: 2 :: 3 :: 1) [2,5,8,11,14,17]
```

# filter?

```
filter :: (Elt e ) => (E -> Bool) -> Array DIM1 e -> Array DIM1 e
```

can't be shape polymorphic

the shape of the output depends on the value of the input

filtering rows in a matrix might give different lengths (but we only deal with rectangular arrays)

# Matrix Multiplication

$$(A \cdot B)_{i,j} = \sum_{k=1}^n A_{i,k} \cdot B_{k,j}$$

---

$a_{11}$	$a_{12}$	$a_{13}$
$a_{21}$	$a_{22}$	$a_{23}$
$a_{31}$	$a_{32}$	$a_{33}$
$a_{41}$	$a_{42}$	$a_{43}$

 $\cdot$ 

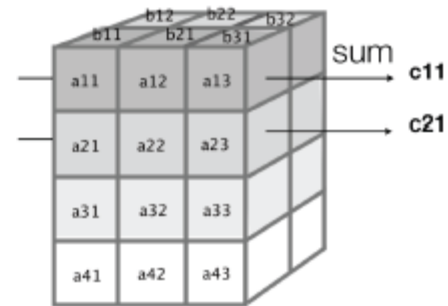
$b_{11}$	$b_{12}$
$b_{21}$	$b_{22}$
$b_{31}$	$b_{32}$

 $=$ 

$c_{11}$	$c_{12}$
$c_{21}$	$c_{22}$
$c_{31}$	$c_{32}$
$c_{41}$	$c_{42}$

# Matrix Multiplication

---



```
mmMult
```

```
  :: (Num e, Elt e)
```

```
 => Array DIM2 e
```

```
 -> Array DIM2 e -> Array DIM2 e
```

```
mmMult arr brr
```

```
 = sum (zipWith (*) arrRepl brrRepl)
```

```
 where
```

```
   trr = transpose2D brr
```

```
   arrR = replicate (Z :: All    :: colsB :: All) arr
```

```
   brrR = replicate (Z :: rowsA :: All    :: All) trr
```

```
   (Z :: colsA :: rowsA) = extent arr
```

```
   (Z :: colsB :: rowsB) = extent brr
```

# Fusion

---

- It's nice to program with bulk operations  
.. but we usually want them to be fused.
- We imagine replicating the source arrays being replicated when writing the program, but we don't want this at runtime.
- Fusion eliminates the intermediate arrays and the corresponding memory traffic.



## Manifest and Delayed Arrays

---

```
data Array sh e
  = Manifest sh (UArr e)
  | Delayed   sh (sh -> e)
```

- **Manifest** wraps a bona-fide unboxed array.  
Bulk-strict semantics. Forcing one element forces them all.
- **Delayed** wraps an element producing function, perhaps an index transformation that references some other array.
- Delayed functions are inlined and fused by the existing GHC optimiser (and lots of rewrite rules).

## Manifest and Delayed Arrays

```
data Array  
  = Manifest  
  | Delayed
```

- **Manifest** wraps a boxed array. It has Bulk-strict semantics. For

- **Delayed** wraps an element of an array. It has an index transformation that references some other array.

- Delayed functions are inlined and fused by the existing GHC optimiser (and lots of rewrite rules).

worker-wrapper transformation, hoisting etc.

End up with the index transformations nicely composed

This is what gives tight loops in the resulting code (and good performance)

## Manifest and Delayed Arrays

---

```
data Array sh e
  = Manifest sh (UArr e)
  | Delayed sh (sh -> e)
```

- **Manifest** wraps a bona-fide array  
Bulk-strict semantics

- **Delayed** wraps an e  
index transformation

- Delayed functions are  
optimiser (and lots of

Note on our research 😊

we have a similar symbolic array representation in Obsidian (our DSL for GPU programming in Haskell (Svensson, Claessen, Sheeran))

and in Feldspar (DSL for DSP algorithm design (Axelsson, Persson, Svenningsson, Sheeran))

## Sharing and *force*

---

```
let arr = ...  
    brr = map f arr  
in mmMult brr brr
```

## Sharing and force

---

```
let arr = ...  
    brr = map f arr  
in mmMult brr brr
```

```
data Array sh e  
  = Manifest sh (UArr e)  
  | Delayed   sh (sh -> e)
```

## Sharing and force

---

```
force :: Array sh e
      -> Array sh e

data Array sh e
  = Manifest sh (UArr e)
  | Delayed   sh (sh -> e)

let arr = ...
      brr = force (map f arr)
in mmMult brr brr
```

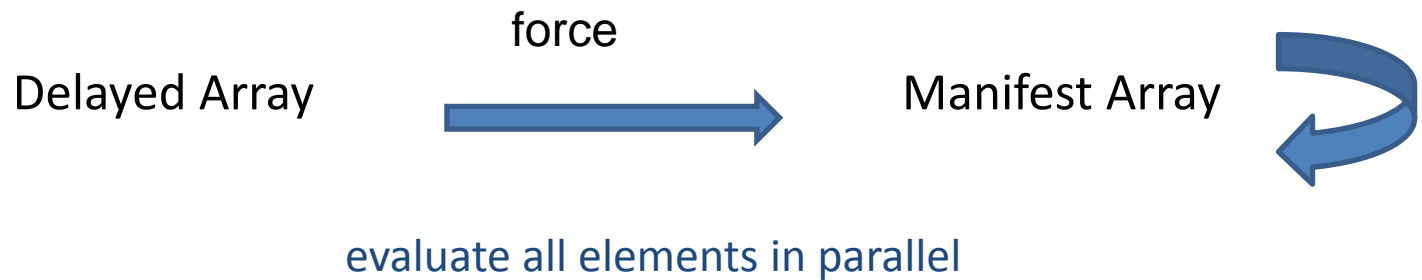
- For **Manifest** arrays, force is the identity.
- For **Delayed** arrays, it evaluates all the elements in parallel, producing a manifest array.
- The programmer must add force manually.

```
force :: (Shape sh, Elt a) => Array sh a -> Array sh a
```



evaluate all elements in parallel

`force :: (Shape sh, Elt a) => Array sh a -> Array sh a`



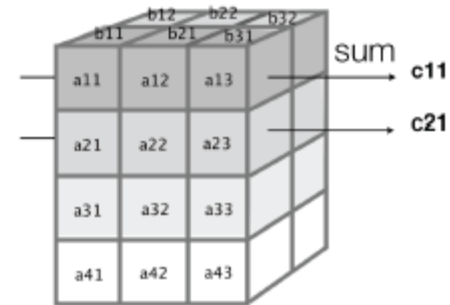


```
force :: (Shape sh, Elt a) => Array sh a -> Array sh a
```

Delayed Array

if you index into a delayed array without forcing it first, then each indexing operation costs a function call. It also *recomputes* the value of the array element at that index.

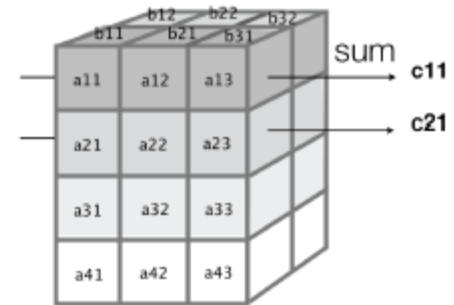
## Using the force ...



```
mmMult
  :: (Num e, Elt e)
  => Array DIM2 e
  -> Array DIM2 e -> Array DIM2 e
```

```
mmMult arr brr
= sum (zipWith (*) arrRepl brrRepl)
where
  trr = force (transpose2D brr)
  arrR = replicate (Z ::All ::colsB ::All) arr
  brrR = replicate (Z ::rowsA ::All ::All) trr
  (Z :: colsA :: rowsA) = extent arr
  (Z :: colsB :: rowsB) = extent brr
```

## Using the force ...

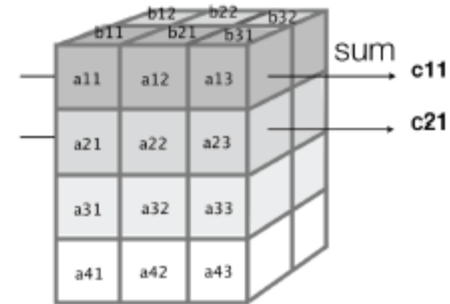


```
mmMult
  :: (Num e, Elt e)
  => Array DIM2 e
  -> Array DIM2 e -> Array DIM2 e
```

```
mmMult arr brr
= sum (zipWith (*) arrRepl brrRepl)
where
  trr = force (transpose2D brr)
  arrR = replicate (Z :: All    :: colsB :: All) arr
  brrR = replicate (Z :: rowsA :: All    :: All) trr
  (Z :: col  rowsA) = extent arr
  (Z :: col  rowsB) = extent brr
```

better cache performance when  
accessing the elements of b in  
row major order, so force the  
transposed version

## Using the force ...



```
mmMult
  :: (Num e, Elt e) => Array DIM2 e
  -> Array DIM2 e -> Array DIM2 e
```

```
mmMult arr brr
= sum (zipWith (*) arrRepl brr) [1..rowsA]
```

where

```
trr = force (transpose arr)
arrR = replicate (Z :. colA) trr
brrR = replicate (Z :. colB) trr
(Z :. colA) rowsA
(Z :. colB) rowsB
```

2 uses of force  
to get parallelism  
to improve locality

better cache performance when  
accessing the elements of b in  
row major order, so force the  
transposed version



# same prescan in Repa (my fastest so far)

```
-- assumes input of length a power of 2
prescan :: (Elt a) => (a -> a -> a) -> a -> (Array (Z :: Int) a) -> (Array (Z :: Int) a)
{-# INLINE prescan #-}
prescan f !i !as = sc as
  where
    sc as | size (extent as) == 1 = force $ fromList (Z :: (1 :: Int)) [i]
    sc as | otherwise =
      let es = force $ selEven as
          os = force $ selOdd as
          ss = force $ sc (A.zipWith f es os)
      in as `deepSeqArray` interleave2M ss (A.zipWith f ss es)
```

5 or 6 times faster for `sumAll . prescan (+) (0::Int)` on  $2^{20}$  inputs  
still 3-4 times slower than `scanl1` ☹️ but good speedup on 2 cores -N4  
and hopefully on more

# more operations

```
map :: (Shape sh, Elt a, Elt b) => (a -> b) -> Array sh a -> Array sh b
```

Doesn't care about shape of array. Just applies the function to each element.

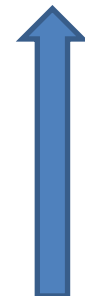
plain Haskell

```
foldl :: (a -> b -> a) -> a -> [b] -> a
```

Repa

```
foldl :: (Shape sh, Elt a, Elt b) => (a -> b -> a) -> a -> Array (sh :. Int) b -> Array sh a
```

reduce shape by one dimension



```
*Main> let y = fromList ((Z :: 2 :: 3 :: 3) :: DIM3) [1..18]
```

```
*Main> A.fold (+) 0 y
```

```
Array (Z :: 2 :: 3) [6.0,15.0,24.0,33.0,42.0,51.0]
```

```
*Main> A.transpose y
```

```
Array (Z :: 2 :: 3 :: 3)
```

```
[1.0,4.0,7.0,2.0,5.0,8.0,3.0,6.0,9.0,10.0,13.0,16.0,11.0,14.0,17.0,12.0,15.0,18.0]
```

```
*Main> A.fold (+) 0 (A.transpose y)
```

```
Array (Z :: 2 :: 3) [12.0,15.0,18.0,39.0,42.0,45.0]
```

each fold is sequential, but they are all done at once



# more operations

`map :: (Shape sh, Elt a, Elt b) => (a -> b) -> Array sh a -> Array sh b`

Doesn't care about the function's return type. Applies function to each element.

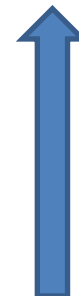
Note: in this version, there is no map over (say) the inner dimension (e.g. each row of a DIM2 array)

That might cause NESTEDNESS  
Note, though, that later Repa versions have chunked arrays and also a notion of regions in an array

plain Haskell

Repa `foldl :: (Shape sh, Elt a, Elt b) => (a -> a -> b) -> a -> Array sh a -> Array sh a`

reduce shape by one dimension

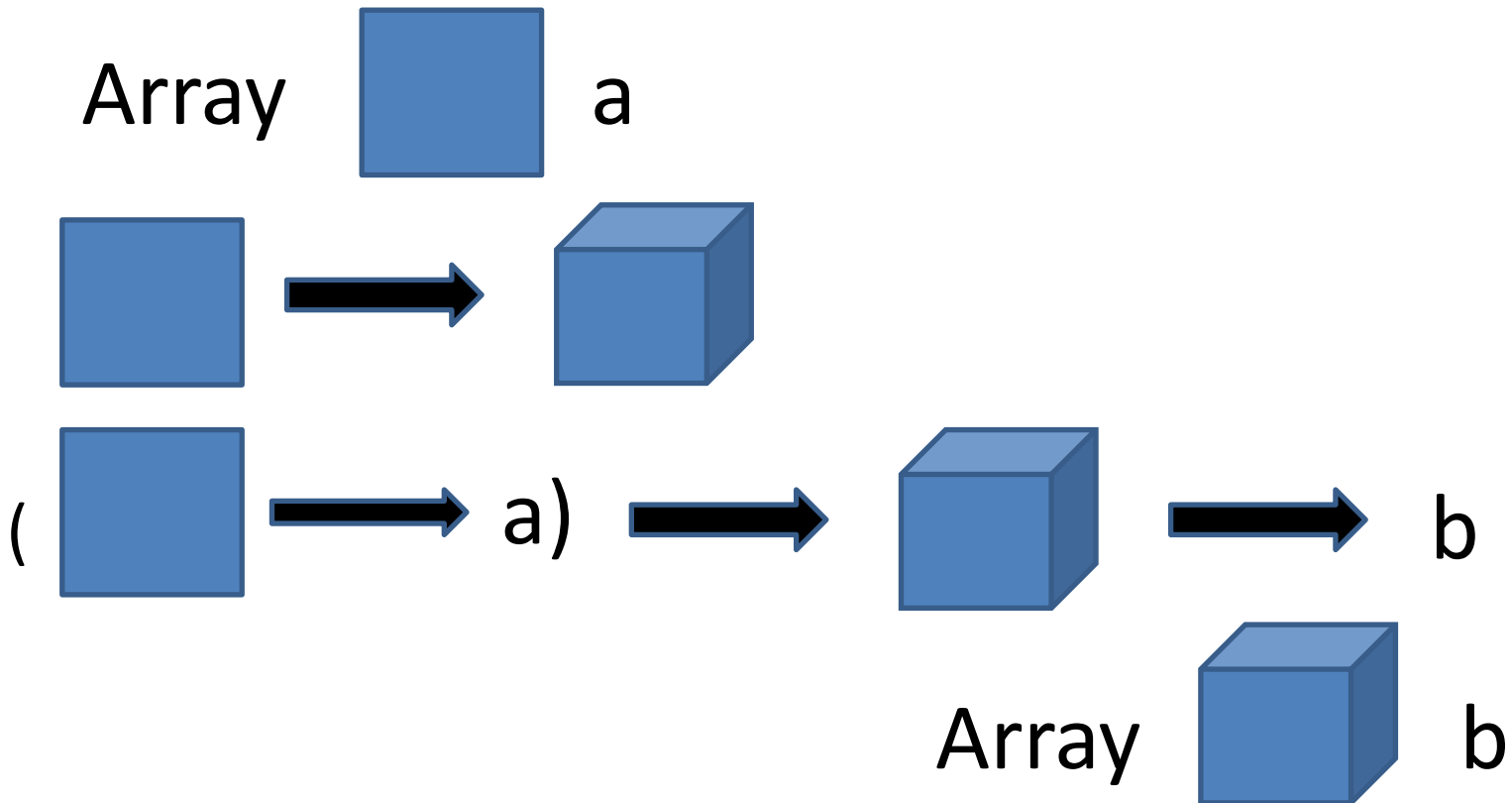


# key function: traverse

traverse

:: (Shape sh', Shape sh, Elt a) =>

Array shin a -> (shin -> shout) -> ((shin -> a) -> shout -> b) -> Array shout b

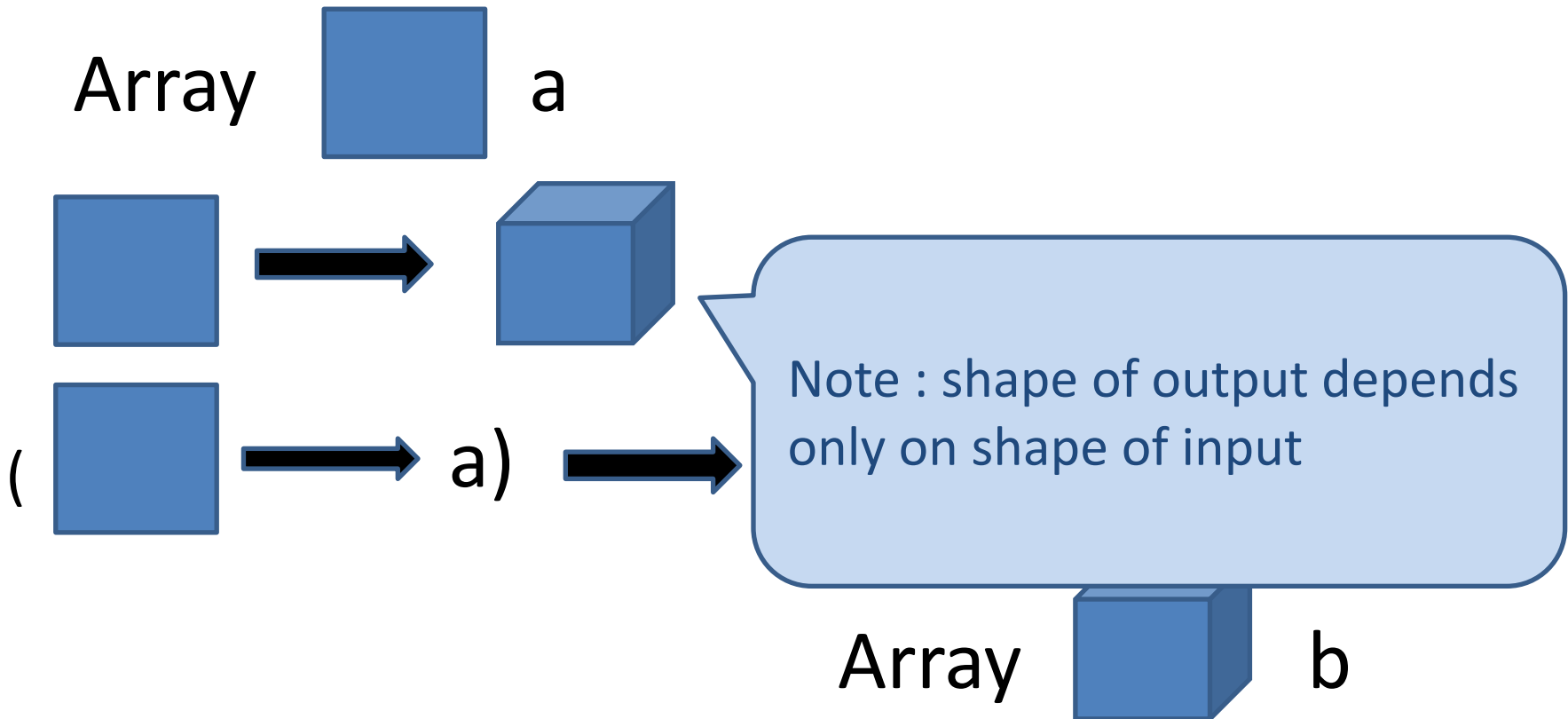


# key function: traverse

traverse

:: (Shape sh', Shape sh, Elt a) =>

Array shin a -> (shin -> shout) -> ((shin -> a) -> shout -> b) -> Array shout b

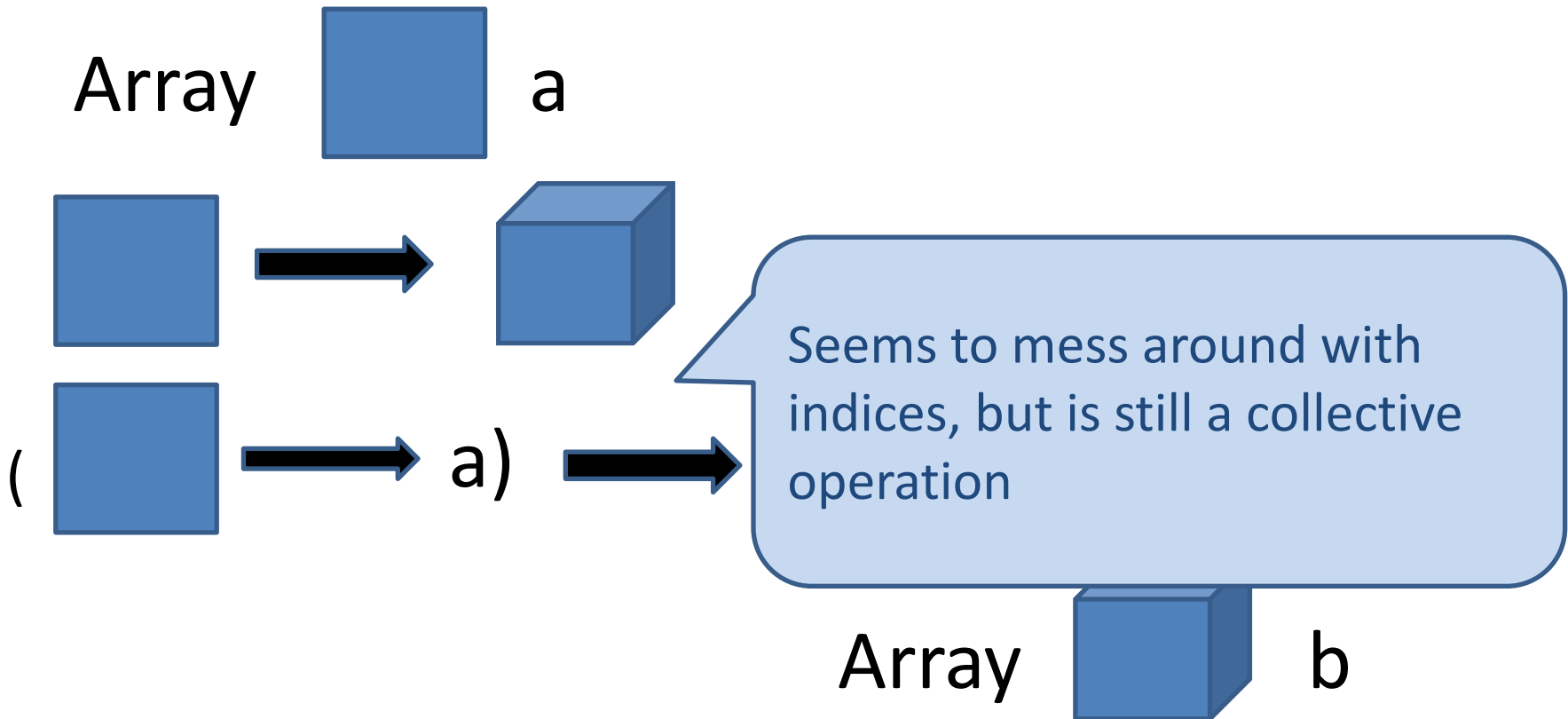


# key function: traverse

traverse

:: (Shape sh', Shape sh, Elt a) =>

Array shin a -> (shin -> shout) -> ((shin -> a) -> shout -> b) -> Array shout b

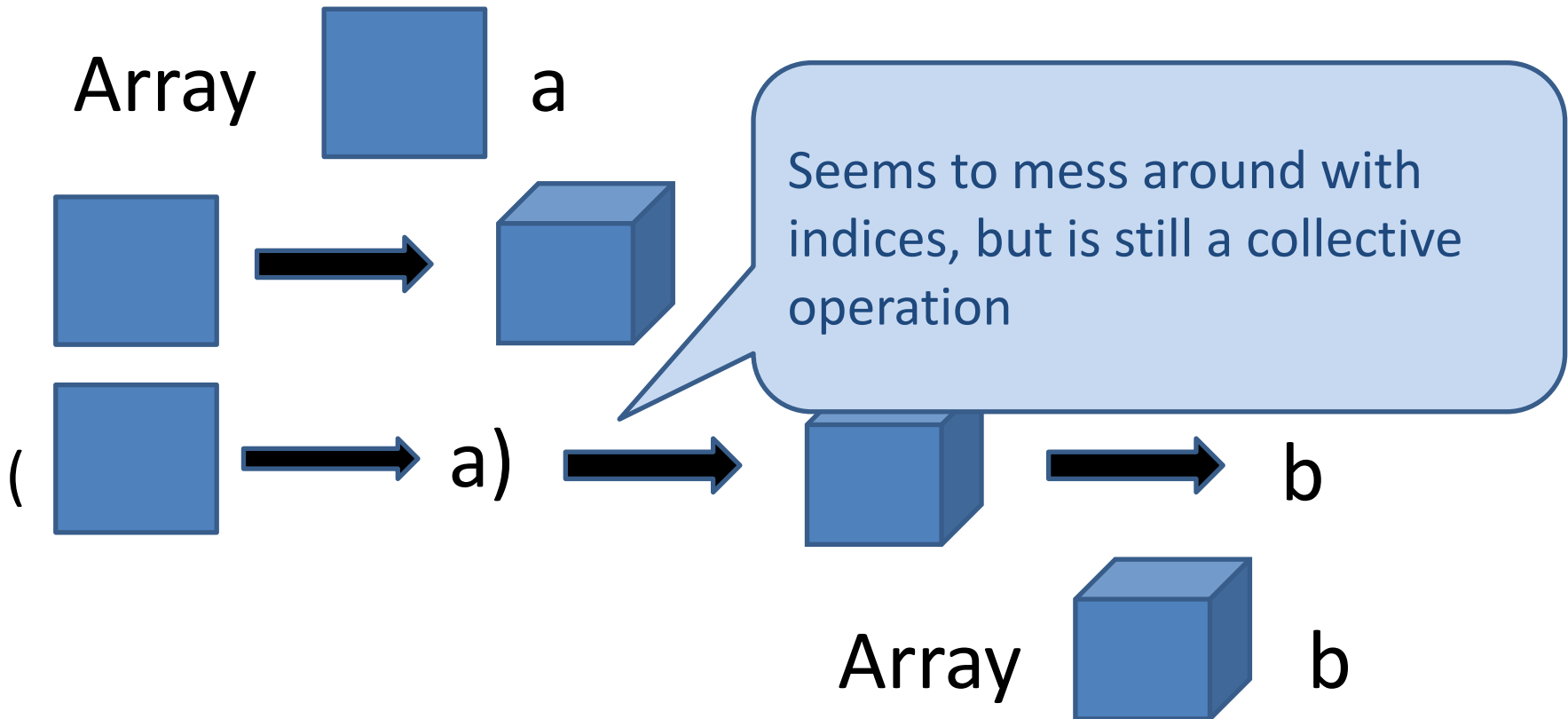


# key function: traverse

traverse

:: (Shape sh', Shape sh, Elt a) =>

Array shin a -> (shin -> shout) -> ((shin -> a) -> shout -> b) -> Array shout b




there is also a version called `unsafeTraverse` that skips bounds checking and so is faster

# use of traverse

```
backpermute :: (Shape sh', Shape s, Elt e) =>  
             shout -> (shout -> shin) -> Array shin e -> Array shout e  
backpermute shout perm arr = traverse arr (const shout) (. perm)
```

output shape  
shout



input shape -> output shape  
shin -> shout

(ignore input shape)

perm :: (shout -> shin)  
(. perm) :: (shin -> a) -> shout -> a

# use of traverse

$A.\text{map} :: (\text{Shape } sh, \text{Elt } b, \text{Elt } a) \Rightarrow (a \rightarrow b) \rightarrow \text{Array } sh \ a \rightarrow \text{Array } sh \ b$

$\text{map } f \text{ arr} = \text{traverse arr id } (f \ .)$

# unsafeTraverse

```
{-# INLINE bfly #-}  
bfly !k !as  
  = unsafeTraverse as id (\f (s :: i) -> let a = f (s :: i)  
                                           b = f (s :: (flipBit i k))  
                                           in if (testBit i k) then (b-a) else (a+b))
```





```
{-# INLINE interleave2M #-}  
interleave2M arr1 arr2  
= arr1 `deepSeqArray` arr2 `deepSeqArray`  
  unsafeTraverse2 arr1 arr2 shapeFn elemFn  
where  
  shapeFn dim1 dim2  
  | sh :: len1 <- dim1  
  , sh :: len2 <- dim2  
  = sh :: (len1 + len2)  
  
  elemFn get1 get2 (sh :: ix)  
  = case ix `mod` 2 of  
      0      -> get1 (sh :: ix `div` 2)  
      1      -> get2 (sh :: ix `div` 2)
```

```
*Main> let w = fromList (Z :: 2 :: 3 :: 3 :: DIM3) [1..(18 ::Int)]
```

```
*Main> selEven w
```

```
Array (Z :: 2 :: 3 :: 2) [1,3,4,6,7,9,10,12,13,15,16,18]
```

```
*Main> selOdd w
```

```
Array (Z :: 2 :: 3 :: 1) [2,5,8,11,14,17]
```

```
*Main> interleave2 (selEven w) (selOdd w)
```

```
Array (** Exception: Data.Array.Repa.interleave2: arrays must  
have same extent
```

```
*Main> interleave2M (selEven w) (selOdd w)
```

```
Array (Z :: 2 :: 3 :: 3)
```

```
[1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18]
```

# unsafeTraverse

```
{-# INLINE bfly #-}  
bfly !k !as  
  = unsafeTraverse as id (\f (s :: i) -> let a = f (s :: i)  
                                           b = f (s :: (flipBit i k))  
                                           in if (testBit i k) then (b-a) else (a+b))
```

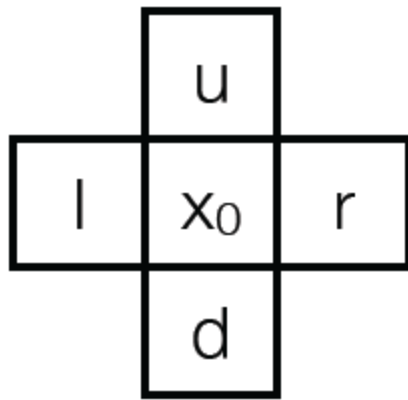
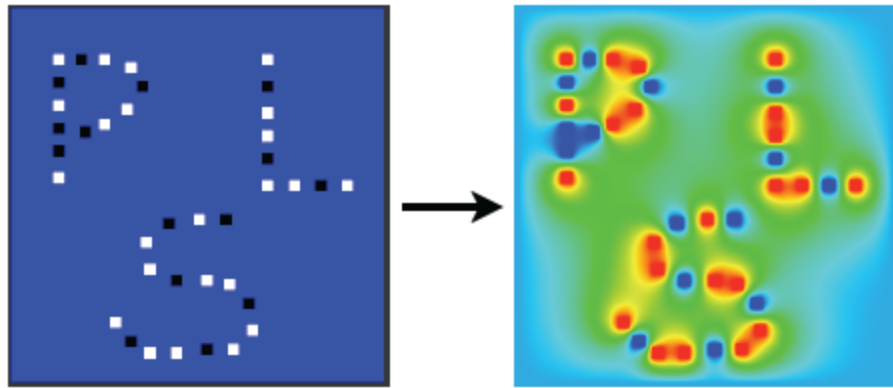
```
{-# INLINE twids #-}  
twids !k !as  
  = let k2 = 2^k  
      k2' = 2*k2 in  
      unsafeTraverse as id (\f (s :: i) -> let a = f (s :: i)  
                                              t = tw (i `mod` k2) k2'  
                                              in if (testBit i k) then t*a else a)
```

```
{-# INLINE fft4 #-}  
fft4 !n !as = foldr1 (.) [ force . twids k . bfly k | k <- [0..(n-1)]] as
```

# Example Applications

---

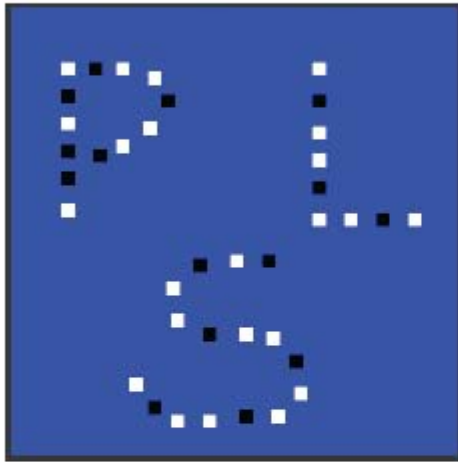
Solving the  
Laplace Equation



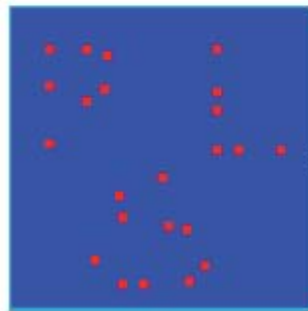
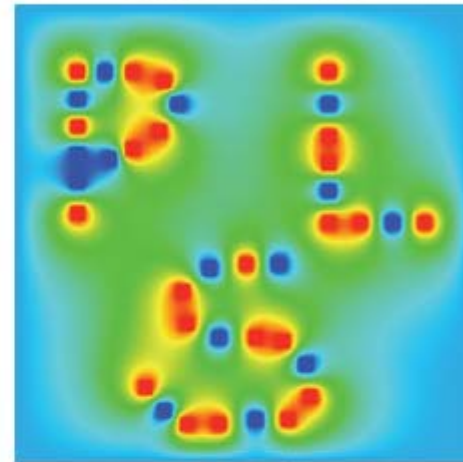
$$x_0' = (l + r + u + d) / 4$$

# Laplace Equation

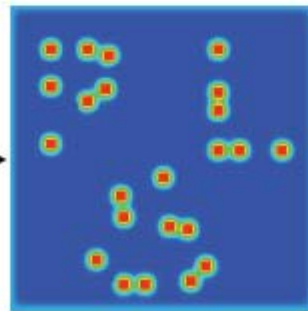
**boundary conditions**



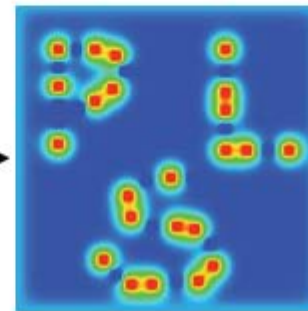
**5000 steps**



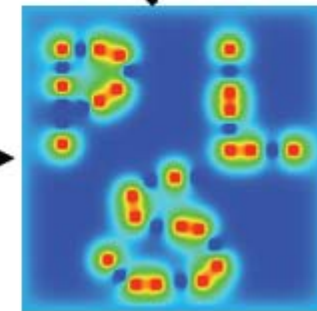
**0 steps**



**100 steps**



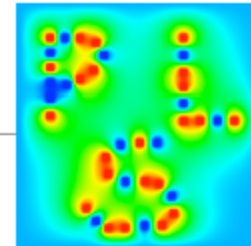
**500 steps**



**1000 steps**

# Laplace Equation

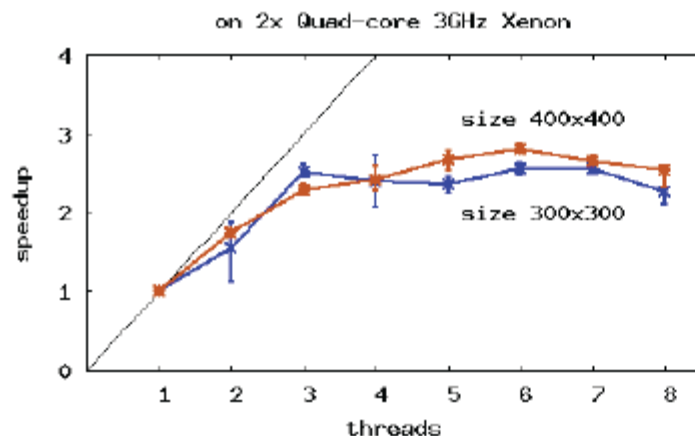
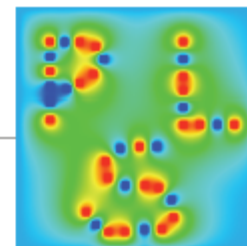
---



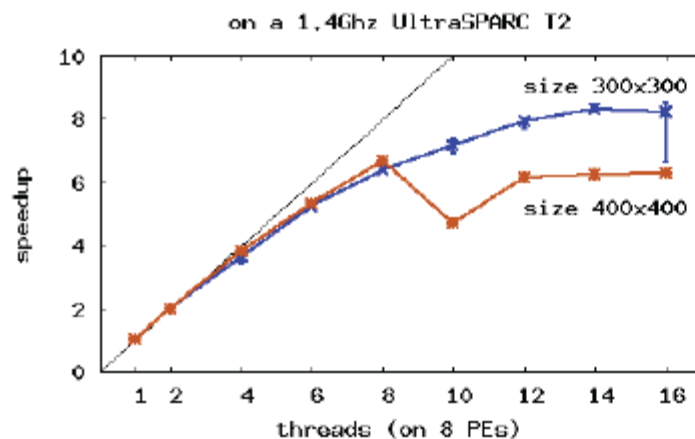
```
stencil :: Array DIM2 Double  
        -> Array DIM2 Double
```

```
stencil arr  
  = traverse arr id update  
  where  
    _ :: height :: width = extent arr  
  
    update get d@(sh :: i :: j)  
      = if isBoundary i j  
        then get d  
        else (get (sh :: (i-1)) :: j)  
            + get (sh :: i      :: (j-1))  
            + get (sh :: (i+1)) :: j)  
            + get (sh :: i      :: (j+1))) / 4
```

# Laplace Equation



	<b>GCC</b>	<b>single thread</b>	<b>fastest parallel</b>
<b>Xenon</b>	0.70	1.7s	0.68s
<b>T2</b>	6.5s	32s	3.8s



- GHC native code generator does no instruction reordering on SPARC. No LLVM 'port.
- Single threaded on T2 is slow



# Conclusions

Based on DPH technology

Good speedups!

Neat programs

Good control of Parallelism

**BUT CACHE AWARENESS** needs to be tackled

# Next lecture (Monday)

I would like to have a couple of student talks next wednesday (having talked to a couple of you earlier). Please contact to me again to confirm!

Student talks on topics related to the course would be most welcome!