

# Finite Automata and Formal Languages

## TMV026/DIT321 – LP4 2012

### DFA and NFA

#### Week 2

In these exercises, book sections, exercise numbers and pages refer to those in the third edition of the course book.

Those of you that are interested can implement the DFA or the NFA below.

#### DFA

1. Build a DFA that recognises the words over  $\{0, 1\}^*$  ending with the string 0100.
2. if  $\mathcal{L} \subseteq \Sigma^*$  is a language, we define  $\text{Prefix}(\mathcal{L})$  to be the set of words that are prefix of a word in  $\mathcal{L}$ . Show that if  $\mathcal{L}$  is regular then so is  $\text{Prefix}(\mathcal{L})$ .
3. Let  $\Sigma = \{0, 1\}$  and  $\text{Bool} = \{\text{True}, \text{False}\}$ . We define  $f, g, h : \Sigma^* \rightarrow \text{Bool}$  as follows:

$$\begin{array}{lll} f \epsilon = \text{False} & g \epsilon = \text{False} & h \epsilon = \text{True} \\ f(0x) = g x & g(0x) = h x & h(0x) = f x \\ f(1x) = h x & g(1x) = f x & h(1x) = h x \end{array}$$

Show that the language  $\{x \in \Sigma^* \mid f x = \text{True}\}$  is a *regular* language.

4. Suppose  $\Sigma = \{a, b\}$ . Build a DFA that accepts the words containing *bba* as a subword. Build then a DFA that accepts the words *not* containing *bba* as a subword.
5. Let  $\Sigma = \{a, b, c\}$ . Build a DFA  $D_1$  that accepts the words containing *ac* as a subword. Build a DFA  $D_2$  that accepts the words containing *ab* as a subword.  
Using the product construction, build then a DFA that accepts the words containing both *ab* and *ac* as subwords, and another DFA that accepts the words containing *ac* but *not ab* as a subword.
6. In a factory, we have the possible events  $a, b, c$ . A constraint  $L_1$  is that if the event  $b$  occurs after the event  $a$ , then the event  $c$  should occur in between. Represent this constraint  $L_1$  as a DFA  $D_1$ .

Suppose that there is another constraint  $L_2$  stating that if the event  $b$  occurs after the event  $c$ , then the event  $a$  should occur in between.

Explain intuitively why, if we have both constraints  $L_1$  and  $L_2$  then the event  $b$  cannot occur after the event  $a$ .

Represent the constraint  $L_2$  as a DFA  $D_2$ .

Do the product construction of  $D_1$  and  $D_2$  produce an automaton representing the conjunction of the constraint  $L_1$  and  $L_2$ ? Verify on this automaton that  $b$  cannot occur after  $a$  or  $c$ .

7. Do exercises 2.2.1, 2.2.4, 2.2.7, 2.2.8, 2.2.9, 2.2.10 and 2.2.11.

## NFA and $\epsilon$ -NFA

1. Show that if  $\mathcal{L} \subseteq \Sigma^*$  is regular then so is  $\mathcal{L}^R = \{rev(x) \mid x \in \mathcal{L}\}$ .  
(Hint: given an automaton for  $\mathcal{L}$  build a NFA for  $\mathcal{L}^R$ )
2. A ship attempts to transmit data to shore at random intervals. The receiver must continually listen and recognise when an actual transmission starts so that it can record the data that follows. Let us assume that the start of the transmission is signaled by the string 010010 and the end of the transmission is signaled by the string 000111. Represent this behaviour with a DFA.
3. Do exercises 2.3.1, 2.3.2 and 2.3.3.
4. Do exercises 2.5.1, 2.5.2 and 2.5.3.