Modelling & Datatypes

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Software

Software = Programs + Data

Modelling Data

- A big part of designing software is *modelling the data* in an appropriate way
- Numbers are not good for this!
- We model the data by defining *new* types

Modelling a Card Game

- Every card has a *suit*

Hearts,
Whist,
Plump,
Bridge, ...

- Model by a *new* type:

```
data Suit = Spades | Hearts | Diamonds | Clubs
```

The new type

The values of this type

Investigating the new type

```
Main> :i Suit
-- type constructor
data Suit

-- constructors:
Spades :: Suit
Hearts :: Suit
Diamonds :: Suit
Clubs :: Suit

Main> :i Spades
Spades :: Suit  -- data constructor
```

The new type

The new values
-- *constructors*

Types and constructors start with a capital letter

Printing Values

```
Main> Spades
ERROR - Cannot find "show" function for:
*** Expression : Spades
*** Of type    : Suit

Main> :i show
show :: Show a => a -> String  -- class member
```

Needed to print values

- **Fix**

```
data Suit = Spades | Hearts | Diamonds | Clubs
    deriving Show
```

```
Main> Spades
Spades
```

The Colours of Cards

- Each suit has a colour – *red* or *black*
- Model colours by a type

```
data Colour = Black | Red
   deriving Show
```

- Define functions by *pattern matching*

```
colour :: Suit -> Colour
colour Spades = Black
colour Hearts = Red
colour Diamonds = Red
colour Clubs = Black
```

One equation per value

```
Main> colour Hearts
Red
```

The Ranks of Cards

- Cards have ranks: $\underbrace{2..10}$, J, Q, K, A

Numeric ranks

- Model by a new type

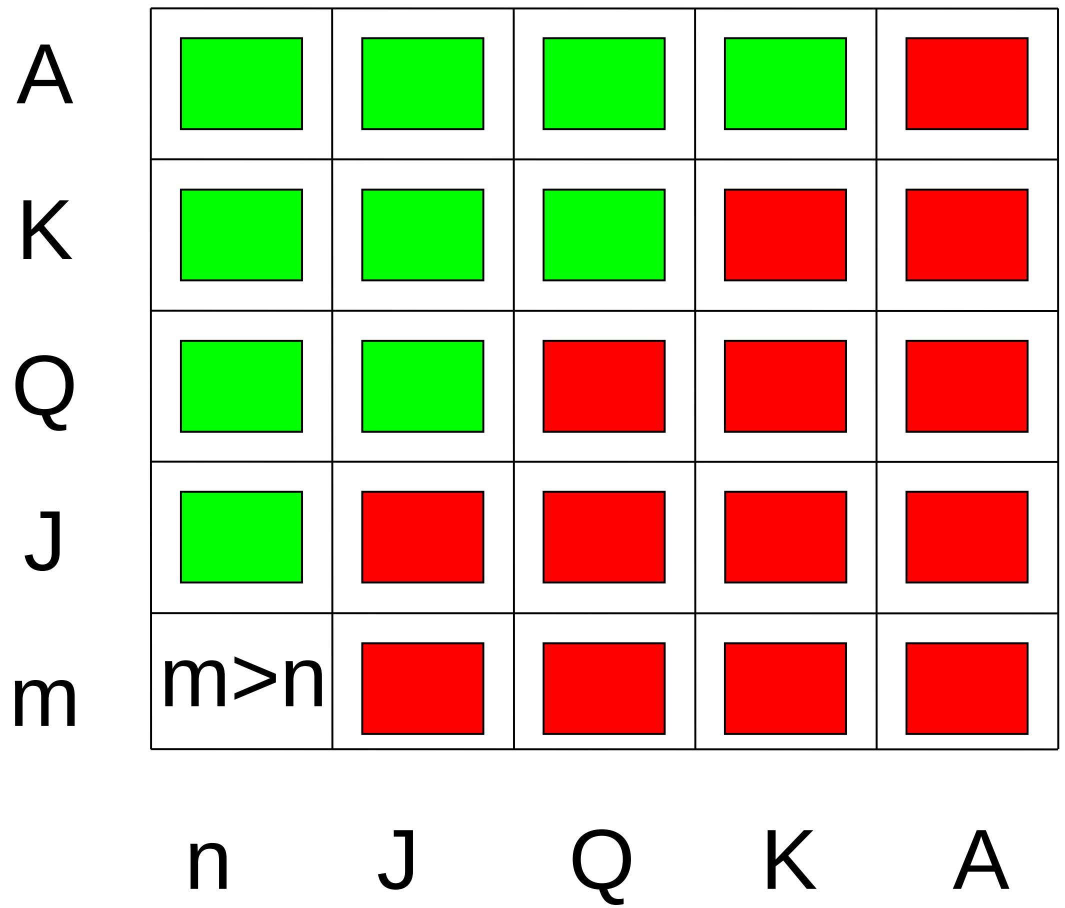
data Rank = Numeric Integer | Jack | Queen | King | Ace
    deriving Show

Numeric ranks
contain an Integer

```
Main> :i Numeric
Numeric :: Integer -> Rank  -- data constructor
Main> Numeric 3
Numeric 3
```

Rank Beats Rank

- When does one rank beat another?

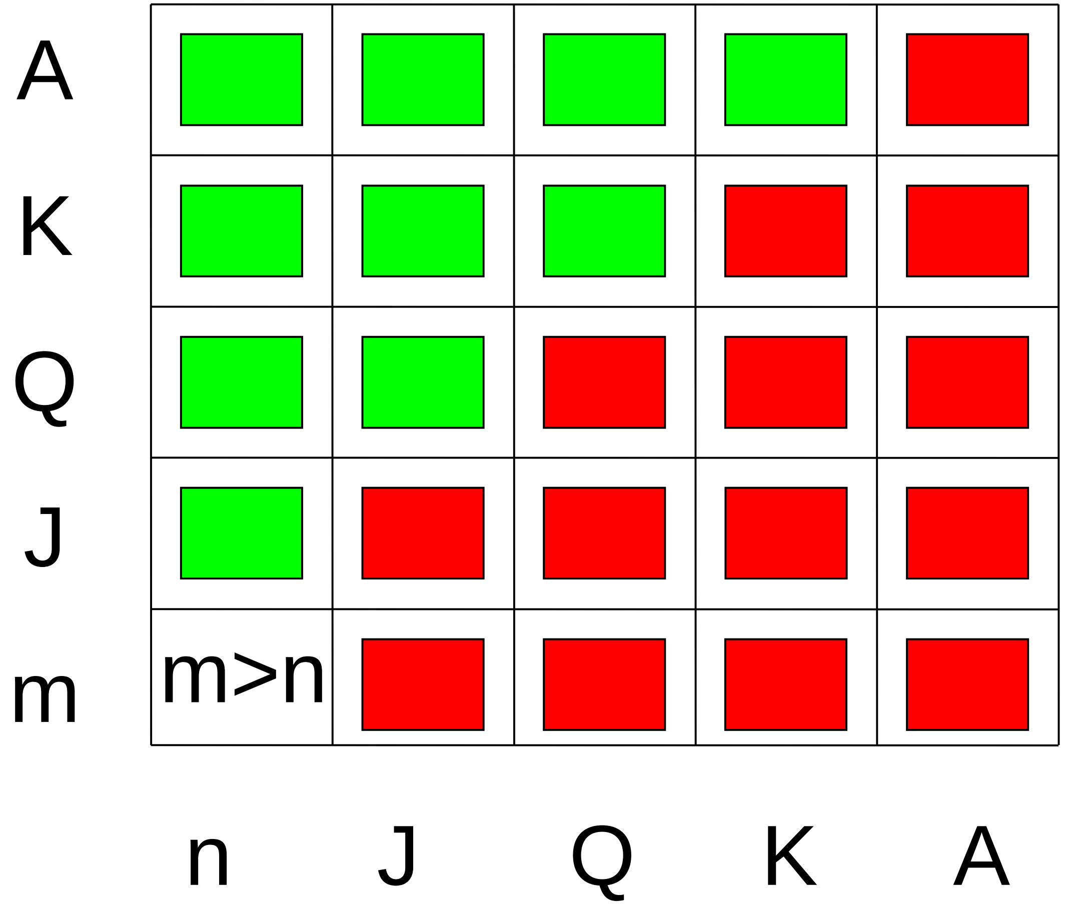


Rank Beats Rank

```
rankBeats :: Rank -> Rank -> Bool
```

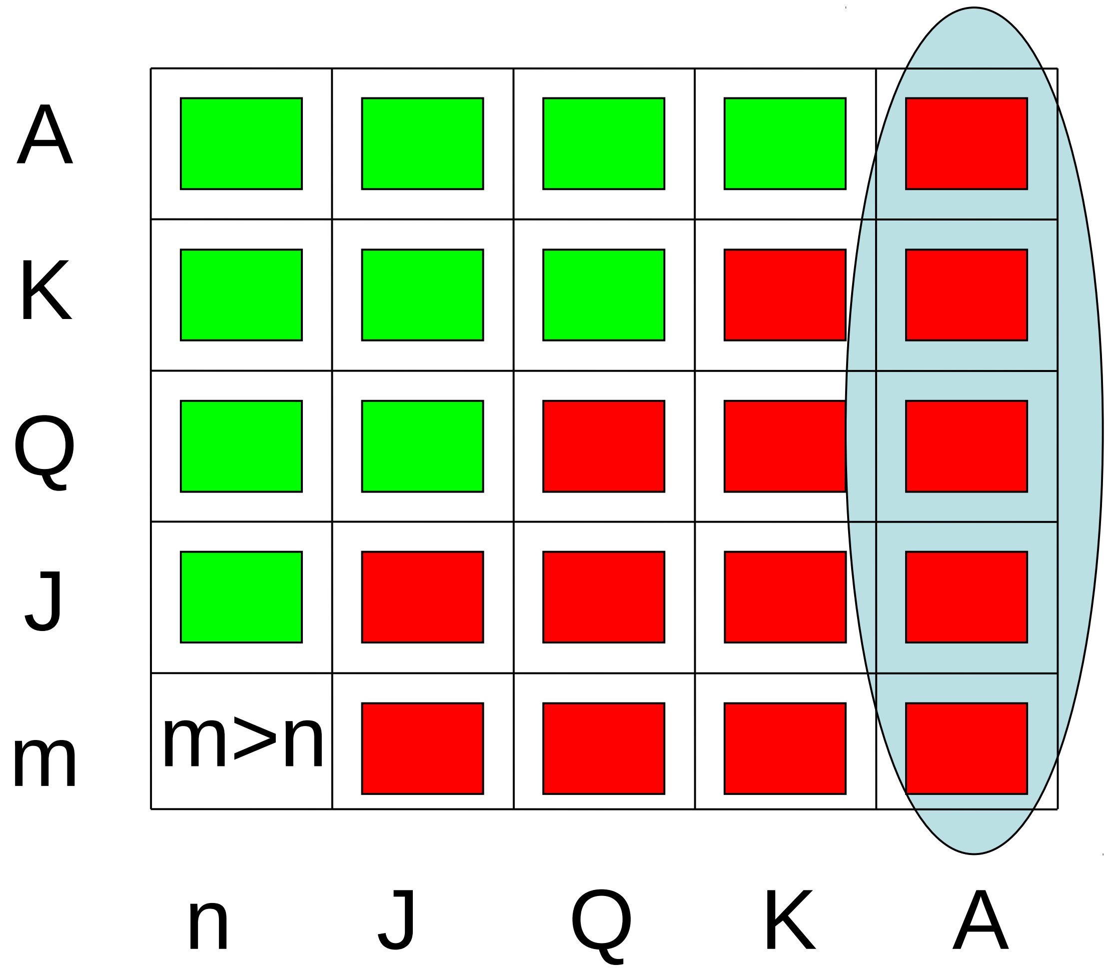
Rank Beats Rank

- When does one rank beat another?



Rank Beats Rank

- When does one rank beat another?



Rank Beats Rank

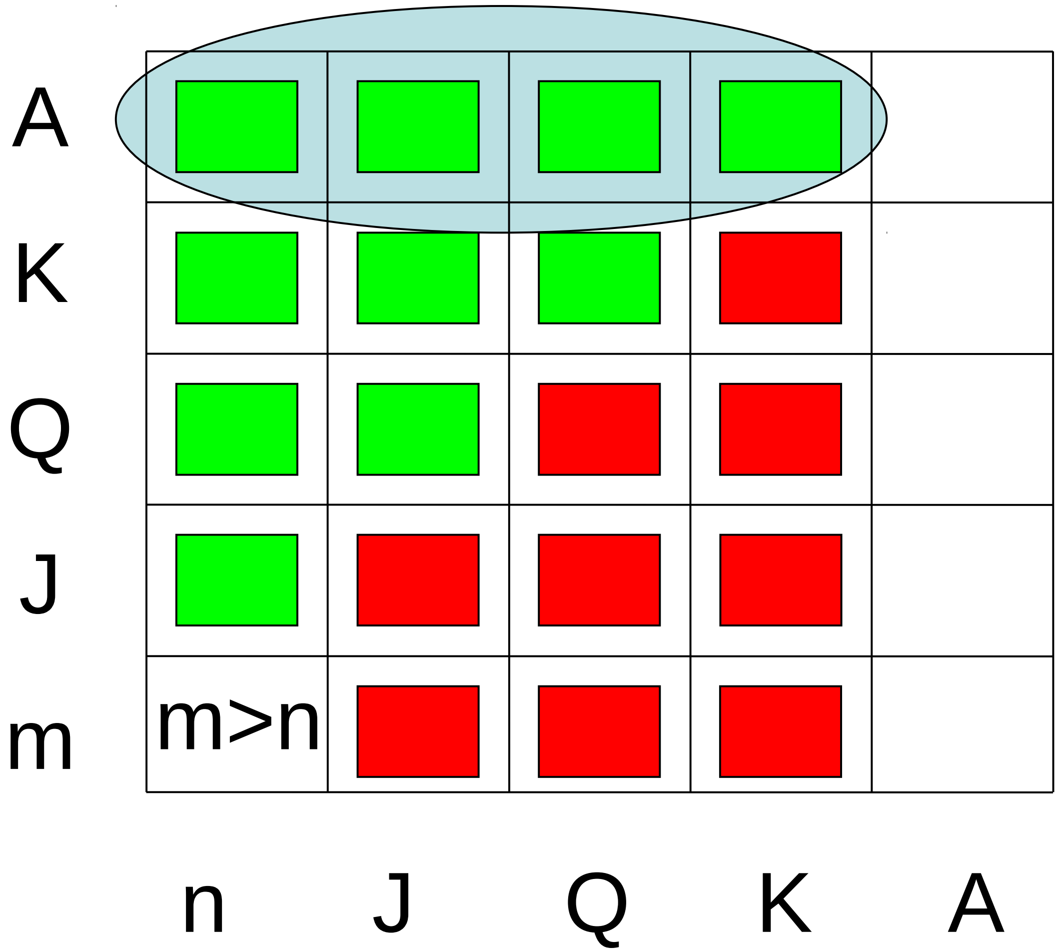
```
rankBeats :: Rank -> Rank -> Bool
rankBeats _ Ace = False
```

Nothing beats an Ace

Matches anything at all

Rank Beats Rank

- When does one rank beat another?



Rank Beats Rank

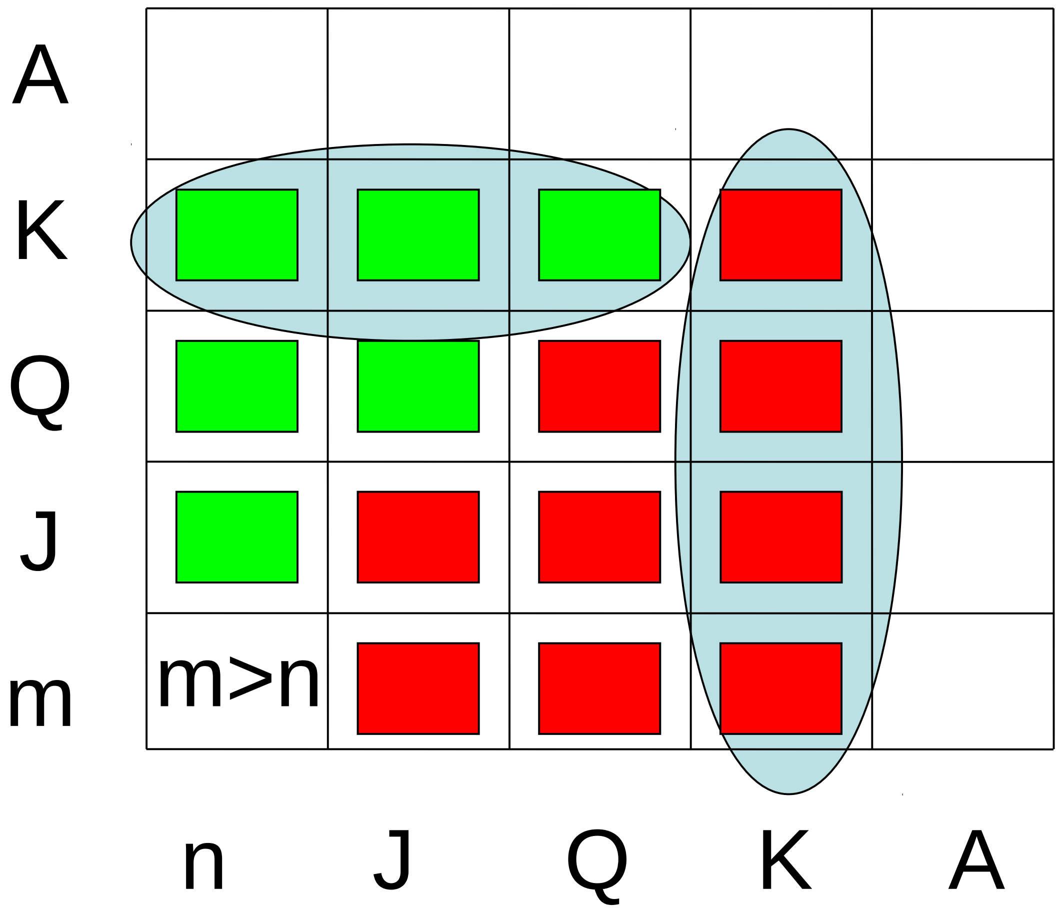
```
rankBeats :: Rank -> Rank -> Bool
rankBeats _ Ace = False
rankBeats Ace _ = True
```

An Ace beats anything else

Used only if the first equation does not match.

Rank Beats Rank

- When does one rank beat another?

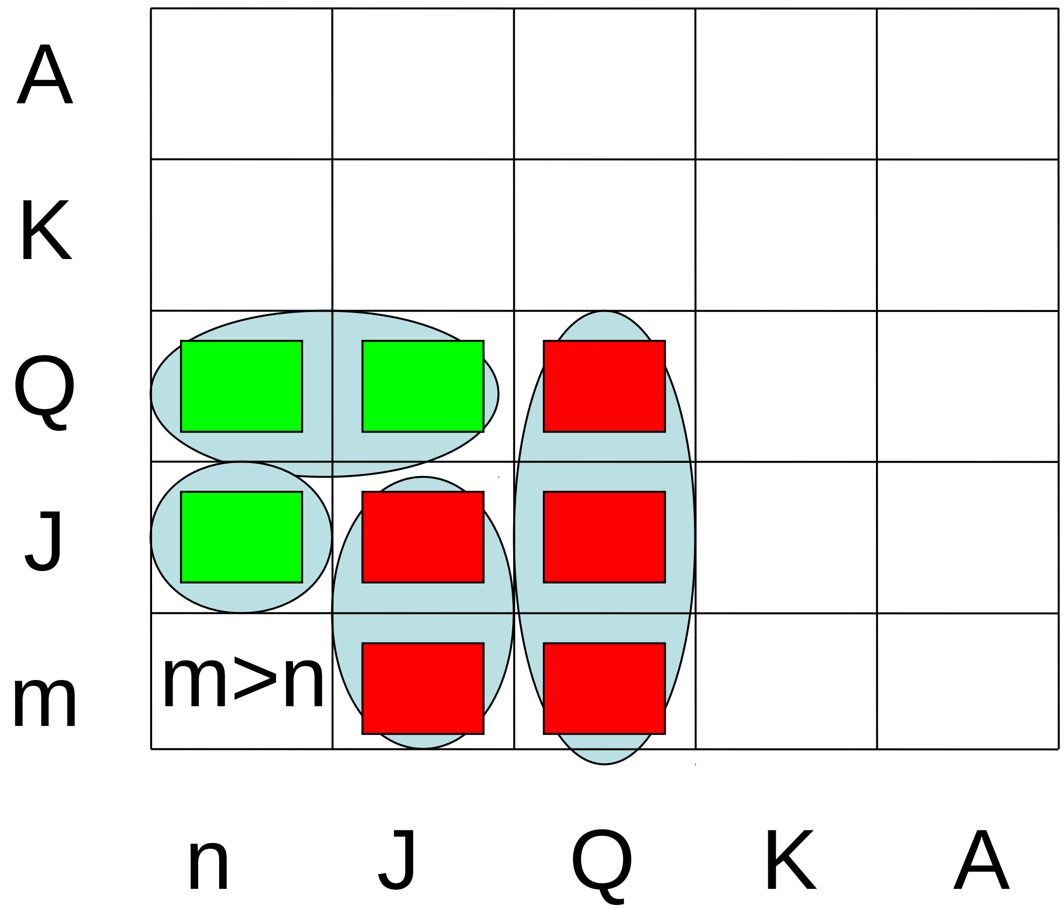


Rank Beats Rank

```
rankBeats :: Rank -> Rank -> Bool
rankBeats _ Ace = False
rankBeats Ace _ = True
rankBeats _ King = False
rankBeats King _ = True
```

Rank Beats Rank

- When does one rank beat another?



Rank Beats Rank

```
rankBeats :: Rank -> Rank -> Bool
rankBeats _ Ace = False
rankBeats Ace _ = True
rankBeats _ King = False
rankBeats King _ = True
rankBeats _ Queen = False
rankBeats Queen _ = True
rankBeats _ Jack = False
rankBeats Jack _ = True
```

Rank Beats Rank

- When does one rank beat another?

A					
K					
Q					
J					
m	m>n				
	n	J	Q	K	A

Rank Beats Rank

```
rankBeats :: Rank -> Rank -> Bool
rankBeats _ Ace = False
rankBeats Ace _ = True
rankBeats _ King = False
rankBeats King _ = True
rankBeats _ Queen = False
rankBeats Queen _ = True
rankBeats _ Jack = False
rankBeats Jack _ = True
rankBeats (Numeric m) (Numeric n) = m > n
```

Match Numeric 7, for example

Names the number in the rank

Examples

```
Main> rankBeats Jack (Numeric 7)
True
Main> rankBeats (Numeric 10) Queen
False
```

Modelling a Card

- A Card has both a Rank and a Suit

```
data Card = Card Rank Suit
    deriving Show
```

- Define functions to inspect both

```
rank :: Card -> Rank
rank (Card r s) = r

suit :: Card -> Suit
suit (Card r s) = s
```

A Useful Abbreviation

- Define type and inspection functions together, as follows

```
data Card = Card {rank :: Rank, suit :: Suit}
    deriving Show
```

When does one card beat another?

- When both cards have the same suit, and the rank is higher

can be written down simpler...

```
cardBeats :: Card -> Card -> Bool
cardBeats c c'
   | suit c == suit c'   = rankBeats (rank c) (rank c')
   | otherwise           = False

data Suit = Spades | Hearts | Diamonds | Clubs
   deriving (Show, Eq)
```

When does one card beat another?

- When both cards have the same suit, and the rank is higher

```
cardBeats :: Card -> Card -> Bool
cardBeats c c' =  suit c == suit c’
                  && rankBeats (rank c) (rank c')
```

Intermezzo: Figures

- Modelling geometrical figures
  - triangle
  - rectangle
  - circle

```
data Figure = Triangle ...
            | Rectangle ...
            | Circle ...
```

```
circumference :: Figure -> Double
circumference = ...
```

Intermezzo: Figures

```
data Figure = Triangle Double Double Double
            | Rectangle Double Double
            | Circle Double
```

```
circumference :: Figure -> Double
circumference (Triangle a b c) = a + b + c
circumference (Rectangle x y) = 2* (x + y)
circumference (Circle r)       = 2 * pi * r
```

Intermezzo: Figures

```haskell
data Figure = Triangle Double Double Double
            | Rectangle Double Double
            | Circle Double

-- types
Triangle :: Double -> Double -> Double -> Figure
Rectangle :: Double -> Double -> Figure
Circle :: Double -> Figure
```

```haskell
square :: Double -> Figure
square s = Rectangle s s
```

Modelling a Hand of Cards

- A hand may contain any number of cards from zero up!

```
data Hand = Cards Card … Card
    deriving Show
```

We can't use …!!!

- The solution is… *recursion!*

Modelling a Hand of Cards

- A hand may contain any number of cards from zero up!
  - A hand may be empty
  - It may consist of a *first card* and th
    - The rest is another hand of cards!

very much like a list...

```
data Hand = Empty | Add Card Hand
  deriving Show
```

A *recursive* type!

Solve the problem of modelling a hand with one fewer cards!

When can a hand beat a card?

- An empty hand beats nothing
- A non-empty hand can beat a card if the first card can, *or* the rest of the hand can!

```
handBeats :: Hand -> Card -> Bool
handBeats Empty     card = False
handBeats (Add c h) card =
    cardBeats c card || handBeats h card
```

- A *recursive* function!

What Did We Learn?

- Modelling the problem using datatypes with components
- Using *recursive datatypes* to model things of varying size
- Using *recursive functions* to manipulate recursive datatypes
- Writing properties of more complex algorithms