Software Engineering using Formal Methods

Reasoning about Programs with Loops and Method Calls

Wolfgang Ahrendt

17 October 2013

Program Logic Calculus – Repetition

Calculus realises symbolic interpreter:

- works on first active statement

$$\Gamma \Longrightarrow \langle \texttt{i=j++;}\mathbf{if}\texttt{(isValid)\{ok=}\mathbf{true}\texttt{;\}...}\rangle\phi$$

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- control flow branching induces proof splitting
- application of update computes weakest precondition of $\mathcal{U}'$ wrt. ϕ

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- all components of an array `arr` of length 2 have value 0?

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- all components of an array arr of length 2 have value 0?
  $$\{\texttt{arr[0]} := \texttt{0} \parallel \texttt{arr[1]} := \texttt{0}\}\phi$$
- all components of an array arr of length n have value 0?

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  $$\{\mathtt{arr[0]} := 0 \parallel \mathtt{arr[1]} := 0\}\phi$$
- all components of an array `arr` of length `n` have value 0?

For example to deal with things like

$$\langle \mathbf{int}[]\ \mathtt{a} = \mathbf{new}\ \mathbf{int}[\mathtt{n}];\rangle$$
$$\forall\, \mathbf{int}\ x;\ (0 \leq x < \mathtt{a.length} \rightarrow \mathtt{a}[x] \doteq 0)$$

Quantified Updates

Definition (Quantified Update)

For T well-ordered type (no ∞ descending chains): quantified update:

$$\{\texttt{\textbackslash for}\ T\ x; \texttt{\textbackslash if}\ \phi(x); l(x) := r(x)\}$$

- For all objects d in T such that $\phi(d)$
  perform the updates $\{l(d) := r(d)\}$ in parallel
- If there are several l with conflicting d then choose T-minimal one

- The conditional expression is optional
- Typically, x occurs in ϕ, l, and r (but doesn't need to)
- There is a normal form for updates computed efficiently by KeY

Quantified Updates Cont'd

Example (Initialization of field `a` for all objects in class `C`)

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Example (Integer types are well-ordered in KeY)

$$\{\texttt{\textbackslash for int}\ i;\, \mathrm{a}[0] := i\}(\mathrm{a}[0] \doteq 0)$$

- Non-standard order for $\mathbb{Z}$ (with 0 smallest and preserving $<$ for arguments of same sign)
- Proven automatically by update simplifier

Loop Invariants

Symbolic execution of loops: unwind

$$\text{unwindLoop}\ \frac{\Gamma \Longrightarrow \mathcal{U}[\pi\,\texttt{\textbf{if}}\,\texttt{(b)\{p;}\ \texttt{\textbf{while}}\texttt{(b) p\}}\ \omega]\phi, \Delta}{\Gamma \Longrightarrow \mathcal{U}[\pi\,\texttt{\textbf{while}}\texttt{(b) p}\ \omega]\phi, \Delta}$$

(We omitted $\mathcal{U}$ previous lectures, for simplicity.)

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How to handle a loop with...

- 0 iterations?

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How to handle a loop with...

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- 10 iterations?

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- 0 iterations? Unwind $1\times$
- 10 iterations? Unwind $11\times$

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- an unknown number of iterations?

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We need an invariant rule (or some form of induction)

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- Context Γ, Δ, $\mathcal{U}$ must be omitted in 2nd and 3rd premise:
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- Relevant context information must be added to *Inv* ☹

Example

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int i = 0;
while(i < a.length) {
    a[i] = 1;
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(Implicit) Class Invariant: $a \doteq\!\!\!\!/\ \mathtt{null}$

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$\{\mathtt{i} := c\}$ (c fresh constant symbol)

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$\mathcal{V} = \{\mathtt{i} := c \parallel \mathtt{\backslash for}\ x;\ \mathtt{a}[x] := f(x)\}$
(c, f fresh constant resp. function symbol)

Loop Invariants Cont'd

Improved Invariant Rule

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Loop Invariants Cont'd

Improved Invariant Rule

$$\frac{\begin{array}{ll} \Gamma \Longrightarrow \mathcal{U}\,\mathit{Inv}, \Delta & \text{(valid when entering loop)} \\ \Gamma \Longrightarrow \mathcal{U}\mathcal{V}(\mathit{Inv}\ \&\ b \doteq \mathtt{TRUE} \rightarrow [\mathrm{p}]\mathit{Inv}), \Delta & \text{(preserved by p)} \\ \Gamma \Longrightarrow \mathcal{U}\mathcal{V}(\mathit{Inv}\ \&\ b \doteq \mathtt{FALSE} \rightarrow [\pi\ \omega]\phi), \Delta & \text{(assumed after exit)} \end{array}}{\Gamma \Longrightarrow \mathcal{U}[\pi\, \mathtt{\textbf{while}(b)\ p}\ \omega]\phi, \Delta}$$

- Context is kept as far as possible
- Invariant not 'responsible' for un-assignable locations
- Missing assignable clause (equiv. to **`assignable \everything`**):
  - $\mathcal{V} = \{* := *\}$ wipes out **all** information
  - Equivalent to basic invariant rule
  - Avoid this! Always give a specific **`assignable`** clause

Example with Improved Invariant Rule

```
int i = 0;
while(i < a.length) {
    a[i] = 1;
    i++;
}
```

Example with Improved Invariant Rule

(Implicit) Class Invariant: $a \doteq\!\!\!\!/\ \texttt{null}$

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int i = 0;
while(i < a.length) {
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(Implicit) Class Invariant: $\texttt{a} \not\doteq \textbf{\texttt{null}}$

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int i = 0;
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Postcondition: $\forall\, \textbf{\texttt{int}}\ x;\ (0 \leq x < \texttt{a.length} \rightarrow \texttt{a}[x] \doteq 1)$

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Loop invariant: $0 \leq \mathtt{i}\ \ \&\ \ \mathtt{i} \leq \mathtt{a.length}$

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Example with Improved Invariant Rule

(Implicit) Class Invariant: $\mathtt{a} \not\doteq \mathbf{null}$ not needed for loop invariant

```
int i = 0;
while(i < a.length) {
    a[i] = 1;
    i++;
}
```

Postcondition: $\forall\, \mathbf{int}\ x; (0 \leq x < \mathtt{a.length} \rightarrow \mathtt{a}[x] \doteq 1)$

Loop invariant: $0 \leq \mathtt{i}\ \ \&\ \ \mathtt{i} \leq \mathtt{a.length}$
$\&\ \ \forall\, \mathbf{int}\ x; (0 \leq x < \mathtt{i} \rightarrow \mathtt{a}[x] \doteq 1)$

Example in JML/Java – Loop.java

```
public int[] a;
/*@ public normal_behavior
  @  ensures (\forall int x; 0<=x && x<a.length; a[x]==1);
  @  diverges true;
  @*/
public void m() {
  int i = 0;
  /*@ loop_invariant
    @  (0 <= i && i <= a.length &&
    @   (\forall int x; 0<=x && x<i; a[x]==1));
    @ assignable i, a[*];
    @*/
  while(i < a.length) {
    a[i] = 1;
    i++;
  }
}
```

Example from last week

$\forall$ `int` x;
 $(x \doteq \mathtt{n} \land x >= 0 \rightarrow$
  $[$
```
i = 0; r = 0;
while (i<n) { i = i + 1; r = r + i;}
r=r+r-n;
```
  $]\mathtt{r} \doteq ?)$

How can we prove that the above formula is valid
(i.e. satisfied in all states)?

Example from last week

$\forall$ `int` x;
  $(x \doteq \mathtt{n} \wedge x >= 0 \rightarrow$
    [`i = 0; r = 0;`

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while (i<n) { i = i + 1; r = r + i;}
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How can we prove that the above formula is valid
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Solution:

```
@ loop_invariant
@     i>=0 && 2*r == i*(i + 1) && i <= n;
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```

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File: Loop2.java

Hints

Proving `assignable`

- The invariant rule assumes that **`assignable`** is correct
  E.g., with **`assignable \nothing;`** one can prove nonsense
- Invariant rule of KeY generates proof obligation that ensures correctness of **`assignable`**

Hints

Proving **assignable**

- The invariant rule assumes that **assignable** is correct
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Setting in the KeY Prover when proving loops

- Loop treatment: Invariant
- Quantifier treatment: No Splits with Progs
- If program contains `*`, `/`:
  Arithmetic treatment: DefOps
- Is search limit high enough (time out, rule apps.)?
- When proving partial correctness, add **`diverges true;`**

Total Correctness

Find a decreasing integer term v (called variant)

Add the following premisses to the invariant rule:

- $v \geq 0$ is initially valid
- $v \geq 0$ is preserved by the loop body
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- Remove directive `diverges true;`
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Example (The `array` loop)

```
@ decreasing
```

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```
@ decreasing  a.length - i;
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Files:

- `LoopT.java`
- `Loop2T.java`

Method Calls – Repetition

Method Call with actual parameters $arg_0, \ldots, arg_n$

$$\{arg_0 := t_0 \,||\, \ldots \,||\, arg_n := t_n \,||\, c := t_c\}\langle c.\mathtt{m}(arg_0, \ldots, arg_n);\rangle\phi$$

where $\mathtt{m}$ declared as $\mathtt{\textbf{void}}\ \mathtt{m}(\mathtt{T_0\ p_0}, \ldots, \mathtt{T_n\ p_n})$

Actions of rule methodCall

- for each formal parameter $\mathtt{p_i}$ of $\mathtt{m}$:
  declare and initialize new local variable $\mathtt{T_i\ p\#i} = arg_i;$

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Actions of rule methodCall

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  if implementation cannot be uniquely determined
- create method invocation $c.\mathtt{m}(\mathtt{p\#0}, \ldots, \mathtt{p\#n})@C$

Method Calls Cont'd

Method Body Expand

1. Execute code that binds actual to formal parameters $\texttt{T}_\texttt{i}\ \texttt{p\#i} = arg_i;$
2. Call rule methodBodyExpand

$$\frac{\Gamma \Longrightarrow \langle \pi\ \texttt{method-frame(source=C, this=c)\{ body \}}\ \omega\rangle\phi, \Delta}{\Gamma \Longrightarrow \langle \pi\ \texttt{c.m(p\#0,...,p\#n)@C;}\ \omega\rangle\phi, \Delta}$$

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Symbolic Execution

Only static information available, proof splitting;

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File: `inlineDynamicDispatch.key`

Problem

Formal specification of JAVA API and other called methods

How to perform symbolic execution when JAVA API method is called?

1. Method has reference implementation in JAVA
   Inline method body and execute symbolically

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   Problems Reference implementation not always available
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   Impossible to deal with recursion

2. Use method contract instead of method implementation

Method Contract Rule – Normal Behavior Case

Warning: Simplified version

```
/*@ public normal_behavior
  @ requires normalPre;
  @ ensures normalPost;
  @ assignable mod;
  @*/
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Method Contract Rule – Exceptional Behavior Case

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Understanding Proof Situations

Reasons why a proof may not close

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- bug in program
- maximal number of steps reached: restart or increase # of steps
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Understanding open proof goals

- follow the taken control-flow from the root to the open goal
- branch labels may give useful hints
- identify (part of) the post-condition or invariant that cannot be proven
- sequent remains always in "pre-state". I.e., constraints like $\mathtt{i} \geq 0$ refer to the value of $\mathtt{i}$ before executing the program (exception: sub-formulae prefixed by update or modality)
- remember: $\Gamma \Longrightarrow \mathrm{o} \doteq \mathtt{null}, \Delta$ is equivalent to $\Gamma, \mathrm{o} \not\doteq \mathtt{null} \Longrightarrow \Delta$

Summary

- Most JAVA features covered in KeY
- Several of remaining features available in experimental version
  - Simplified multi-threaded JMM
  - Floats
- Degree of automation for loop-free programs is high
- Proving loops requires user to provide invariant
  - Automatic invariant generation sometimes possible
- Symbolic execution paradigm lets you use KeY w/o understanding details of logic

Literature for this Lecture

Essential

KeY Book Verification of Object-Oriented Software (see course web page), Chapter 10: Using KeY

KeY Book Verification of Object-Oriented Software (see course web page), Chapter 3: Dynamic Logic, Sections 3.1, 3.2, 3.4, 3.5, 3.6.1, 3.6.2, 3.6.3, 3.6.4, 3.6.5, 3.6.7, 3.7