Parsing Expressions

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Expressions

- Such as
  - – 5*2+12
  - – 17+3*(4*3+75)
- Can be modelled as a datatype

```
data Expr
   = Num Int
   | Add Expr Expr
   | Mul Expr Expr
```

Showing and Reading

- We have seen how to write

```
showExpr :: Expr -> String
```

built-in show function produces ugly results

```
Main> showExpr (Add (Num 2) (Num 4))
"2+4"
Main> showExpr (Mul (Add (Num 2) (Num 3)) (Num 4)
(2+3)*4
```

- This lecture: How to write

```
readExpr :: String -> Expr
```

built-in read function does not match showExpr

Parsing

- Transforming a ”flat” string into something with a richer structure is called *parsing*
  - expressions
  - programming languages
  - natural language (swedish, english, dutch)
  - ...
- Very common problem in computer science
  - Many different solutions

Expressions

```
data Expr
  = Num Int
  | Add Expr Expr
  | Mul Expr Expr
```

- Let us start with a simpler problem
- How to parse

```
data Expr
  = Num Int
```

but we keep in mind that we want to parse real expressions...

Parsing Numbers

```
number :: String -> Int
```

```
Main> number ”23”
23
Main> number ”apa”
?
Main> number ”23+17”
?
```

Parsing Numbers

- Parsing a string to a number, ther cases:
  - (1) the string is a number, e.g. ”23”
  - (2) the string is not a number at all, e.g. ”apa”
  - (3) the string *starts* with a number, e.g. ”17+24”

Case (1) and (3) are similar...

how to model these?

```
type Parser a = String -> Maybe (a, String)
```

Parsing Numbers

```
number :: Parser Int
```

```
Main> number ”23”
Just (23, ””)
Main> number ”117junk”
Just (117, ”junk”)
Main> number ”apa”
Nothing
Main> number ”23+17”
Just (23, ”+17”)
```

how to implement?

Parsing Numbers

a helper function

with an extra argument

```
number :: Parser Int
number (c:s) | isDigit c = Just (digits 0 (c:s))
number _                 = Nothing
```

```
digits :: Int -> String -> (Int,String)
digits n (c:s) | isDigit c = digits (10*n + digitToInt c) s
digits n s                 = (n,s)
```

```
import Data.Char
```

at the top of your file

Parsing Numbers

```
number :: Parser Int
```

```
num :: Parser Expr
num s = case number s of
             Just (n, s’) -> Just (Num n, s’)
             Nothing      -> Nothing
```

a *case expression*

```
Main> num ”23”
Just (Num 23, ””)
Main> num ”apa”
Nothing
Main> num ”23+17”
Just (Num 23, ”+17”)
```

Expressions

```
data Expr
   = Num Int
   | Add Expr Expr
```

- Expressions are now of the form
  - ”23”
  - ”3+23”
  - ”17+3+23+14+0”

a *chain* of numbers with ”+”

Parsing Expressions

```
expr :: Parser Expr
```

```
Main> expr ”23”
Just (Num 23, ””)
Main> expr ”apa”
Nothing
Main> expr ”23+17”
Just (Add (Num 23) (Num 17), ””)
Main> expr ”23+17mumble”
Just (Add (Num 23) (Num 17), ”mumble”)
```

Parsing Expressions

start with a number?

is there a + sign?

can we parse *another* expr?

```
expr :: Parser Expr
expr s1 = case num s1 of
            Just (a,s2) -> case s2 of
                             ’+’:s3 -> case expr s3 of
                                         Just (b,s4) -> Just (Add a b, s4)
                                         Nothing     -> Just (a,s2)
                             _      -> Just (a,s2)
            Nothing     -> Nothing
```

Expressions

```
data Expr
   = Num Int
   | Add Expr Expr
   | Mul Expr Expr
```

- Expressions are now of the form
  - ”23”
  - ”3+23*4”
  - ”17*3+23*5*7+14”

a chain of *terms* with ”+”

a chain of *factors* with ”*”

Expression *Grammar*

- expr ::= term “**+**” ... “**+**” term
- term ::= factor “*****” ... “*****” factor
- factor ::= number

Parsing Expressions

```
expr :: Parser Expr
expr s1 = case term s1 of
            Just (a,s2) -> case s2 of
                            '+':s3 -> case expr s3 of
                                      Just (b,s4) -> Just (Add a b, s4)
                                      Nothing     -> Just (a,s2)
                            _      -> Just (a,s2)
            Nothing     -> Nothing
```

```
term :: Parser Expr
term = ?
```

Parsing Terms

```
term :: Parser Expr
term s1 = case factor s1 of
            Just (a,s2) -> case s2 of
                             ’*’:s3 -> case term s3 of
                                         Just (b,s4) -> Just (Mul a b, s4)
                                         Nothing     -> Just (a,s2)
                             _      -> Just (a,s2)
            Nothing     -> Nothing
```

just **copy** the code from expr and make some **changes**!

NO!!

Parsing Chains

```
chain :: Parser a -> Char -> (a->a->a) -> Parser a
```

```
chain p op f s1 =
  case p s1 of
    Just (a,s2) -> case s2 of
                     c:s3 | c == op -> case chain p op f s3 of
                                         Just (b,s4) -> Just (f a b, s4)
                                         Nothing     -> Just (a,s2)
                     _              -> Just (a,s2)
    Nothing     -> Nothing
```

argument op

recursion

argument f

argument p

a *higher-order function*

```
expr, term :: Parser Expr
expr  = chain term ’+’ Add
term  = chain factor ’*’ Mul
```

Factor?

```
factor :: Parser Expr
factor = num
```

Parentheses

- So far no parentheses
- Expressions look like
  - 23
  - 23+5*17
  - 23+5*(17+23*5+3)

a factor can be a parenthesized expression again

Expression Grammar

- expr ::= term “**+**” ... “**+**” term

- term ::= factor “*” ... “*” factor

- factor ::= number
  | “**(**“ expr “**)**”

Factor

```
factor :: Parser Expr
factor (’(’:s) =
   case expr s of
      Just (a, ’)’:s1) -> Just (a, s1)
      _                -> Nothing

factor s = num s
```

Reading an Expr

```
Main> readExpr ”23”
Just (Num 23)
Main> readExpr ”apa”
Nothing
Main> readExpr ”23+17”
Just (Add (Num 23) (Num 17))
```

```
readExpr :: String -> Maybe Expr
readExpr s = case expr s of
               Just (a,””) -> Just a
               _           -> Nothing
```

Summary

- Parsing becomes easier when
  - Failing results are explicit
  - A parser also produces the *rest* of the string
- Case expressions
  - To look at an intermediate result
- Higher-order functions
  - Avoid copy-and-paste programming

The Code (1)

```
readExpr :: String -> Maybe Expr
readExpr s = case expr s of
                      Just (a,””) -> Just a
                      _           -> Nothing

expr, term :: Parser Expr
expr  = chain term ’+’ Add
term  = chain factor ’*’ Mul

factor :: Parser Expr
factor (’(’:s) =
   case expr s of
      Just (a, ’)’:s1) -> Just (a, s1)
      _                -> Nothing
factor s = num s
```

The Code (2)

```
chain :: Parser a -> Char -> (a->a->a) -> Parser a
chain p op f s1 =
   case p s1 of
     Just (a,s2) -> case s2 of
                      c:s3 | c == op -> case chain p op f s3 of
                                          Just (b,s4) -> Just (f a b, s4)
                                          Nothing     -> Just (a,s2)
                      _              -> Just (a,s2)
     Nothing     -> Nothing
```

```
number :: Parser Int
number (c:s) | isDigit c = Just (digits 0 (c:s))
number _                 = Nothing

digits :: Int -> String -> (Int,String)
digits n (c:s) | isDigit c = digits (10*n + digitToInt c) s
digits n s                 = (n,s)
```

Testing readExpr

```
prop_ShowRead :: Expr -> Bool
prop_ShowRead a =
    readExpr (show a) == Just a
```

```
Main> quickCheck prop_ShowRead
Falsifiable, after 3 tests:
-2*7+3
```

negative numbers?

Fixing the Number Parser

```
number :: Parser Int
number (c:s) | isDigit c = Just (digits 0 (c:s))
number ('-':s)            = fmap neg (number s)
number _                  = Nothing
```

```
fmap :: (a -> b) -> Maybe a -> Maybe b
fmap f (Just x) = Just (f x)
fmap f Nothing = Nothing
```

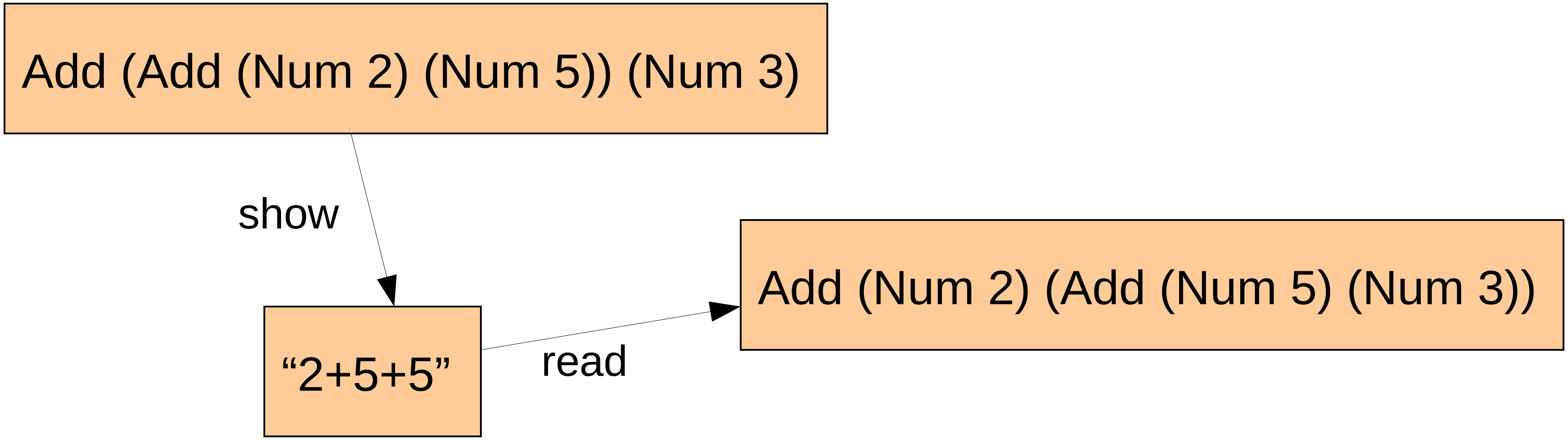
```
neg :: (Int,String) -> (Int,String)
neg (x,s) = (-x,s)
```

Testing again

```
Main> quickCheck prop_ShowRead
Falsifiable, after 5 tests:
2+5+3
```

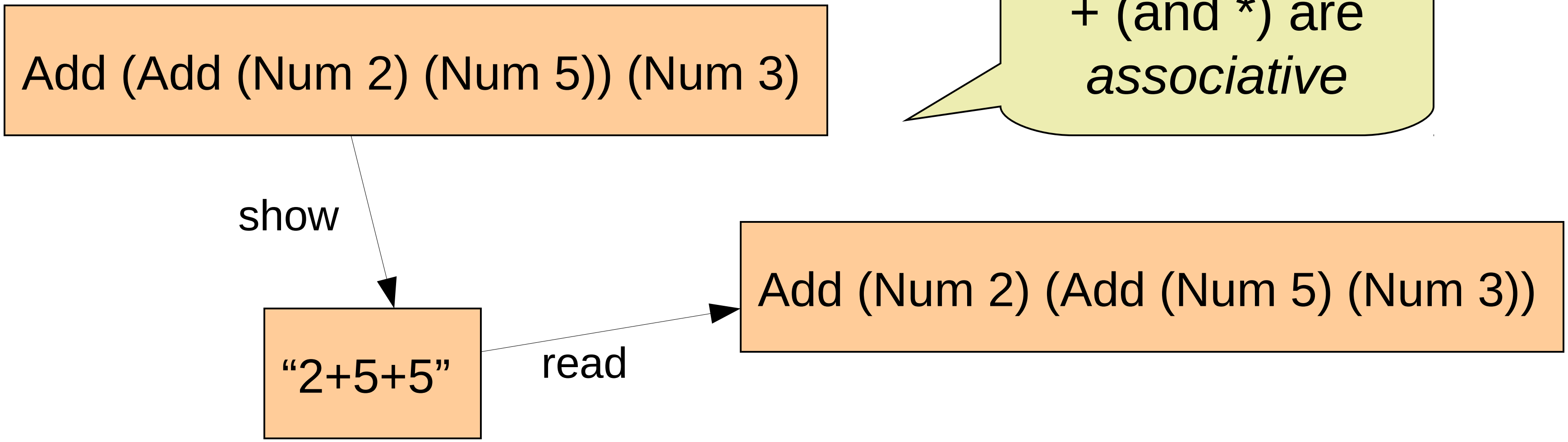
Testing again

```
Main> quickCheck prop_ShowRead
Falsifiable, after 5 tests:
2+5+3
```


Add (Add (Num 2) (Num 5)) (Num 3)
show
“2+5+5”
read
Add (Num 2) (Add (Num 5) (Num 3))

Testing again

```
Main> quickCheck prop_ShowRead
Falsifiable, after 5 tests:
2+5+3
```



Add (Add (Num 2) (Num 5)) (Num 3)

show

“2+5+5”

read

Add (Num 2) (Add (Num 5) (Num 3))

+ (and *) are *associative*

Fixing the Property (1)

The result does not have to be *exactly* the same, as long as the *value* does not change.

```
prop_ShowReadEval :: Expr -> Bool
prop_ShowReadEval a =
   fmap eval (readExpr (show a)) == Just (eval a)
```

```
Main> quickCheck prop_ShowReadEval
OK, passed 100 tests.
```

Fixing the Property (2)

The result does not have to be *exactly* the same, only after rearranging associative operators

```
prop_ShowReadAssoc :: Expr -> Bool
prop_ShowReadAssoc a =
  readExpr (show a) == Just (assoc a)
```

non-trivial recursion and pattern matching

```
assoc :: Expr -> Expr
assoc (Add (Add a b) c) = assoc (Add a (Add b c))
assoc (Add a b)         = Add (assoc a) (assoc b)
assoc (Mul (Mul a b) c) = assoc (Mul a (Mul b c))
assoc (Mul a b)         = Mul (assoc a) (assoc b)
assoc a                 = a
```

(study this definition and what this function does)

```
Main> quickCheck prop_ShowReadAssoc
OK, passed 100 tests.
```

Properties about Parsing

- We have checked that readExpr correctly processes anything produced by showExpr
- Is there any other property we should check?
  - What can still go wrong?
  - How to test this?

Very difficult!

Summary

- Testing a parser:
  - Take any expression,
  - convert to a String (show),
  - convert back to an expression (read),
  - check if they are the same
- Some structural information gets lost
  - associativity!
  - use “eval”
  - use “assoc”