

# Compiling functional languages

<http://www.cse.chalmers.se/edu/year/2011/course/CompFun/>

Lecture 7

Lazy evaluation

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# Laziness

- Main principle: names stand for arbitrary expressions, not just their final values
- Consequence: an expression bound to a name must not be evaluated until the name is first referenced
- Optimization: an expression bound to a name must not be re-evaluated after the name is first referenced

# Laziness

- Note: a lazy language
  - is fundamentally distinct from our strict core, with a different semantics
  - may however be implemented by translation into our strict core
  - may also reuse the syntax, type system, etc, of our strict core
- To emphasize distinction, we write a lazy  $e$  as  $e$

# Translating laziness

- Delaying evaluation can be implemented by creating 0-arity closures (a.k.a. thunks)
- Must be done for
  - function arguments (bound to parameters)
  - constructor arguments (bound to var-patterns)
  - RHS of let-bindings
- Avoiding re-evaluation will require some form of memoization/mutation

# Building/entering thunks

- Building a thunk:

$\text{translate } (\underline{e_0} \ e_1) = (\text{translate } \underline{e_0}) \ (CL \ f \ ys)$

where

$ys = \text{fvs}(e_1)$

and  $f$  is a fresh top-level name defined as

$f = \backslash x_{\text{this}} \rightarrow \text{case } x_{\text{this}} \text{ of } CL \ \_ \ ys \rightarrow \text{translate } \underline{e_1}$

- Similarly for RHS of top- and let-bindings
- Entering a thunk:

$\text{translate } \underline{x} = \text{case } x \text{ of } CL \ f \rightarrow f \ x$

# Avoiding re-evaluation

- After translation, a variable will denote a thunk (a closure expecting no arguments) on the heap
- Referencing a variable means entering its closure, which triggers the delayed evaluation
- After evaluation, the thunk should mutate:
  - (a) either into a thunk that just returns the value
  - (b) or into the value itself!

# Thunk mutation (a)

- Concretely, a thunk  $CL\ f\ ys$ , with

$f = \lambda x_{this} \rightarrow \text{case } x_{this} \text{ of } CL\ \_ \ ys \rightarrow e$

should mutate into  $CL\ f_{done}\ v$  when  $e$  has evaluated to  $v$ , and where

$f_{done} = \lambda x_{this} \rightarrow \text{case } x_{this} \text{ of } CL\ \_ \ v \rightarrow v$

is a a common run-time system function

- Advantage: uniform code for referencing a variable before/after mutation
- Disadvantage: persistent overhead of a fun-call

# Thunk mutation (b)

- Here, a thunk `CL f ys`, with

`f = \xthis -> case xthis of CL _ ys -> e,`

may mutate into `K vs` when `e` evaluates into `v` and `v` is a pointer to (a sufficiently small) `K vs` on the heap

- Advantage: no overhead of indirect jumps after evaluation
- Disadvantage: slightly more complicated variable referencing code:

translate `x` = `case x of CL f -> f x; _ -> x`



# Thunk mutation

- Scheme (a)  $\approx$  push/enter (with no push!)
- Scheme (b)  $\approx$  eval/apply (with no apply!)
- Both schemes can however coexist with an overall eval/apply arity-matching strategy
- Scheme (a) might also be necessary as a last resort, unless all thunks are made big enough to hold the largest possible heap value
- Moreover, scheme (a) allows update with unboxed non-pointer values

# Thunk mutation

- Scheme (a) in concrete C code:

```
WORD f (WORD xthis) {  
    WORD y1 = xthis[1]; ...; WORD ym = xthis[m];  
    WORD v = <code for evaluating e>;  
    xthis[0] = fdone;  
    xthis[1] = v;  
    return v;  
}
```

- Scheme (b) is similar, but must resort to (a) if  $v$  points to a node bigger than  $m+1$  words

# Function thunks

- What if a thunk evaluates to a closure?

`closureConvert (translate ( $x e_1 \dots e_m$ )) =`

`case (case  $x$  of CL  $f \rightarrow f x$ ) of CL  $f n$`

`|  $m == n \rightarrow f x e_1 \dots e_m$`

`|  $m < n \rightarrow CL \text{pap}_{n-m,m} (n-m) x e_1 \dots e_m$`

`|  $m > n \rightarrow \text{apply}_{m-n} (f x e_1 \dots e_n) e_{n+1} \dots e_m$`

- Observation: the ordinary closure-entry code works just as well for thunks ( $n = 0$ )! Equivalent:

`case  $x$  of CL  $f n$`

`|  $m == n \rightarrow f x e_1 \dots e_m$`

`|  $m < n \rightarrow CL \text{pap}_{n-m,m} (n-m) x e_1 \dots e_m$`

`|  $m > n \rightarrow \text{apply}_{m-n} (f x e_1 \dots e_n) e_{n+1} \dots e_m$`

- Requires that thunks store a zero arity as param 2

# Constructor thunks

- A common case with scheme (b):

translate ( $h = \lambda x \rightarrow \text{case } x \text{ of } K_i \text{ } x_{s_i} \rightarrow e_i$ ) =

$h = \lambda x \rightarrow \text{case (case } x \text{ of CL } f \rightarrow f \text{ } x; \_ \rightarrow x) \text{ of}$   
 $K_i \text{ } x_{s_i} \rightarrow \text{translate } \underline{e_i}$

- Might be optimized into

$h = \lambda x \rightarrow \text{case } x \text{ of CL } f \rightarrow h (f \text{ } x); K_i \text{ } x_{s_i} \rightarrow \text{translate } \underline{e_i}$

- That is, the cost of checking evaluatedness can be hidden in the ordinary code for branching
- Requires that closures are also given a tagged representation, with a globally unique tag

# Simple optimizations

- A very common pattern:

$$\text{translate } (\underline{e_0} \ x) = (\text{translate } \underline{e_0}) \ (\text{CL } f \ x)$$

where

$$f = \lambda x_{\text{this}} \rightarrow \text{case } x_{\text{this}} \text{ of } \text{CL } \_ \ x \rightarrow \\ \text{case } x \text{ of } \text{CL } f \rightarrow f \ x$$

- But a closure that just enters  $x$  is equal to  $x$ !
- Thus:

$$\text{translate } (\underline{e_0} \ x) = (\text{translate } \underline{e_0}) \ x$$

# Simple optimizations

- Literals are already evaluated:

$$\text{translate } (\underline{e_0} \ n) = (\text{translate } \underline{e_0}) \ (\text{CL } f_{\text{done}} \ n)$$

- Variables known to be evaluated may be treated the same way (scheme (a)):

let  $x = K \ es$  in ...

$$\text{translate } (\underline{e_0} \ x) = (\text{translate } \underline{e_0}) \ (\text{CL } f_{\text{done}} \ x)$$

- Or using scheme (b):

let  $x = K \ es$  in ...

$$\text{translate } (\underline{e_0} \ x) = (\text{translate } \underline{e_0}) \ x$$

# Exploiting strictness

- Consider a function  $h = \lambda x \rightarrow \text{case } x \text{ of } K_i \ xs_i \rightarrow e_i$  and a call  $h \ (y \ \top)$
- After translation we would have
$$h = \lambda x \rightarrow \text{case } x \text{ of } CL \ f \rightarrow h \ (f \ x); K_i \ xs_i \rightarrow e_i$$
and the call would have become  $h \ (CL \ f \ y)$ where  $f = \lambda x_{\text{this}} \rightarrow \text{case } x_{\text{this}} \text{ of } CL \ \_ \ y \rightarrow y \ \top$
- Clearly the thunk given to  $h$  will be entered right away, so an equivalent call is simply  $h \ (y \ \top)$
- A function like  $h$ , which can be called "by value" just as well as lazily, is characterized as strict

# Strictness

- Formally, a function  $h$  is strict if  $h\ e$  diverges for all non-terminating  $e$
- In other words,  $h$  either always diverges, or it needs to inspect the value of its argument:
  - branch according to its constructor tag, or
  - feed it to a primitive operator, or
  - apply it to other arguments
- (For higher arities, we say a function is strict/non-strict in argument 1, 2, ...)



# Strictness analysis

- Despite many examples of obviously strict functions, the strictness property of functions in general is undecidable
- Still, even a coarse approximation to strictness is beneficial to the efficiency of lazy languages
- Many safe strictness analysis techniques exist (and every lazy language compiler implements one)
- The classic approach is based on abstract interpretation (example follows Wadler, 1987)

# Abstract interpretation

- Reduce computations over big or infinite value domains to abstract computations over small and finite abstract domains
- Iterate abstract function behavior until fixpoint
- Choose the abstract domains so that they reveal interesting program properties
- For strictness analysis, let the abstract domains capture varying degrees of definedness

# Strictness analysis

## abstract interpretation

- Let the abstract domain of integers be
  - $\top$  - any concrete integer value
  - $\perp$  - the undefined (non-terminating) integer
- Let the abstract domain of integer lists be
  - $\top_\epsilon$  - a finite list with no undefined elements
  - $\perp_\epsilon$  - a finite list with some undefined elements
  - $\infty$  - an infinite list (with an undefined tail)
  - $\perp$  - the fully undefined list
- Order elements as depicted!

# Strictness analysis

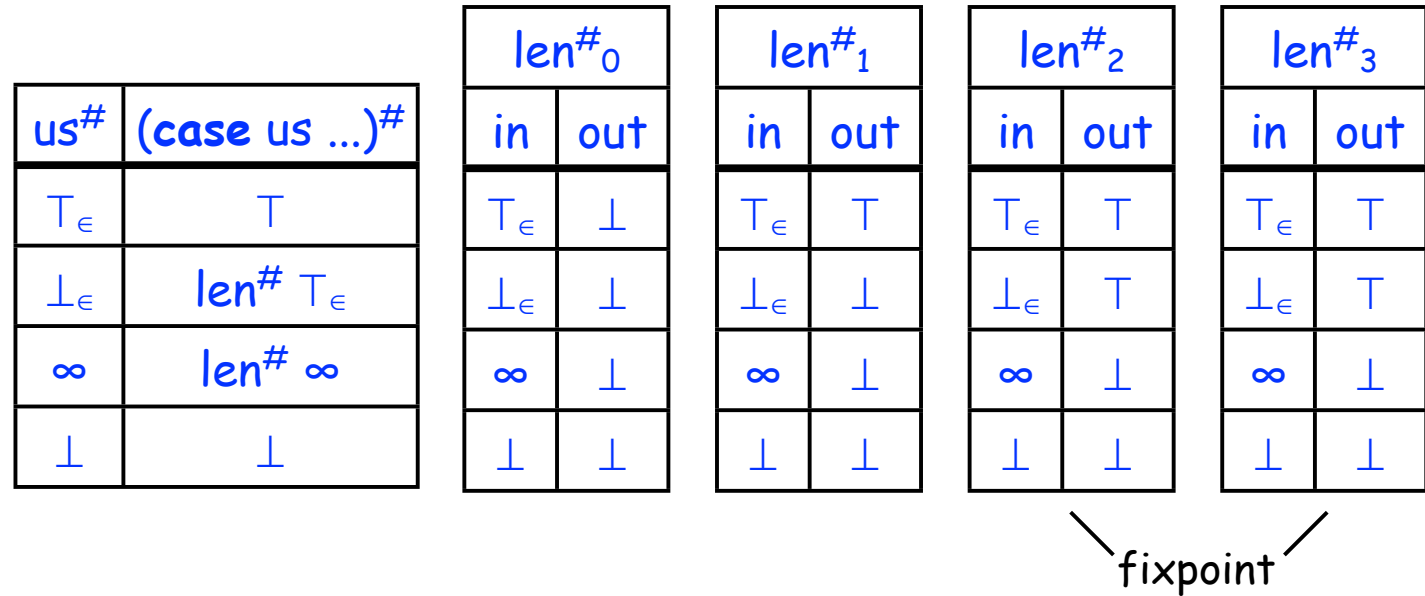
## abstract interpretation

- Let  $x^\#$  be the abstraction of value  $x$
- Some abstract values:  
 $0^\# = \top$      $1^\# = \top$      $99999^\# = \top$      $\perp^\# = \perp$   
 $[]^\# = \top_\epsilon$      $(1:2:[])^\# = \top_\epsilon$      $(1:\perp:[])^\# = \perp_\epsilon$      $(1:\perp)^\# = \infty$
- Let  $f^\#$  be the abstraction of function  $f$
- Calculate abstract function tables using finite value enumeration, monotonicity, least upper / greatest lower bounds, fixpoint iteration, ...

# Example

$x^\#$	$xs^\#$	$(x:xs)^\#$
$\top$	$\top_\epsilon$	$\top_\epsilon$
$\top$	$\perp_\epsilon$	$\perp_\epsilon$
$\top$	$\infty$	$\infty$
$\top$	$\perp$	$\infty$
$\perp$	$\top_\epsilon$	$\perp_\epsilon$
$\perp$	$\perp_\epsilon$	$\perp_\epsilon$
$\perp$	$\infty$	$\infty$
$\perp$	$\perp$	$\infty$

$len = \lambda us \rightarrow \text{case } us \text{ of } [] \rightarrow 0; x:xs \rightarrow 0 + len \ xs$



# Example

- Conclusions from the abstract interpretation:
  - `len` maps  $\perp$  to  $\perp$ , so it's safe to evaluate its argument before the call
  - `len` maps  $\infty$  to  $\perp$ , so it's also safe to evaluate all tails of the argument before the call
  - `len` maps  $\perp_\epsilon$  to  $\top$ , so it's not safe to evaluate any elements of the argument before the call

# Summary

- Laziness is straightforward to implement, but efficiency relies heavily on optimizations
- Strictness analysis is particularly useful, classic technique is based on abstract interpretation
- Course summary:
  - Mapping a FL to C quite simple (modulo GC issues)
  - Challenge lies in exploiting source-to-source transformations (including type-based ones)
  - Hands-on experience is the only lasting value, **complete your compiler projects!**