

Compiling functional languages

<http://www.cse.chalmers.se/edu/year/2011/course/CompFun/>

Lecture 2
C representation

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Recall our core language

```
prog ::= module K where ds
d ::= x = e
e ::= x | K es | lit | e e | \x -> e | let ds in e | case e of alts
alt ::= lit -> e | K xs -> e
```

Good for analysis and optimization,
but still not directly mappable to C

Restrict form even further

```
prog ::= module K where ds
d ::= f = \xs -> b | x = K es | x = e
b ::= let ds in b | case x of alts | b || b | fail | e
e ::= x | f | x es | f es | lit | K
alt ::= K -> b | K xs -> b | lit -> b
```

Main difference:

expression syntax now depends on position

- The right-hand side of a declaration (**d**)
- The body of a function (**b**)
- Arguments to functions and constructors (**e**)

Minor differences: marking known functions (**f**)

+ multi-argument abstraction & application

C correspondence

Declarations:

`f = \xs -> b`

A C function declaration (if on the top level)

`x = K es`

A `malloc()` call followed by assignments

`x = e`

A single assignment

Function bodies:

`let ds in b`

A sequence of assignments (if ds not recursive)

`case x of alts`

A `switch` statement

`e`

A `return` statement

`fail and e || e`

`break` and sequential composition

Expressions:

`x`

Variable `x`

`f es`

A function call to `f` (if arity matches)

`lit`

Literal `lit` (if not a string literal)

`Ki`

Integer literal `i`

`x es and f`

Deferred...

Data layout

```
typedef int *Ptr;
```

Basic assumptions:

```
(Ptr)(int)x = x
```

```
(int)(Ptr)y = y
```

Construction:

```
x = Ki e1 ... en
```

```
Ptr x = malloc((n+1)*sizeof(int));
```

```
x[0] = i;
```

```
x[1] = (int)e1; ... x[n] = (int)en;
```

Deconstruction:

```
case x of
```

```
....
```

```
Ki x1 ... xn -> bodyi
```

```
switch (x[0]) {
```

```
...
```

```
case i: { Ptr x1 = (Ptr)x[1]; ...
```

```
Ptr xn = (Ptr)x[n];
```

```
bodyi }
```

Nullary constructors

Could just use the generic form:

$x = K_i$

case x **of**

...

$K_i \rightarrow$ body

Ptr $x = \text{malloc}(\text{sizeof}(\text{int}));$

$x[0] = i;$

switch ($x[0]$) {

...

case i : { body }

For better memory efficiency, encode as small pointer:

K_i

case x **of**

$K_i \rightarrow$ body _{i}

...

$K_j \ x_1 \dots x_n \rightarrow$ body _{j}

(Ptr) i

switch ((int) x) {

case i : { body _{i} }

...

default: **switch** ($x[0]$) {

case j : { Ptr $x_1 = (\text{Ptr})x[1]; \dots$

Ptr $x_n = (\text{Ptr})x[n];$

body _{j} }

Single constructors

Could just use the generic form:

$x = K_0 e_1 \dots e_n$

case x of

$K_0 x_1 \dots x_n \rightarrow \text{body}_i$

`Ptr x = malloc((n+1)*sizeof(int));`

`x[0] = 0;`

`x[1] = (int)e1; ... x->arg[n] = (int)en;`

switch (`x[0]`) {

case 0: { `Ptr x1 = (Ptr)x[1]; ...`

`Ptr xn = (Ptr)x[n];`

`body0 }`

For better efficiency, encode without a tag:

$x = K_0 e_1 \dots e_n$

case x of

$K_0 x_1 \dots x_n \rightarrow \text{body}_0$

`Ptr x = malloc(n*sizeof(int));`

`x[0] = (int)e1; ...`

`x[n-1] = (int)en;`

`Ptr x1 = (Ptr)x[0]; ...`

`Ptr xn = (Ptr)x[n-1];`

`body0`

On primitives

- Syntactically, a primitive operation is just a named function f the compiler already knows about (implicitly declared)
- However, there are two sets of such names:
 - Primitives of the source language (up to you)
 - Primitives of the target (defined by C)
- Requires a conscious mapping (again up to you!)

Obtaining restricted form

- Collecting multiple arguments:

$$\text{translate } (\dots((x \ e_1) \ e_2) \dots \ e_n) \quad = \quad x \ e_1 \ e_2 \ \dots \ e_n$$

$$\text{translate } (\backslash x_1 \rightarrow \dots \rightarrow \backslash x_n \rightarrow e) \quad = \quad \backslash x_1 \ \dots \ x_n \rightarrow e$$

- Normalizing an e depending on position:

$$\text{normD } (f = \backslash xs \rightarrow e) \quad = \quad f = \backslash xs \rightarrow \text{normB } e$$

$$\text{normD } (x = K \ es) \quad = \quad \dots$$

$$\text{normD } (x = e) \quad = \quad ds \ ++ \ x = e' \quad \text{where } (ds, e') = \text{normE } e$$

$$\text{normB } (\backslash xs \rightarrow e) \quad = \quad \text{let normD } (x = \backslash xs \rightarrow e) \text{ in } x$$

$$\text{normB } (\text{case/let}) \quad = \quad \dots$$

$$\text{normB } e \quad = \quad \text{let } ds \text{ in } e' \quad \text{where } (ds, e') = \text{normE } e$$

Obtaining restricted form

- Normalizing an e depending on position:

$$\begin{aligned} \text{normE } x &= ([], x) \\ \text{normE } \text{lit} &= ([], \text{lit}) \\ \text{normE } K &= ([], K) \\ \text{normE } (f \ e_1 \ \dots \ e_n) &= (ds_1 ++ \dots ++ ds_n, f \ e_1' \ \dots \ e_n') \\ \quad \text{where } (ds_i, e_i') &= \text{normE } e_i \\ \text{normE } (x \ e_1 \ \dots \ e_n) &= \dots \\ \text{normE } (e \ e_1 \ \dots \ e_n) &= (\text{normD } (x = e) ++ ds, e') \\ \quad \text{where } (ds, e') &= \text{normE } (x \ e_1 \ \dots \ e_n) \\ \text{normE } (\text{let } ds \ \text{in } e) &= (\text{normD } ds ++ ds', e') \\ \quad \text{where } (ds', e') &= \text{normE } e \\ \text{normE } (\text{case } \dots) &= (\text{normD } (x = _ \rightarrow \text{case } \dots), x ()) \\ \text{normE } e &= (\text{normD } (x = e), x) \end{aligned}$$

Manipulating scopes

- The hazards of moving and merging decls:

$\text{normE } (\text{let } x = 3 \text{ in let } x = 4 \text{ in } x) = ([x = 3, x = 4], x) \text{ ???}$

$\text{normE } (f (\text{let } x = 3 \text{ in } x) x) = ([x = 3], f x x) \text{ ???}$

- Solution: alpha-convert local scope

$\text{normE } (\text{let } x = 3 \text{ in let } x = 4 \text{ in } x) = ([x = 3, y = 4], y) \quad y \text{ new}$

$\text{normE } (f (\text{let } x = 3 \text{ in } x) x) = ([y = 3], f y x) \quad y \text{ new}$

- Detecting name capture:
 - check against all vars in scope (keep an environment)
 - or check the vars actually free in shadowed exprs

Free variables

- A standard notion in lambda calculus:

$fv\ x$	$=$	$\{x\}$
$fv\ (K\ e_1\ \dots\ e_n)$	$=$	$fv\ e_1\ \cup\ \dots\ \cup\ fv\ e_n$
$fv\ lit$	$=$	$\{\}$
$fv\ (e\ e')$	$=$	$fv\ e\ \cup\ fv\ e'$
$fv\ (\lambda x\ \rightarrow e)$	$=$	$fv\ e\ \setminus \{x\}$
$fv\ (let\ ds\ in\ e)$	$=$	$(fv\ ds\ \cup\ fv\ e)\ \setminus\ dom\ ds$
$fv\ (case\ e\ of\ alt_1\ ;\ \dots\ ;\ alt_n)$	$=$	$fv\ e\ \cup\ fv\ alt_1\ \cup\ \dots\ \cup\ fv\ alt_n$
$fv\ (x_1 = e_1\ ;\ \dots\ ;\ x_n = e_n)$	$=$	$fv\ e_1\ \cup\ \dots\ \cup\ fv\ e_n$
$fv\ (K\ xs\ \rightarrow e)$	$=$	$fv\ e\ \setminus \{xs\}$
$fv\ (lit\ \rightarrow e)$	$=$	$fv\ e$
$dom\ (x_1 = e_1\ ;\ \dots\ ;\ x_n = e_n)$	$=$	$\{x_1, \dots, x_n\}$

After normalization

- A program form corresponding to C syntax, but with some serious caveats:
 - Function declarations must be on top level only
 - Only function declarations may be recursive
 - Function calls must match arity of callee
 - Function names must not be used as values
 - Unknown functions cannot be called
- (Assuming string literals already desugared away...)

Recursive declarations

- The simple case: recursive functions
(corresponds to cross-referencing code-blocks on the assembly level)
- Also a non-issue in lazy languages (where every name denotes a code-block in general)
- Normally forbidden in strict languages:
 - Recursive non-functions: $x = K y; y = L x$
 - Recursive non-values: $x = f y; y = g x$

Recursive declarations

- Natural approach: forbid recursive non-functions and non-values in our language as well (can be checked initially and easily preserved)
- However, checking after sorting according to dependency order adds valuable expressiveness:

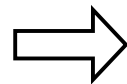
```
let f = \xs -> ... x ... g ...
```

```
  x = 2+3
```

```
  g = \ys -> ... f ...
```

```
  y = f []
```

```
in e
```



```
let x = 2+3
```

```
in let f = \xs -> ... x ... g ...
```

```
      g = \ys -> ... f ...
```

```
in let y = f []
```

```
in e
```

Only top-level functions

- Naive idea: just move the declarations!
- Problem: loss of local scope

$f = \lambda a \rightarrow \text{let } g = \lambda b \rightarrow a + b$
 $\quad \text{in } g \ 7$ \Rightarrow $g = \lambda b \rightarrow \underline{a} + b$
 $f = \lambda a \rightarrow g \ 7$

- A way forward: first turn free variables into parameters!

$f = \lambda a \rightarrow \text{let } g = \lambda a \ b \rightarrow a + b$
 $\quad \text{in } g \ a \ 7$ \Rightarrow $g = \lambda a \ b \rightarrow a + b$
 $f = \lambda a \rightarrow g \ a \ 7$

Lambda-lifting

- An algorithm for lifting functions (lambda-abstractions) out of their scope
[Johnsson (1985), variant in SPJ's book]
- Fact: lifting itself is trivial (just avoid name-clashes) — adding the necessary parameters to functions is the interesting part
- We'll study a formulation that only performs the interesting part!

A lambda-lifter

- Assume fv splits its output as (fs, xs)
- Assume ext maps each f in scope to its extra args
- The interesting cases:

$$\begin{aligned} \text{lift ext } (f \text{ es}) &= f \text{ (ext } f \text{ ++ lift ext es)} \\ \text{lift ext } (\text{let } ds \text{ in } b) &= \text{let } (\text{lift ext}' ds) \text{ in } (\text{lift ext}' b) \\ \text{where } (f_1 \dots f_n, xs) &= fv ds \setminus \text{dom } ds \\ \quad xs' &= xs \cup \text{ext } f_1 \cup \dots \cup \text{ext } f_n \\ \quad \text{ext}' f &= \text{if } (f \text{ `elem` dom } ds) \text{ then } xs' \text{ else ext } f \\ \dots & \\ \text{lift ext } (f = \lambda xs \rightarrow e) &= f = \lambda (\text{ext } f \text{ ++ } xs) \rightarrow \text{lift ext } e \end{aligned}$$

A lambda-lifter

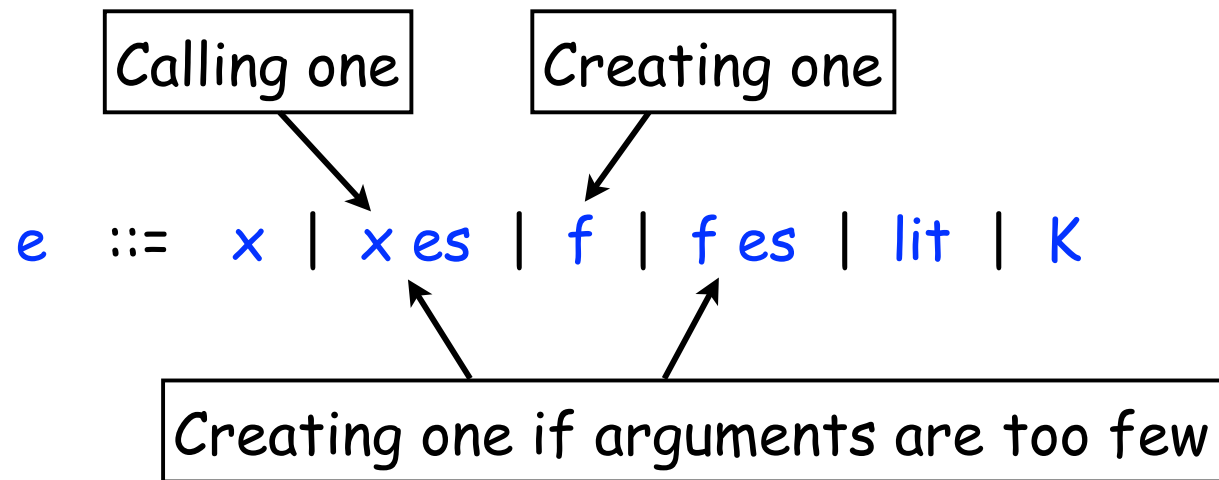
- On the top-level:

`module K where ds` \Rightarrow `module K where lift ext0 ds`
`where ext0 f = []`

- Result: a program where each declaration `f = \xs -> e` has zero free variables
- Such decls can easily be moved to the top
- Variant: exclude the global non-functions when listing free variables in `lift`

Anonymous functions

- Our latest expression grammar:



- Must be supported - not a functional language otherwise!
- Requires the concept of closures!

Closures

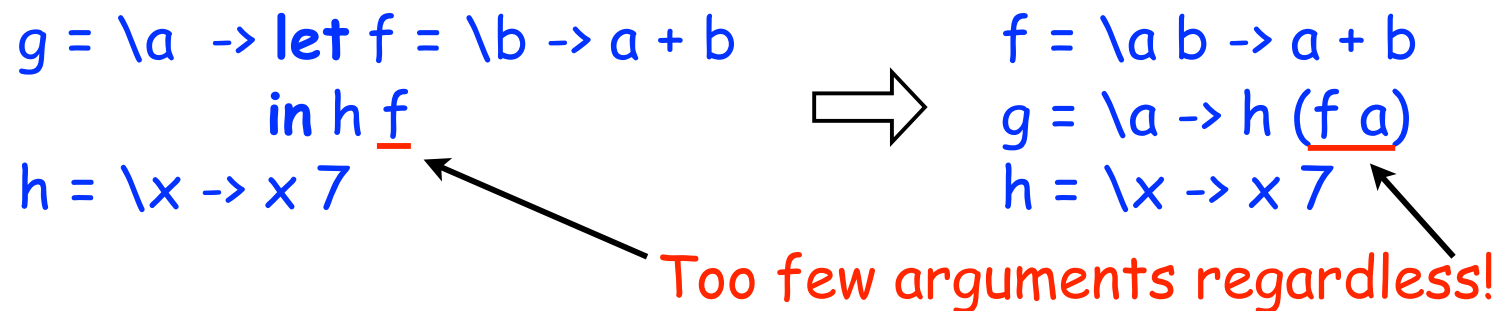
- The generic representation of functions: a function pointer with a list of free variables
- The limits of lambda-lifting:

$g = \lambda a \rightarrow \text{let } f = \lambda b \rightarrow a + b$
 $\quad \quad \quad \text{in } h \text{ } \underline{f}$
 $h = \lambda x \rightarrow x \cdot 7$

\Rightarrow

$f = \lambda a \ b \rightarrow a + b$
 $g = \lambda a \rightarrow h \text{ } \underline{(f \ a)}$
 $h = \lambda x \rightarrow x \cdot 7$

Too few arguments regardless!



- Closures can represent partial applications, even in the presence of free variables
- Nevertheless, lambda-lifting before closure-conversion simplifies the presentation somewhat

Closure-conversion

- Assume a lambda-lifted $f = \lambda x_1 \dots x_n \rightarrow e$

$\text{closureConvert } f = \text{CL } f_0 \ n$

$\text{closureConvert } (f \ e_1 \dots e_m) =$

$\quad | \ m < n \quad = \ \text{CL } f_m \ (n-m) \ e_1 \dots e_m$

...

where f_m is a new top-level function

$f_m = \lambda x_{\text{this}} \ x_{m+1} \dots x_n \rightarrow \text{case } x_{\text{this}} \ \text{of}$

$\quad \text{CL } _ _ \ y_1 \dots y_m \rightarrow f \ y_1 \dots y_m \ x_{m+1} \dots x_n$

$\text{closureConvert } (x \ e_1 \dots e_m) = \text{case } x \ \text{of } \text{CL } f_{\text{unknown}} \ n$
 $\quad | \ m == n \rightarrow f_{\text{unknown}} \ x \ e_1 \dots e_m$

...

Closures

- CL is an ordinary constructor name (a K) and a closure term is just a constructor application that references an f
- After closure conversion, these applications will be our only references to function names outside function calls
- Note: static typing will actually require a CL_k for each closure arity k (as well as existentials and subtyping!), but we're past type-checking here!

Closure-conversion

- Example before and after lambda-lifting:

$g = \lambda a \rightarrow \text{let } f = \lambda b \rightarrow a + b$
 $\text{in } f \ g$
 $h = \lambda x \rightarrow x \ 7$

⇒

$f = \lambda a \ b \rightarrow a + b$
 $g = \lambda a \rightarrow h \ f$
 $h = \lambda x \rightarrow x \ 7$

- And after closure-conversion:

$f = \lambda a \ b \rightarrow a + b$
 $g = \lambda a \rightarrow \text{CL } f_1 \ 1 \ a$
 $h = \lambda x \rightarrow \text{case } x \ \text{of } \text{CL } f_{\text{unknown}} \ 1 \rightarrow f_{\text{unknown}} \ x \ 7$

$f_1 = \lambda x_{\text{this}} \ x_2 \rightarrow \text{case } x_{\text{this}} \ \text{of } \text{CL } _ _ \ y_1 \rightarrow f \ y_1 \ x_2$

- But we're still ignoring arity mismatches...

Matching arities

- Strategies for matching function arity with the number of arguments:
 - "Push/enter": arguments pushed and code entered unconditionally, matching done by called function
 - "Eval/apply": function evaluated and asked for arity by caller, then only applied if enough arguments are present

Push/enter

- Assume arguments $e_1 \dots e_m$ are on the stack
- In prologue to each function $f = \lambda x_1 \dots x_n \rightarrow e$:
 - If $m = n$, return result of call (popping $e_1 \dots e_n$)
 - If $m < n$, pop $e_1 \dots e_m$ and return closure corresponding to $\lambda x_{m+1} \dots x_n \rightarrow f e_1 \dots e_m x_{m+1} \dots x_m$
 - If $m > n$, enter result of current call after popping $e_1 \dots e_n$

Push/enter

- Simple model inspired by OO virtual methods
- Involves rather heavy use of indirect jumps
- Finding & counting all arguments on the stack is hard using C calling conventions
- In use: core characteristic of the original STG-machine (Peyton Jones, 1992), which is the back-end format used by GHC

Eval/apply

- Assume f has arity n
- For each call $f e_1 \dots e_m$:
 - If $m = n$, just return the result of the call
 - If $m < n$, return closure corresponding to $\lambda x_{m+1} \dots x_n \rightarrow f e_1 \dots e_m x_{m+1} \dots x_n$
 - If $m > n$, let x be the result of $f e_1 \dots e_n$ and continue applying call $x e_{n+1} \dots e_m$
- Technique used by common ML implementations (SML-NJ, O'Cam1), but nowadays also GHC

Checking arities

(eval/apply)

- Assuming $f = \lambda x_1 \dots x_n \rightarrow e$

$\text{closureConvert } f = \text{CL } f_0 \ n$

$\text{closureConvert } (f \ e_1 \dots \ e_m) =$

| $m == n$ = $f \ e_1 \dots \ e_m$

| $m < n$ = $\text{CL } f_m \ (n-m) \ e_1 \dots \ e_m$

| $m > n$ = $\text{apply}_{m-n} (f \ e_1 \dots \ e_n) \ e_{n+1} \dots \ e_m$

where each apply_k is a run-time system function TBD

- Note: checks are done at compile-time

Checking arities

(eval/apply)

- The full dynamic case (checks at run-time!):

$\text{closureConvert } (x \ e_1 \ \dots \ e_m) \quad = \quad \text{apply}_m \ x \ e_1 \ \dots \ e_m$

$\text{apply}_m = \lambda x_{\text{this}} \ x_1 \ \dots \ x_m \rightarrow \text{case } x_{\text{this}} \text{ of CL } f_{\text{unknown}} \ n$
| $m == n \rightarrow f_{\text{unknown}} \ x_{\text{this}} \ x_1 \ \dots \ x_m$
| $m < n \rightarrow \text{CL } \text{pap}_{n-m,m} \ (n-m) \ x_{\text{this}} \ x_1 \ \dots \ x_m$
| $m > n \rightarrow \text{apply}_{m-n} \ (f_{\text{unknown}} \ x_{\text{this}} \ x_1 \ \dots \ x_n) \ x_{n+1} \ \dots \ x_m$

$\text{pap}_{k,m} = \lambda x_{\text{this}} \ x_1 \ \dots \ x_k \rightarrow \text{case } x_{\text{this}} \text{ of CL } _ _ \ y_{\text{that}} \ y_1 \ \dots \ y_m \rightarrow$
 $\text{apply}_{m+k} \ y_{\text{that}} \ y_1 \ \dots \ y_m \ x_1 \ \dots \ x_k$

Summary

- C code generation involves
 - 1) Normalization (pretty straightforward)
 - 2) Lambda-lifting (known functions)
 - 3) Closure conversion (anonymous/partial apps)
- 3) supersedes 2) but is generally less efficient
- Challenge: avoid the need for special $\text{pap}_{k,m}$ functions for every combination of k and m
- Idea: make m a closure parameter as well, and write a generic $\text{pap}_{k,m}$ directly in assembly code