Here is the proof of the construction in **exercise 2 (NFA and ϵ-NFA) 1** that didn't work out during the exercise session. Remember in the session we concluded that since the language $\mathcal{L}$ is regular then we have a DFA $D = \{Q, \Sigma, q_0, \delta, F\}$ such that $L(D) = \mathcal{L}$, then we constructed an ϵ-NFA $\bar{D}$ as follows.
$\bar{D} = \{Q \cup \{\bar{q}\}, \Sigma, \bar{q}, \bar{\delta}, \{q_0\}\}$ such that $\bar{q} \notin Q$ and we defined the transition function $\bar{\delta}$ as follows.

$$\begin{aligned}
&\bar{\delta}(\bar{q}, \epsilon) = F \\
&\forall a \in \Sigma.\ \bar{\delta}(\bar{q}, a) = \emptyset \\
&\forall a \in \Sigma.\ \forall q \in Q.\ \bar{\delta}(q, a) = \{p \in Q \ \mid \delta(p, a) = q\} \\
&\forall q \in Q.\ \bar{\delta}(q, \epsilon) = \emptyset
\end{aligned}$$

The claim is that $L(\bar{D}) = \mathcal{L}^R$. Translating this claim into a formal statement we get the statement $S:\ \forall x \in \Sigma^*.\ \hat{\delta}(q_0, x) \in F \leftrightarrow \hat{\bar{\delta}}(\bar{q}, rev(x)) \cap \{q_0\} \neq \emptyset$. Then we noted that if we try to prove this statement we get stuck. for example if we assume $S(x)$ for some string x and try to prove $S(ax)$ then for one direction we are required to prove something like $\forall x \in \Sigma^*.\ \hat{\delta}(q_0, ax) \in F \rightarrow \hat{\bar{\delta}}(\bar{q}, rev(ax)) \cap \{q_0\} \neq \emptyset$. If we proceed with the proof (i.e. assuming the antecedent -left side of the implication- and proving the consequent -right side-), then if we have $\hat{\delta}(q_0, ax) \in F$ we get $\hat{\delta}(\delta(q_0, a), x)$ by the definition of $\hat{\delta}$ but then we are stuck since our induction hypothesis $S(x)$ say nothing about $\hat{\delta}(\delta(q_0, a), x)$ it only mentions the state q_0 not $\delta(q_0, a)$. Thus as we mentioned we need to strengthen our hypothesis. We stopped at this point in the session, and here is the rest of the proof.
As usual when we want to prove a non-trivial statement about an automaton we need a statement for each state (recall the answer to DFA.3), here we don't really have an explicit enumeration of the states Q, nevertheless we can still strengthen the statement S in the form
$T:\ \forall x \in \Sigma^*.\ \forall p, q \in Q.\ \hat{\delta}(p, x) = q \leftrightarrow \hat{\bar{\delta}}(q, rev(x)) \cap \{p\} \neq \emptyset$. Please note the forall quantification over the states Q, now we have something to say about each state. Still it is not quite obvious why a proof of this statement T gives us a proof of the statement S which is the one we actually want to prove. To see this let's see the instance of statement T at state q_0, we'll call this $T(q_0)$, it looks like this $\forall x \in \Sigma^*.\ \forall q \in Q.\ \hat{\delta}(q_0, x) = q \leftrightarrow \hat{\bar{\delta}}(q, rev(x)) \cap \{q_0\} \neq \emptyset$, now we know per our construction (definition of $\bar{\delta}$ above) and by definition of transition function for ϵ-NFA (look the ϵ closure in the book) that $\forall p \in F.\ \forall w.\ \hat{\bar{\delta}}(p, w) = \hat{\bar{\delta}}(\bar{q}, w)$, hence, $\forall q \in F.\ \hat{\bar{\delta}}(q, rev(x)) \cap \{q_0\} \neq \emptyset \leftrightarrow \hat{\bar{\delta}}(\bar{q}, rev(x)) \cap \{q_0\} \neq \emptyset$. By this we know that if we have a proof of $T(q_0)$ we have a proof of S. Taking a second look at the statement T we note that the state q in the statement is just a placeholder, and we can do without it simplifying the statement to be $T:\ \forall x \in \Sigma^*.\ \forall p \in Q.\ \hat{\bar{\delta}}(\hat{\delta}(p, x), rev(x)) \cap \{p\} \neq \emptyset$. Now let's prove the statement T by induction on the string x.

base case ϵ

For $p \in Q$ we have
$\hat{\bar{\delta}}(\hat{\delta}(p, \epsilon), rev(\epsilon)) = \hat{\bar{\delta}}(p, \epsilon)$ by definition of $\hat{\delta}$ for a DFA and rev. Per our definition of $\bar{\delta}$ and the definition of transition function for ϵ-NFA we know that

$\hat{\bar{\delta}}(p, \epsilon) = \{p\}$ since per the definition above the ϵ-closure of any state $p \in Q$ is $\{p\}$. Thus $\hat{\bar{\delta}}(\hat{\delta}(p, \epsilon), rev(\epsilon)) \cap \{p\} \neq \emptyset$ is True.

Inductive step

Assume $T(x) : \forall p \in Q.\ \hat{\bar{\delta}}(\hat{\delta}(p, x), rev(x)) \cap \{p\} \neq \emptyset$ for some $x \in \Sigma^*$.
We have $\hat{\bar{\delta}}(\hat{\delta}(p, ax), rev(ax)) = \hat{\bar{\delta}}(\hat{\delta}(\delta(p, a), x), rev(x)a)$ by definition of $\hat{\delta}$ and rev.
Let $P = \hat{\bar{\delta}}(\hat{\delta}(\delta(p, a), x), rev(x))$ then
(*) $\hat{\bar{\delta}}(\hat{\delta}(\delta(p, a), x), rev(x)a) = \bigcup_{q \in P} \bar{\delta}(q, a)$. by def of $\hat{\bar{\delta}}$
By induction hypothesis (applied to the state $\delta(p, a)$) we know that
$\hat{\bar{\delta}}(\hat{\delta}(\delta(p, a), x), rev(x)) \cap \{\delta(p, a)\} \neq \emptyset$
in other words we know $\delta(p, a) \in P$ and by definition of $\bar{\delta}$ we have
$\bar{\delta}(\delta(p, a), a) = \{q \in Q \mid \delta(q, a) = \delta(p, a)\}$ then $p \in \bar{\delta}(\delta(p, a), a)$ (also intuitively! $\bar{\delta}$ reverses δ)
and since $\delta(p, a) \in P$ by (*) above we get
$p \in \bigcup_{q \in P} \bar{\delta}(q, a)$ and thus
$\hat{\bar{\delta}}(\hat{\delta}(\delta(p, a), x), rev(x)a) \cap \{p\} \neq \emptyset$. By def of $\hat{\delta}$ and rev this is equivalent to $\hat{\bar{\delta}}(\hat{\delta}(p, ax), rev(ax)) \cap \{p\} \neq \emptyset$. Thus completing the inductive step.