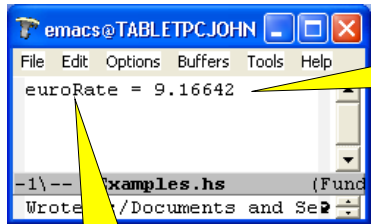


Creating the Definition



Give the name *euroRate* to the value 9.16642

```
euroRate = 9.16642
```

variable

Using the Definition

The prompt changes – we have now loaded a program.

```
Prelude> :l Examples
Main> euroRate
9.16642
Main> 53*euroRate
485.82026
Main>
```

Load the file *Examples.hs* into *ghci* – make the definition available.

We are free to make use of the definition.

A Function to convert Euros to SEK

A definition – placed in the file *Examples.hs*

A comment – to help us understand the program

```
-- convert euros to SEK
sek x = x*euroRate
```

Function name – the name we are defining.

Name for the argument

Expression to compute the result

Using the Function

Reload the file to make the new definition available.

```
Main> :r
Main> sek 53
485.82026
Main> euro (sek 49) == 49
True
```

The operator `==` tests whether two values are equal

Converting Back Again

```
-- convert SEK to euros
euro x = x/euroRate
```

```
Main> :r
Main> euro 485.82026
53.0
Main> euro (sek 49)
49.0
Main> sek (euro 217)
217.0
```

A Test

Automated Testing

- Define a function to perform the test for us

```
prop_EuroSek x = euro (sek x) == x
```

```
Main> prop_EuroSek 53
True
Main> prop_EuroSek 49
True
```

Performs the same tests as before – but now we need only remember the function name!

Writing Properties in Files

- Convention: functions names beginning "prop_" are *properties we expect to be True*
- Writing properties in files
 - Tells us *how* functions should behave
 - Tells us *what* has been tested
 - Lets us *repeat* tests after changing a definition

Automatic Testing

- Testing account for *more than half* the cost of a software project
- We will use a widely used Haskell library for *automatic random* testing

```
import Test.QuickCheck
```

Add *first* in the file of definitions – makes QuickCheck available.

Names are case sensitive.

Running Tests

```
Main> quickCheck prop_EuroSek  
Falsifiable, after 10 tests:  
3.75
```

The value for which the test fails.

It's *not* true!

Runs 100 randomly chosen tests

```
Main> sek 3.75  
34.374075  
Main> euro 34.374075  
3.75
```

Looks OK

The Problem

- There is a very tiny difference between the initial and final values

```
Main> euro (sek 3.75) - 3.75  
4.44089209850063e-016
```

e-016 means $.10^{-16}$

- Calculations are only performed to about 15 significant figures
- The property is wrong!

Fixing the Problem

- The result should be *nearly* the same
- The difference should be small – say <0.000001

```
Main> 2<3  
True  
Main> 3<2  
False
```

We can use $<$ to see whether one number is less than another

Defining "Nearly Equal"

- We can define new *operators* with names made up of symbols

```
x ~== y = x-y < 0.000001
```

Define a new operator $\sim==$

With arguments x and y

Which returns True if the *difference* between x and y is less than 0.000001

Testing ~==

```
Main> 3 ~== 3.0000001
True
Main> 3 ~== 4
True
```

OK

Huh? What's wrong?

```
x ~== y = x - y < 0.000001
```

Fixing the Definition

- A useful function

```
Main> abs 3
3
Main> abs (-3)
3
```

Absolute value

```
x ~== y = abs (x-y) < 0.000001
```

```
Main> 3 ~== 4
False
```

Fixing the Property

```
prop_EuroSek x = euro (sek x) ~== x
```

```
Main> prop_EuroSek 3
True
Main> prop_EuroSek 56
True
Main> prop_EuroSek 2
True
```

Name the Price

- Let's define a name for the price of the game we want

```
price = 53
```

```
Main> sek price
ERROR - Type error in application
*** Expression   : sek price
*** Term        : price
*** Type       : Integer
*** Does not match : Double
```

Every Value has a Type

- The `:i` command prints *information* about a name

```
Main> :i price
price :: Integer
```

Integer (whole number) is the *inferred type* of price

```
Main> :i euroRate
euroRate :: Double
```

Double is the type of *real* numbers
Funny name, but refers to *double the precision* that computers originally used

More Types

```
Main> :i True
True :: Bool -- data constructor
```

```
Main> :i False
False :: Bool -- data constructor
```

```
Main> :i sek
sek :: Double -> Double
```

The type of a function

Type of the result

```
Main> :i prop_EuroSek
prop_EuroSek :: Double -> Bool
```

Type of the argument

Types Matter

- Types determine *how* computations are performed

```
Main> 123456789*123456789 :: Double
1.52415787501905e+016
```

Specify which type to use

```
Main> 123456789*123456789 :: Integer
15241578750190521
```

Correct to 15 figures

GHCi *must know* the type of each expression before computing it.

The exact result – 17 figures (but must be an integer)

Type Checking

- Infers (works out) the type of every expression
- Checks that all types match – *before* running the program

Our Example

```
Main> :i price
price :: Integer

Main> :i sek
sek :: Double -> Double

Main> sek price
ERROR - Type error in application
*** Expression   : sek price
*** Term        : price
*** Type        : Integer
*** Does not match : Double
```

Why did it work before?

```
Main> sek 53
485.82026
Main> 53 :: Integer
53
Main> 53 :: Double
53.0
Main> price :: Integer
53
Main> price :: Double
ERROR - Type error in type annotation
*** Term        : price
*** Type        : Integer
*** Does not match : Double
```

Certainly works to say 53
What is the type of 53?

53 can be used with *several* types – it is *overloaded*

Giving it a name *fixes* the type

Fixing the Problem

- Definitions can be given a *type signature* which *specifies* their type

```
price :: Double
price = 53
```

```
Main> :i price
price :: Double
```

```
Main> sek price
485.82026
```

Always Specify Type Signatures!

- They help the reader (and *you*) understand the program
- The type checker can find your errors more easily, by checking that definitions have the types you say
- Type error messages will be easier to understand
- Sometimes they are necessary (as for price)

Example with Type Signatures

```
euroRate :: Double
euroRate = 9.16642

sek, euro :: Double -> Double
sek x = x*euroRate
euro x = x/euroRate

prop_EuroSek :: Double -> Bool
prop_EuroSek x = euro (sek x) ~== x
```

Function Definition by Cases and Recursion

Example: Absolute Value

- Find the absolute value of a number
 - If x is positive, result is x
 - If x is negative, result is -x

```
-- returns the absolute value of x
absolute :: Integer -> Integer
absolute x | x >= 0 = x
absolute x | x < 0  = -x
```

Notation

- We can abbreviate repeated left hand sides

```
absolute x | x >= 0 = x
absolute x | x < 0  = -x
```

```
absolute x | x >= 0 = x
           | x < 0  = -x
```

- Haskell also has **if then else**

```
absolute x = if x >= 0 then x else -x
```

Recursion

- First example of a *recursive* function
 - Compute x^n (without using built-in x^n)

```
power x 0 = 1
power x n | n > 0 = x * power x (n-1)
```

- Calculate to find the answer:
 $power\ 2\ 2 = 2 * power\ 2\ (2-1)$
 $= 2 * power\ 2\ 1 = 2 * 2 * power\ 2\ (1-1)$
 $= 2 * 2 * power\ 2\ 0 = 2 * 2 * 1 = 4$

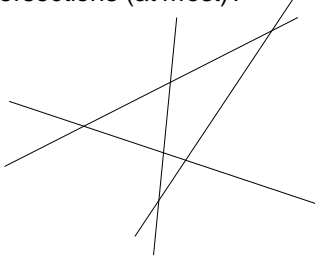
Recursion

- Reduce a problem (e.g. power x n) to a *smaller* problem of the same kind
- So that we eventually reach a "smallest" *base case*
- Solve base case separately
- Build up solutions from smaller solutions

You should have seen recursion before.
If not, ask for a refund on your bachelor degree.

Example: Counting intersections

- n non-parallel lines. How many intersections (at most)?



The Solution

- Always pick the base case as simple as possible!

```
intersect :: Integer -> Integer
intersect 0 = 0
intersect n
  | n > 0 = intersect (n - 1) + n - 1
```