Functional Programming and Erlang

Lecture 6 of TDA384/DIT391

Principles of Concurrent Programming

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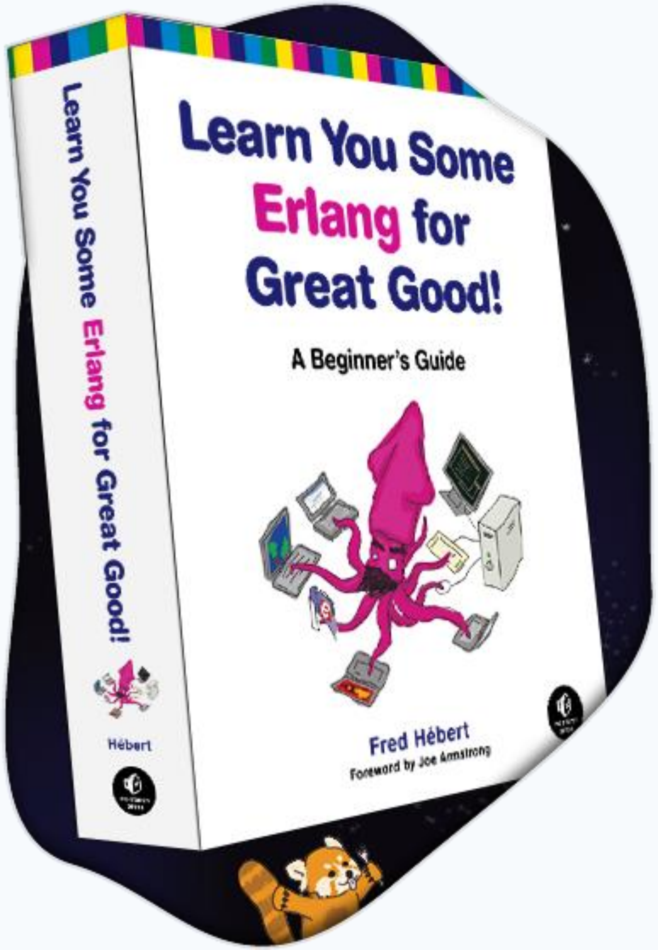
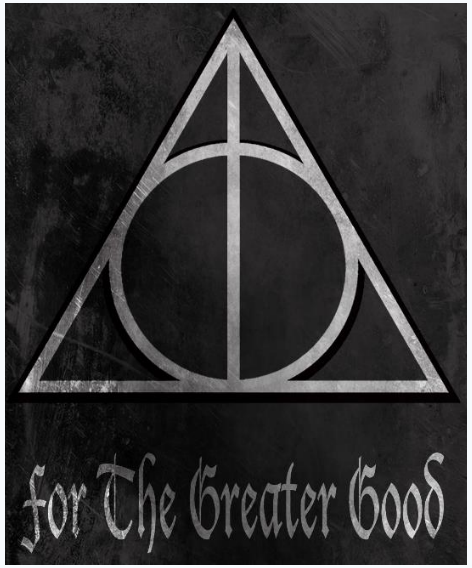
Today's menu

- What is Erlang?
- Types
- Expressions and patterns
- Function definitions
- Recursion
- Impure and higher-order functions

Don't forget\dots





(http://learnyousomeerlang.com/)

What is Erlang?

What is Erlang?

Erlang combines a functional language with message-passing features:

- The functional part is sequential, and is used to define the behavior of processes
- The message-passing part is highly concurrent: it implements the actor model, where actors are Erlang processes

This lecture covers the functional/sequential part of Erlang

Erlang: A minimal history

1973 *Hewitt* and others develop the actor model – a formal model of concurrent computation

1985 *Agha* further refines the actor model

Mid 1980s *Armstrong* and others at Ericsson prototype the first version of Erlang (based on the actor model)

Late 1980s Erlang’s implementation becomes efficient; Erlang code used in production at Ericsson

1998 Ericsson bans Erlang, which becomes open-source

Late 2000s Erlang and the actor model make a come-back in mainstream programming

Erlang in the real world

Erlang has made a significant impact in the practice of concurrent programming by making the formal actor model applicable to real-world scenarios

- Initially, Erlang was mainly used for telecommuncation software:
  - Ericsson’s AXD301 switch – includes over one million lines of Erlang code; achieves “nine 9s” availability (99.9999999%)
  - Cellular communication infrastructure (services such as SMSs)
- Recently, it has been rediscovered for Internet communication apps:
  - WhatsApp’s communication services are written in Erlang
  - Facebook Chat (in the past)

Why Erlang?

> *We've faced many challenges in meeting the ever-growing demand for [the WhatsApp] messaging services, but [...] Erlang continues to prove its capability as a versatile, reliable, high-performance platform.*
>
> *Rick Reed, 2014 - That's 'Billion' with a 'B': Scaling to the next level at WhatsApp*

> *The language itself has many pros and cons, but we chose Erlang to power [Facebook] Chat because its model lends itself well to concurrent, distributed, and robust programming.*
>
> *Chris Piro, 2010 – Chat Stability and Scalability*

What is a functional language?

Functional languages are based on elements quite **different from** those imperative languages are based on

Imperative languages (such as Java) are based on:

- state – variables
- state modifications - assignments
- iteration – loops

Functional languages (such as Erlang) are based on:

- data – values
- functions on data – without side effects
- functional forms – function composition, higher-order functions

What is a functional language?

Functional languages are based on elements quite **different from** those imperative languages are based on

Imperative languages (such as Java) are based on:

An imperative program is a sequence of state modifications on variables

```
// compute x n
int power(int x, int n) {
      int result = 1;
      for (int i = n; i < n; i++)
        result *= x;
      return result;
}
```

Functional languages (such as Erlang) are based on:

A functional program is the side-effect-free application of functions on values

```
% compute X N
power(X, 0) -> 1;
power(X, N) -> X * power(X, N-1)
```

In functional programs, variables store immutable values, which can be copied but not modified

The Erlang shell

You can experiment with Erlang using its shell, which can evaluate expressions on the fly without need to define complete programs

```
$ erl
Erlang R16B03 (erts-5.10.4) [source] [64-bit] [smp:2:2]

Eshell V5.10.4  (abort with ^G)

1> 1 + 2.                 % evaluate expression `1 + 2'
3
2> c(power).              % compile file `power.erl'
{ok,power}
3> power:power(2, 3).     % evaluate power(2, 3)
8
```

- Notice you have to terminate all expressions with a period
- Functions are normally defined in external files, and then used in the shell
- Compilation targets bytecode by default

Types

Types, dynamically

A type constrains:

1. The (kinds) of values that an expression can take
2. The functions that can be applied to expressions of that type

For example, the integer type:

1. includes integer values (`1`, `-100`, `234`), but not, say, decimal numbers (`10.3`, `-4.3311`) or strings (`"hello!"`, `"why not"`)
2. supports functions such as sum `+`, but not, say, logical **`and`**

- Erlang is dynamically typed:
  - programs do **not** use type declarations
  - the type of an expression is only determined at runtime
    - when the expression is evaluated
  - if there is a type mismatch (for example `3 + false`) expression evaluation fails
- Erlang types include primitive and compound data types

An overview of Erlang types

Erlang offers eight primitive types:

- Integers: arbitrary-size integers with the usual operations
- Atoms: roughly corresponding to identifiers
- Floats: 64-bit floating point numbers
- References: globally unique symbols
- Binaries: sequences of bytes
- Pids: process identifiers
- Ports: for communication
- Funs: function closures

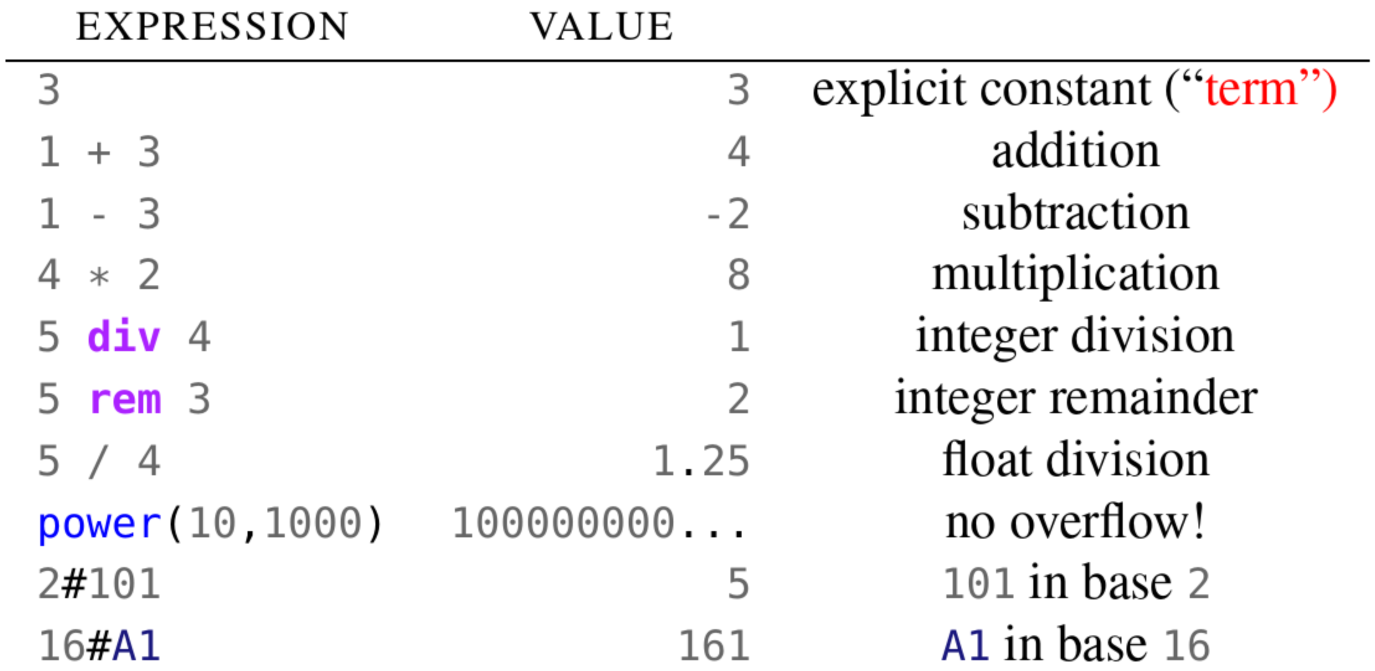
And three + two compound types (a.k.a. type constructors):

- Tuples: fixed-size containers
- Lists: dynamically-sized containers
- Maps: key-value associative tables (a.k.a. dictionaries) –recent feature, experimental in Erlang/OTP R17
- Strings: syntactic sugar for sequences of characters
- Records: syntactic sugar to access tuple elements by name

Numbers

Numeric types include integers and floats

- We will mainly use *integers*, which are arbitrary-size, and thus do not overflow



EXPRESSION	VALUE	
`3`	`3`	explicit constant (“term”)
`1 + 3`	`4`	addition
`1 - 3`	`-2`	subtraction
`4 * 2`	`8`	multiplication
`5 div 4`	`1`	integer division
`5 rem 3`	`2`	integer remainder
`5 / 4`	`1.25`	float division
`power(10,1000)`	`100000000...`	no overflow!
`2#101`	`5`	`101` in base `2`
`16#A1`	`161`	`A1` in base `16`

Atoms

Atoms are used to denote distinguished values

(they are similar to symbolic uninterpreted constants)

An atom can be:

- A sequence of alphanumeric characters and underscores, starting with a lowercase letter
- An arbitrary sequence of characters (including spaces and escape sequences) between single quotes
  - An atom is to be enclosed in single quotes (') if it does not begin with a lower-case letter or if it contains other characters than alphanumeric characters, underscore (_), or @

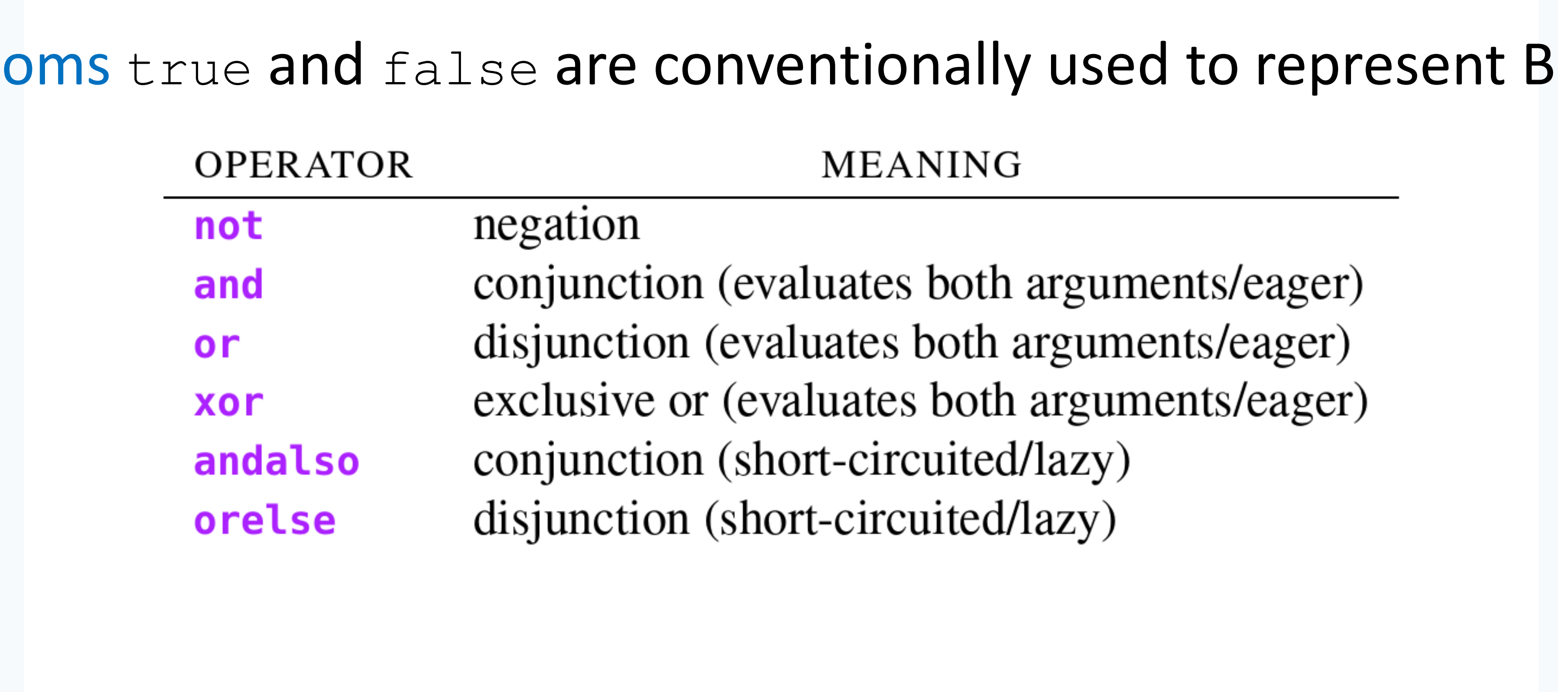
Examples of valid atoms:

```
x
a_Longer_Atom
'Uppercase_Ok_in_quotes'
'This is crazy!'
true
```

Booleans

In Erlang there is **no** Boolean type

Instead, the atoms `true` and `false` are conventionally used to represent Boolean values



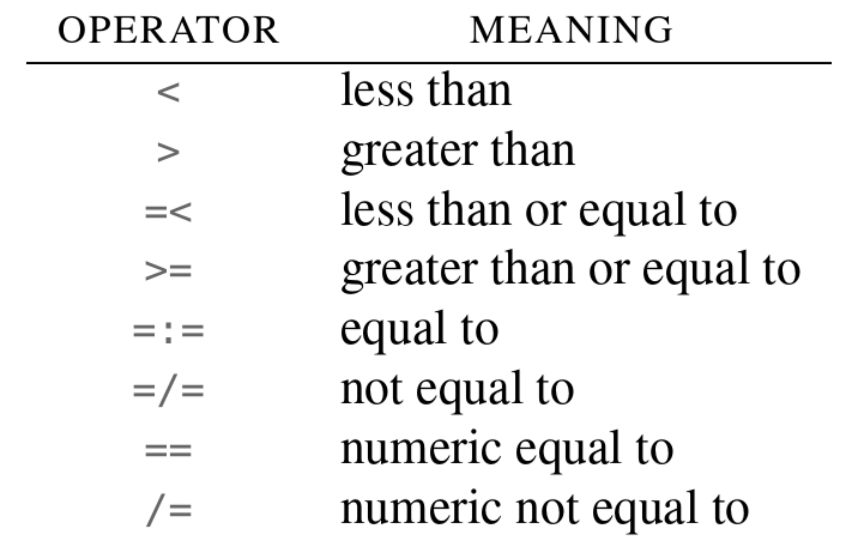
OPERATOR	MEANING
`not`	negation
`and`	conjunction (evaluates both arguments/eager)
`or`	disjunction (evaluates both arguments/eager)
`xor`	exclusive or (evaluates both arguments/eager)
`andalso`	conjunction (short-circuited/lazy)
`orelse`	disjunction (short-circuited/lazy)

Examples:

```
true or     (10 + false)  % error: type mismatch in second argument
true orelse (10 + false)  % true: only evaluates first argument
```

Relational operators

Erlang’s relational operators have a few syntactic differences with those of most other programming languages



OPERATOR	MEANING
<	less than
>	greater than
=<	less than or equal to
>=	greater than or equal to
=:=	equal to
=/=	not equal to
==	numeric equal to
/=	numeric not equal to

Examples:

```
3 =:= 3     % true: same value, same type
3 =:= 3.0   % false: same value, different type
3 == 3.0    % true: same value, type not checked
```

Order between different types

Erlang defines an order relationship between values of any type

When different types are compared, the following order applies:

$$number < atom < reference < fun < port < pid < tuple < map < list$$

Thus, the following inequalities hold:

```
3 < true                     % number < atom
3 < false                    % number < atom
999999999 < infinity         % number < atom
100000000000000 < epsilon    % number < atom
```

When comparing tuples to tuples:

- comparison is by <u>size</u> first
- two tuples with the same size or two lists are compared <u>element</u> by element, and satisfy the comparison only if all (existing) pairs satisfy it

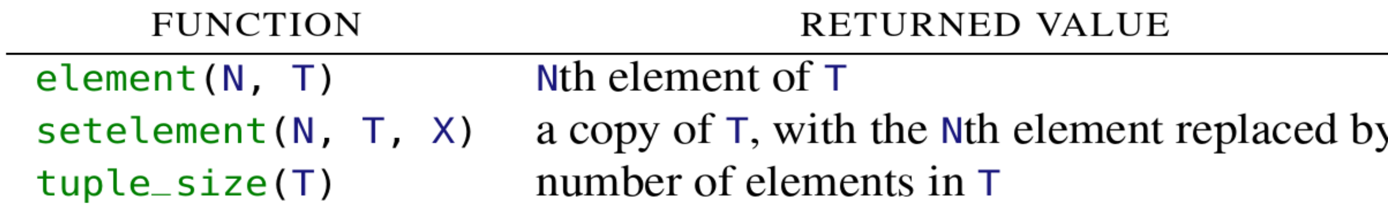
Tuples

Tuples denote **ordered** sequences with a **fixed** (but arbitrary for each tuple instance) number of elements (They are written as comma-separated sequences enclosed in **curly braces**)

Examples of valid tuples:

```
{ }                          % empty tuple
{ 10, 12, 98 }
{ 8.88, false, aToM }        % elements may have different types
{ 10, { -1, true } }         % tuples can be nested
```

Functions on a tuple `T`:



FUNCTION	RETURNED VALUE
`element(N, T)`	`N`th element of `T`
`setelement(N, T, X)`	a copy of `T`, with the `N`th element replaced by
`tuple_size(T)`	number of elements in `T`

Examples:

```
element(2, {a, b, c})        % b: tuples are numbered from 1
setelement(1, {a, b}, z)     % {z, b}
tuple_size({ })              % 0
```

Lists

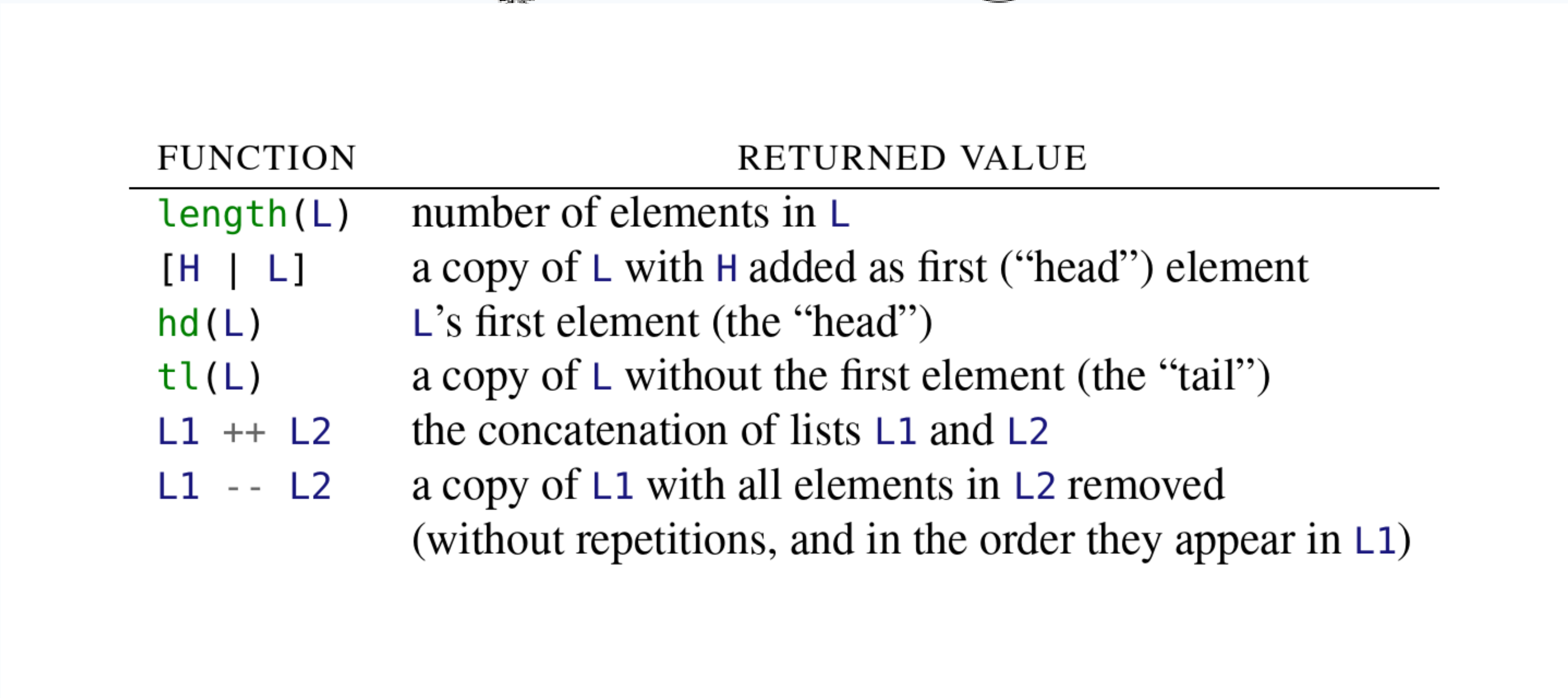
Lists denote **ordered** sequences with a **variable** (but immutable for any list instance) number of elements (They are written as comma-separated sequences enclosed in **square brackets**)

Examples of valid lists:

```
[ ]                         % empty list
[ 10, 12, 98 ]
[ 8.88, false, {1, 2} ]     % elements may have different type
[ 10, [ -1, true ] ]        % lists can be nested
```

List operators

Some useful functions on lists `L`:



FUNCTION	RETURNED VALUE
`length(L)`	number of elements in `L`
`[H | L]`	a copy of `L` with `H` added as first (“head”) element
`hd(L)`	`L`’s first element (the “head”)
`tl(L)`	a copy of `L` without the first element (the “tail”)
`L1 ++ L2`	the concatenation of lists `L1` and `L2`
`L1 -- L2`	a copy of `L1` with all elements in `L2` removed (without repetitions, and in the order they appear in `L1`)

Operator `|` is also called `cons`; using it, we can define any list:

```
[1, 2, 3, 4] =:= [1 | [2 | [3 | [4 | []]]]]
hd([H | T]) =:= H
tl([H | T]) =:= T
  % this is an example of --
[1, 2, 3, 4, 2] -- [1, 5, 2] =:= [3, 4, 2]
```

Strings

Strings are sequences of characters enclosed between double quotation marks

- Strings are just *syntactic sugar* for lists of character codes

String concatenation is implicit whenever multiple strings are juxtaposed without any operators in the middle

Using strings (`$c` denotes the integer code of character `c`):

```
""                 % empty string =:= empty list
"hello!"
"hello" "world"    % =:= "helloworld"
"xyz" =:= [$x, $y, $z] =:= [120, 121, 122] % true
[97, 98, 99]       % evaluates to "abc"!
```

Records

Records are **ordered sequences** with a fixed number of elements, where each element has an atom as name

- Records are just *syntactic sugar* for tuples where positions are named

```
    % define `person' record type
    % with two fields:   `name' with default value "add name"
    %                    `age' without default value (undefined)
 -record(person, { name="add name", age })
   % `person' record value with given name and age
 #person{name="Joe", age=55}
 #person{age=35, name="Jane"} % fields can be given in any order
   % when a field is not initialized, the default applies
 #person{age=22} =:= #person{name="add name", age=22}
   % evaluates to `age' of `Student' (of record type `person')
 Student#person.age
```

- Erlang's shell does not know about records, which can only be used in modules
  - In the shell: `#person{age=7,name="x"}` is `{person, "x", 7}`.

Expressions and patterns

Variables

Variables are identifiers that can be bound to values

(they are similar to constants in an imperative programming language)

A variable name is a sequence of alphanumeric characters, underscores, and `@`, starting with an uppercase letter or an underscore

In the shell, you can directly bind values to variable:

- Evaluating `Var = `*expr* binds the value of expression *expr* to variable `Var`, and returns such value as value of the whole binding expression
- Each variable can only be bound once
- To clear the binding of variable `Var` evaluate `f(Var)`
- Evaluating `f()` clears all variable bindings
- The anonymous variable _ ("any") is used like a variable whose value can be ignored

In modules, variables are used with pattern matching (which we present later)

Expressions and evaluation

- Expressions are evaluated exhaustively to a value – sometimes called (ground) term: a number, an atom, a list, …

The order of evaluation is given by the usual precedence rules

(using **parentheses** forces the evaluation order to be inside-out of the nesting structure)

Some precedence rules to be aware of:

- `and` has higher precedence than `or`
- `andalso` has higher precedence than `orelse`
- when lazy (`andalso`, `orelse`) and eager (`and`, `or`) Boolean operators are mixed, they all have the same precedence and are left-associative
- `++` and `--` are right-associative (concatenation and substraction in lists)
- relational operators have lower precedence than Boolean operators; thus you have to use parentheses in expressions such as `(3 > 0) and (2 == 2.0)`

Precedence rules: Examples

```
3 + 2 * 4                          % is 11
3 + (2 * 4)                        % is 11
(3 + 2) * 4                        % is 20
true or false and false            % is true
true orelse false andalso false    % is true
true or false andalso false        % is false
true orelse false and false        % is true (why?)
```

After evaluating the first “true”
there is no need to evaluate the rest

Patterns

Pattern matching is a flexible and concise mechanism to bind values to variables

It is widely used in functional programming languages to define functions on data (especially lists); Erlang is no exception

A pattern has the same structure as a term, but in a pattern some parts of the term **may** be replaced by free variables

Examples of patterns:

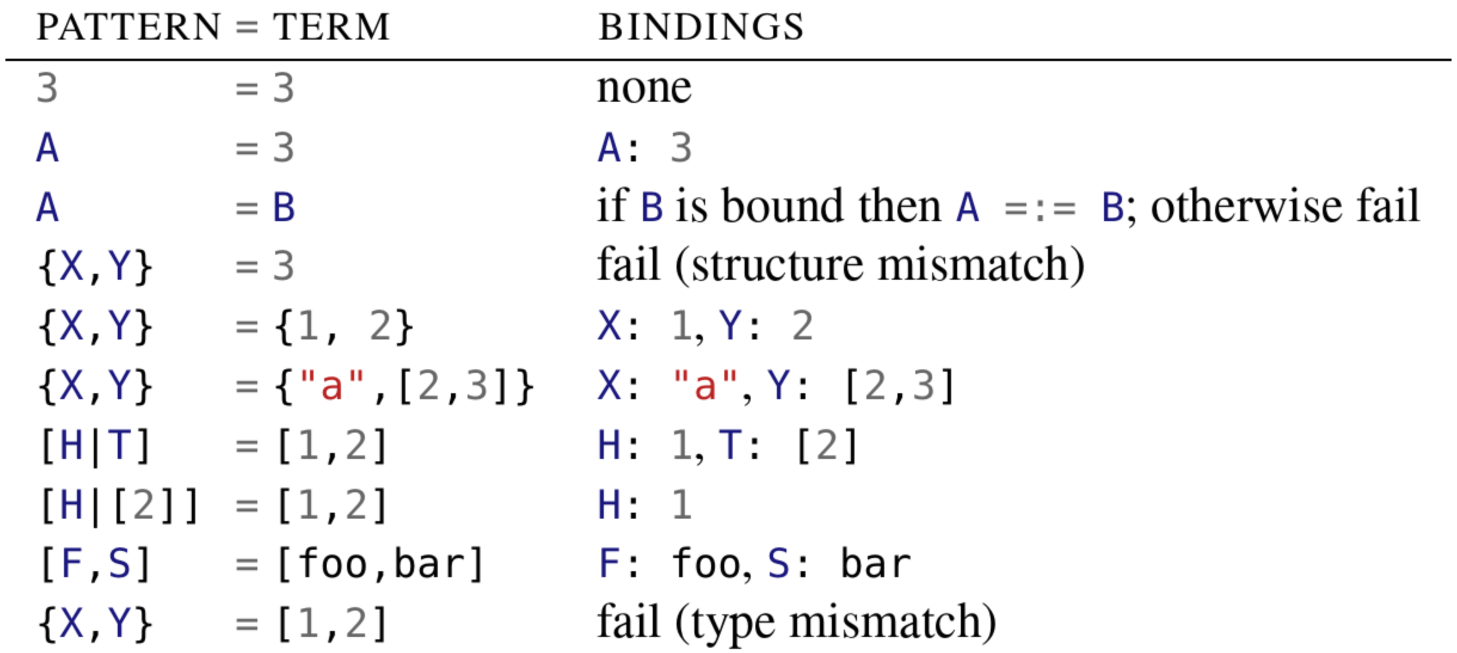
```
3
A
{X, Y}
{X, 3}
[H | T]
[H | [2]]
```

- Note that a pattern may contain bound variables
  - in this case, evaluating the pattern implicitly evaluates its bound variables

Pattern matching

Pattern matching is the process that, given a pattern `P` and a term `T`, binds the variables in `P` to match the values in `T` according to `P` and `T`'s structure

If `P`'s structure (or type) cannot match `T`'s, pattern matching fails



PATTERN	= TERM	BINDINGS
`3`	= `3`	none
`A`	= `3`	`A: 3`
`A`	= `B`	if `B` is bound then `A =:= B`; otherwise fail
`{X,Y}`	= `3`	fail (structure mismatch)
`{X,Y}`	= `{1, 2}`	`X: 1`, `Y: 2`
`{X,Y}`	= `{"a",[2,3]}`	`X: "a"`, `Y: [2,3]`
`[H|T]`	= `[1,2]`	`H: 1`, `T: [2]`
`[H|[2]]`	= `[1,2]`	`H: 1`
`[F,S]`	= `[foo,bar]`	`F: foo`, `S: bar`
`{X,Y}`	= `[1,2]`	fail (type mismatch)

Pattern matching: Notation

Given a pattern `P` and a term `T`, we write `⟨P≜T⟩` to denote the pattern match of `T` to `P`

- If the match is successful, it determines bindings of the variables in `P` to terms
- Given an expression `E`, we write `E⟨P≜T⟩` to denote the term obtained by applying the bindings of the pattern match `⟨P≜T⟩` to the variables in `E` with the same names
- If the pattern match fails, `E⟨P≜T⟩` is undefined

Examples:

- `(X + Y)⟨{X, Y}≜{3, 2}⟩` is `5`
- `(T ++ [2])⟨[H|T]≜[8]⟩` is `[2]`
- `H⟨[H|T]≜[ ]⟩` is undefined

NOTE: The notation `E⟨P≜T⟩` is **not** valid Erlang, but we use it to illustrate Erlang's semantics

Multiple expressions

Multiple expressions `E1`, …, `En` can be combined in a compound expression obtained by separating them using commas

- Evaluating the compound expression entails evaluating all component expressions in the order they appear, and returning the value of the last component expression as the value of the whole compound expression
- A single failing evaluation makes the whole compound expression evaluation fail

Examples:

```
3 < 0, 2.                  % evaluates 3 < 0
                           % returns 2

3 + true, 2.               % evaluates 3 + true
                           % fails

R=10, Pi=3.14, 2*Pi*R.     % binds 10 to R,
                           % binds 3.14 to Pi
                           % returns 62.8…
```

Multiple expression blocks

Using blocks delimited by `begin...end`, we can introduce multiple expressions where commas would normally be interpreted in a different way

This may be useful in function calls:

```
power(2, begin X=3, 4*X end)  % returns power(2, 12)
```

Without `begin...end`, the expression would be interpreted as calling a function `power` with three arguments

List comprehensions

List comprehensions provide a convenient syntax to define lists using pattern matching

It is an expression of the form: [Expression || P1 <- L1, ..., Pm <- Ln, C1, ..., Cn] where:

- each `Pk` is a pattern
- each `Lk` is a list expression
- each `Ck` is a condition (a Boolean expression)

- Intuitively, each pattern `Pk` is matched to every element of `Lk`, thus determining a binding `B`
  - if substituting all bound values makes all conditions evaluate to true, the value obtained by substituting all bound values in `Expression` is accumulated in the list result;
  - otherwise the binding is ignored

Examples:

```
[X*X || X <- [1, 2, 3, 4]]              % is [1, 4, 9, 16]
[X   || X <- [1, -3, 10], X > 0]        % is [1, 10]
[{A, B} || A <- [carl, sven], B <- [carlsson, svensson]]
  % is [{carl, carlsson}, {carl, svensson},
  %      {sven, carlsson}, {sven, svensson}]
```

Modules

A module is a collection of function definitions grouped in a file. Indeed, modules are the only places where functions can be defined – they cannot directly be defined in the shell

The main elements of a module are as follows:

```
-module(foo).   % module with name `foo' in file `foo.erl'
-export([double/1,up_to_5/0]). % exported functions
  % each f/n refers to the function with name `f' and arity `n'
-import(lists, [seq/2]). % functions imported from module `lists'
  % function definitions:
double(X) -> 2*X.
up_to_5() -> seq(1, 5).   % uses imported lists:seq
```

Compiling and using a module in the shell:

```
1> c(foo).        % compile module `foo' in current directory
{ok,foo}.         % compilation successful
2> foo:up_to_5(). % call `up_to_5' in module `foo'
[1,2,3,4,5]
```

Function definitions

Function definitions: basics

In Erlang (and all functional prog. lang.) functions are the fundamental units of computation

- A function defines how to map values to other values
  - Unlike in imperative programming languages, most functions in Erlang have no side effects: they do not change the state of the program executing them (especially their arguments)

The basic definition of an `n`-argument function `f` (arity `n`), denoted by `f/n`, has the form:

$$\overbrace{f(P1,\ldots, Pn)}^{\text{Head}} \;\text{->}\; \overbrace{E}^{\text{Body}}.$$

- The function name `f` is an atom
- The function's formal arguments `P1`,..., `Pn` are patterns
- The body `E` is an expression – normally including variables that appear in the arguments

Examples:

```
identity(X) -> X.          % the identity function
sum(X, Y)   -> X + Y.      % the sum function
```

Examples of function definitions

The basic definition of an `n`-argument function `f` (arity `n`), denoted by `f/n`, has the form:

f(P1,…, Pn) -> E.

More examples:

```
zero()        -> 0.                            % integer zero
identity(X) -> X.                              % identity
sum(X, Y)     -> X + Y.                        % sum
head([H|_]) -> H.                              % head
tail([_|T]) -> T.                              % tail
second({_, Y}) -> Y.                           % 2nd of pair
positives(L)    -> [X || X <- L, X > 0]. % filter positive
```

Function call/evaluation

Given the definition of a function `f/n`:

```
f(P1,…,Pn) -> E.
```

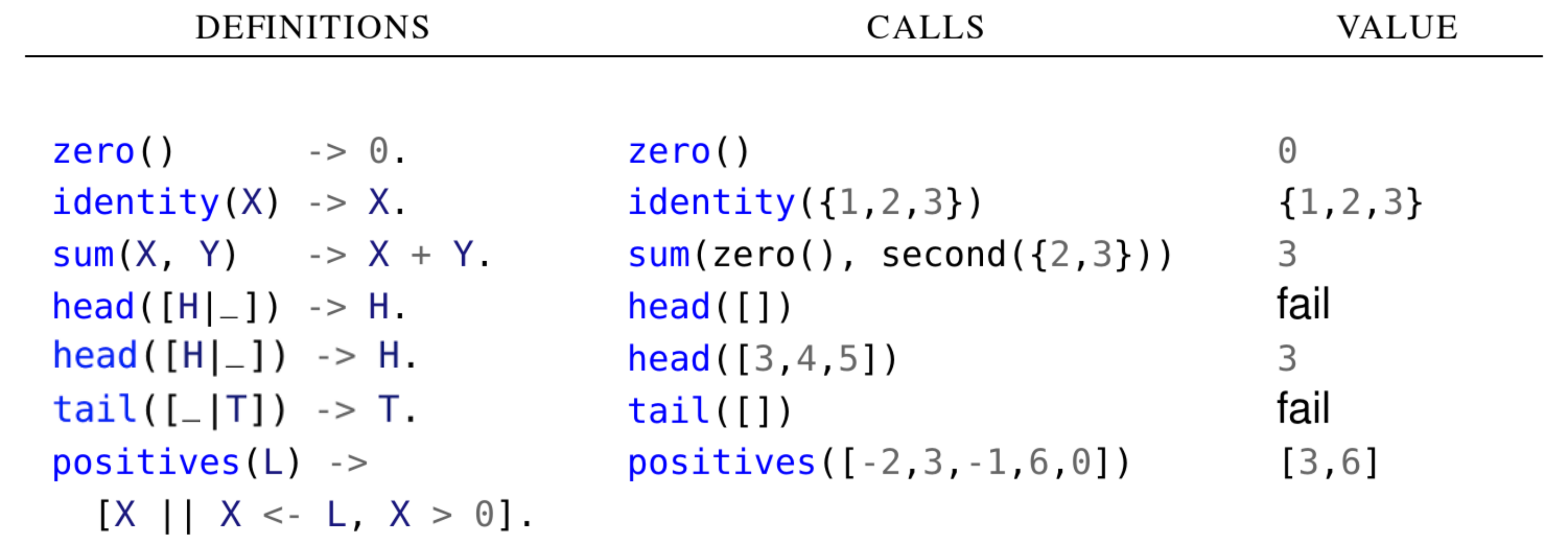
a call expression to `f/n` has the form:

```
f(A1,…,An)
```

and is evaluated as follows:

1. for each `1 ≤ K ≤ n`, evaluate `Ak`, which gives a term `Tk`
2. for each `1 ≤ K ≤ n`, pattern match `Tk` to `Pk`
3. if all pattern matches are successful, the call expression evaluates to `E(P1,...,Pn ≜ T1,...,Tn)`
4. otherwise, the evaluation of the call expression fails

Examples of function calls



DEFINITIONS	CALLS	VALUE
`zero()      -> 0.`	`zero()`	0
`identity(X) -> X.`	`identity({1,2,3})`	{1,2,3}
`sum(X, Y)   -> X + Y.`	`sum(zero(), second({2,3}))`	3
`head([H|_]) -> H.`	`head([])`	fail
`head([H|_]) -> H.`	`head([3,4,5])`	3
`tail([_|T]) -> T.`	`tail([])`	fail
`positives(L) ->` `[X || X <- L, X > 0].`	`positives([-2,3,-1,6,0])`	[3,6]

Function definition: clauses

Function definitions can include multiple clauses, separated by semicolons:

```
f(P11,…,P1n) -> E1;
f(P21,…,P2n) -> E2;
             .
             .
             .
f(Pm1,…,Pmn) -> Em.
```

A call expression is evaluated against each clause in textual order; the first successful *match* is returned as the result of the call

Therefore, we should enumerate clauses from more to less specific

```
lazy_or(true, _)  -> true;
lazy_or(_, true)  -> true;
lazy_or(_, _)     -> false.
```

This function does not work as expected unless this clause is listed last

Pattern matching with records

Pattern matching an expression `R` of record type `rec`

```
#rec{f1=P1, ..., fn=Pn} = R
```

succeeds if, for all `1 ≤ k ≤ n`, field `fk` in `R`'s evaluation – that is, `R#name.fk` – matches to pattern `Pk`

If record type `rec` has fields other than `f1`, …, `fn`, they are ignored in the match

Thanks to this behavior, using arguments of record type provides a simple way to extend data definitions without having to change the signature of all functions that use that datatype

Flexible arguments with records: Example

```
-record(error, {code}).
error_message(#error{code=100}) -> io.format("Wrong address");
error_message(#error{code=101}) -> io.format("Invalid username");
   ...
error_message(_)                -> io.format("Unknown error").
```

If we want to add more information to the type `error`, we only have to change the record definition, and the clauses using the new information:

```
-record(error, {code, line_number}).
error_message(#error{code=100}) -> io.format("Wrong address");
error_message(#error{code=101}) -> io.format("Invalid username");
   ...
error_message(#error{code=C, line_number=L}) -> io.format("Unknown error p", [C, L]).
```

Compare this to the case where we would have had to change `error_message` from a unary to a binary function!

Function definition: guards

Clauses in function definitions can include any number of guards (also called conditions):

```
f(Pk1, . . . , Pkn) when Ck1, Ck2, . . . -> Ek;
```

A guarded clause is selected only if all guards Ck1, Ck2,… evaluate to true under the match, that is if `Cki⟨Pk1,...,Pkn ≜ Tk1,...,Tkn⟩` evaluates to true for all guards `Cki` in the clause

More generally, two guards can be separated by either a comma or a semicolon: commas behave like lazy and (both guards have to hold); semicolon behave like lazy or (at least one guard has to hold)

```
can_drive(Name, Age) when Age >= 18 -> Name ++ " can drive";
can_drive(Name, _)                  -> Name ++ " cannot drive".

same_sign(X, Y) when X > 0, Y > 0; X < 0, Y < 0 -> true;
same_sign(_, _)                                 -> false.
```

Type checking -- at runtime

Since Erlang is dynamically typed, there are cases where we have to test the actual type of an expression

- For example, because a certain operation is only applicable to values of a certain type

To this end, Erlang provides several test functions whose names are self-explanatory:

```
is_atom/1
is_boolean/1
is_float/1
is_integer/1
is_list/1
is_number/1
is_pid/1
is_port/1
is_tuple/1
```

Use these only when necessary: in most cases defining implicitly partial functions is enough

Function definition: local binding

The expression body in a function definition can include compound expressions with bindings:

```
f(Pk1,…, Pkn) -> V1=E1,…, Vw=Ew, Ek;
```

Such bindings are only visible within the function definition

They are useful to define shorthands in the definition of complex expressions

```
volume({cylinder, Radius, Height}) ->
    Pi=3.1415,
    BaseArea=Pi*Radius*Radius,
    Volume=BaseArea*Height,
    Volume.
```

If expressions (guard patterns)

Ifs provide a way to express conditions alternative to guards (in fact, ***ifs*** are called – somewhat confusingly – *guard patterns* in Erlang)

An `if` expression:

```
if
    C1 -> E1;
     ⋮
    Cm -> Em
end
```

evaluates to the expression `Ek` of the first guard `Ck` in textual order that evaluates to true; if no guard evaluates to true, evaluating the `if` expression fails

```
age(Age) ->
  if Age > 21 -> adult;
     Age > 11 -> adolescent;
     Age > 2  -> child;
     true     -> baby end.
```

Case expressions

Cases provide an additional way to use pattern matching to define expressions. A `case` expression:

```
case E of
   P1 -> E1;
      ⋮
   Pm -> Em
end
```

evaluates to `Ek⟨Pk≜T⟩`, where `E` evaluates to `T`, and `Pk` is the first pattern in textual order that `T` matches to; if `T` matches no pattern, evaluating the `case` expression fails

Patterns may include `when` clauses, with the same meaning as in function definitions

```
years(X) ->
 case X of {human, Age} -> Age;
           {dog, Age}   -> 7*Age;
           _            -> cant_say
end.
```

Which one should I use?

Having several different ways of defining a function can be confusing. There are no absolute rules, but here are some guidelines that help you write idiomatic code:

- the first option to try is using pattern matching directly in a function's arguments, using different clauses for different cases
- if parts of a pattern expression depend on others, you may consider using `case` expressions to have nested patterns
- you do not need `if` expressions very often (but it's good to know what they mean, and sometimes they may be appropriate)

Recursion

Recursion in programming

- Recursion is a style of programming where functions are defined in terms of themselves

The definition of a function `f` is recursive if it includes a call to `f` (directly or indirectly)

```
% compute X^n
power(X, 0) -> 1;
power(X, N) -> X * power(X, N-1).
```

↑

Recursive call

Recursion in mathematics

Recursion is a style of programming where functions are defined in terms of themselves

The definition of a function `f` is recursive if it includes a call to `f` (directly or indirectly)

Definition of natural numbers:

- 0 is a natural number;
- if n is a natural number then $n+1$ is a natural number.

↑

Recursive/inductive definition

Recursion: from math to programming

Recursion in programming provides a natural way of implementing recursive definitions in mathematics

Factorial of a nonnegative integer n:

$$n! = \underbrace{n \,.\, (n - 1) \ldots 1}_{n \text{ terms}} = n \,.\, \underbrace{(n - 1) \ldots 1}_{n\text{-}1 \text{ terms}}$$

$$n! = \begin{cases} 1 & \text{if } 0 \le n \le 1 \qquad \longleftarrow \text{ Base case} \\ n.\,(n-1)! & \text{if } n > 1 \qquad \longleftarrow \text{ Recursive/inductive case} \end{cases}$$

Recursion: from math to programming

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Factorial of a nonnegative integer n:

$$n! = \begin{cases} 1 & \text{if } 0 \leq n \leq 1 \quad \longleftarrow \text{Base case} \\ n.\,(n-1)! & \text{if } n > 1 \quad \longleftarrow \text{Recursive/inductive case} \end{cases}$$

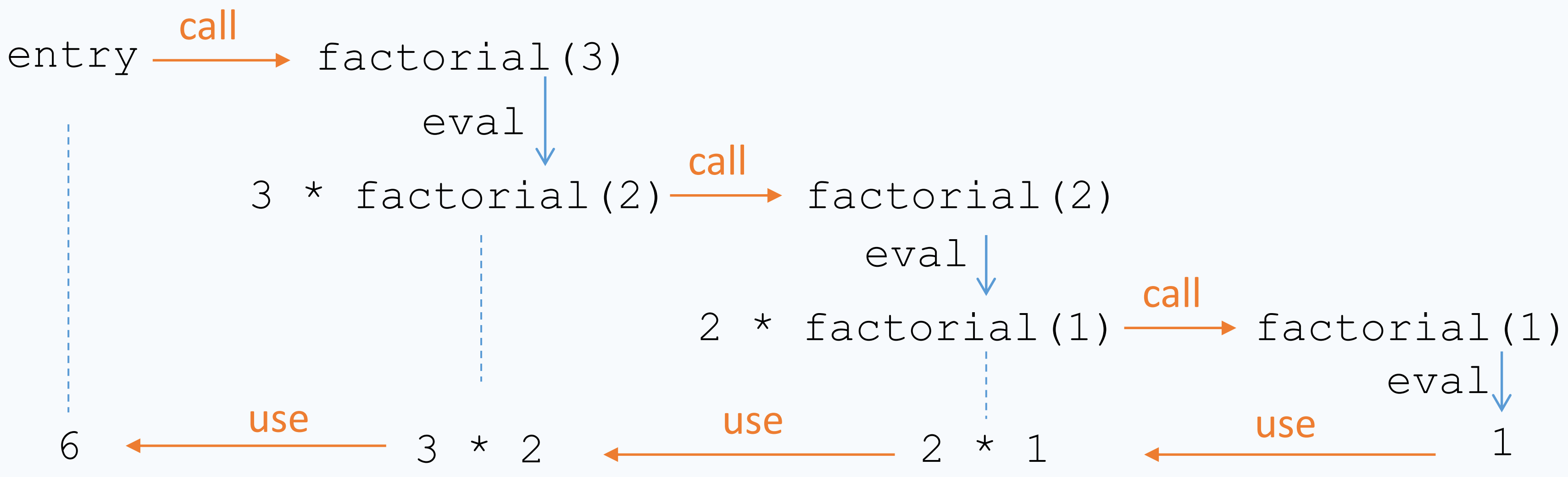
```
factorial(N) when N =< 1 -> 1;                  % base case
factorial(N)             -> N *factorial(N-1). % recursive case
```

↑ Recursive call

How does recursion work?

Each recursive call triggers an independent evaluation of the recursive function
(Independent means that it works on its own private copy of actual argument expressions)

When a recursive instance terminates evaluation, its value is used in the calling instance for its own evaluation



entry —call→ factorial(3)
eval ↓
3 * factorial(2) —call→ factorial(2)
eval ↓
2 * factorial(1) —call→ factorial(1)
eval ↓
6 ←use— 3 * 2 ←use— 2 * 1 ←use— 1

Recursion as a design technique

Recursion as a programming technique is useful to design programs using the divide and conquer approach:

To solve a problem instance P, split P into problem instances $P_1, \ldots, P_n$ chosen such that:

1. Solving $P_1, \ldots, P_n$ is simpler than solving P directly
2. The solution to P is a simple combination of the solutions to $P_1, \ldots, P_n$

In functional programming, recursion goes hand in hand with pattern matching:

- Pattern matching splits a function argument’s into smaller bits according to the input’s structure
- Recursive function definitions define the base cases directly, and combine simpler cases into more complex ones

Recursive functions: Sum of list

Define a function `sum(L)` that returns the sum of all numbers in `L`

1. The base case (the simplest possible) is when `L` is empty: `sum([]) -> 0`
2. Let now `L` be non-empty: a non empty list matches the pattern `[H|T]`
    - `H` is a single number, which we must add to the result
    - `T` is a list, which we can sum by calling `sum` recursively

```
sum([])     -> 0;            % base case
sum([H|T]) -> H + sum(T).   % recursive case
```

Can we switch the order of clauses?

In this case, YES

To make the function more robust, we can skip over all non-numeric elements:

```
sum([])                      -> 0;          % base case
sum([H|T]) when is_number(H) -> H + sum(T); % recursive case 1
sum([_|T])                   -> sum(T).     % recursive case 2
```

Recursive functions: Last list element

Define a function `last(L)` that returns the last element of `L`

1. When `L` is empty, `last` is undefined, so we can ignore this case
2. The simplest case is then when `L` is one element: `last([E]) -> E`
3. Let now `L` be non-empty: a non empty list matches the pattern `[H|T]`
    - `E` is the first element, which we throw away
    - `T` is a list, whose last element we get by calling `last` recursively

```
last([E])    -> E;          % base case
last([_|T])  -> last(T).    % recursive case
```

Can `T` match the empty list?

No, because neither of the clauses match the empty list

To make this explicit, we could write:

```
last([E|[]])-> E;           % base case
last([_|T]) -> last(T).     % recursive case
```

Tail recursion

A recursive function `f` is tail recursive if the evaluation of `f`’s body evaluates the recursive call last

```
% general recursive:
power(_, 0) ->
  1;
power(X, N) ->
  X * power(X, N-1).
```

```
% tail recursive:
power(X, N) ->
  power(X, N, 1).
power(_, 0, Accumulator) ->
  Accumulator;
power(X, N, Accumulator) ->
  power(X, N-1, X*Accumulator).
```

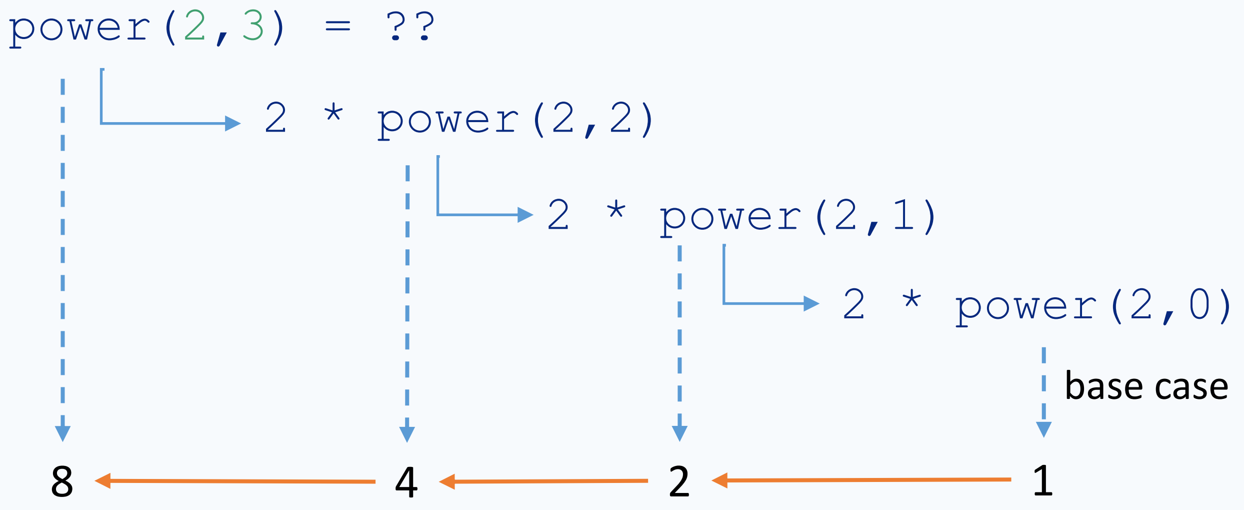
Overloading:
two functions **`power/2`** and **`power/3`**

- Tail-recursive functions are generally more efficient than general-recursive functions
- When efficiency is not an issue, there is no need to use a tail-recursive style; but we will use tail-recursive functions extensively (and naturally) when implementing servers

General Recursion vs Tail Recursion

General recursion:

```
% general recursive:
power(_, 0) -> 1;
power(X, N) -> X * power(X, N-1).
```



power(2,3) = ??

2 * power(2,2)

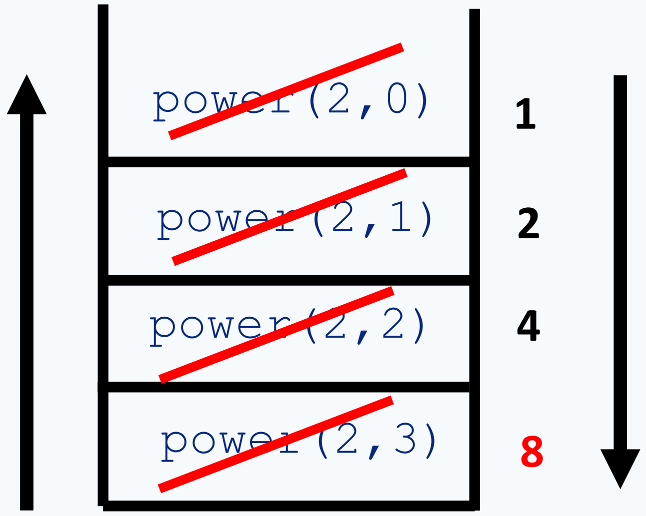
2 * power(2,1)

2 * power(2,0)

base case

8 4 2 1

Stack



~~power(2,0)~~ 1

~~power(2,1)~~ 2

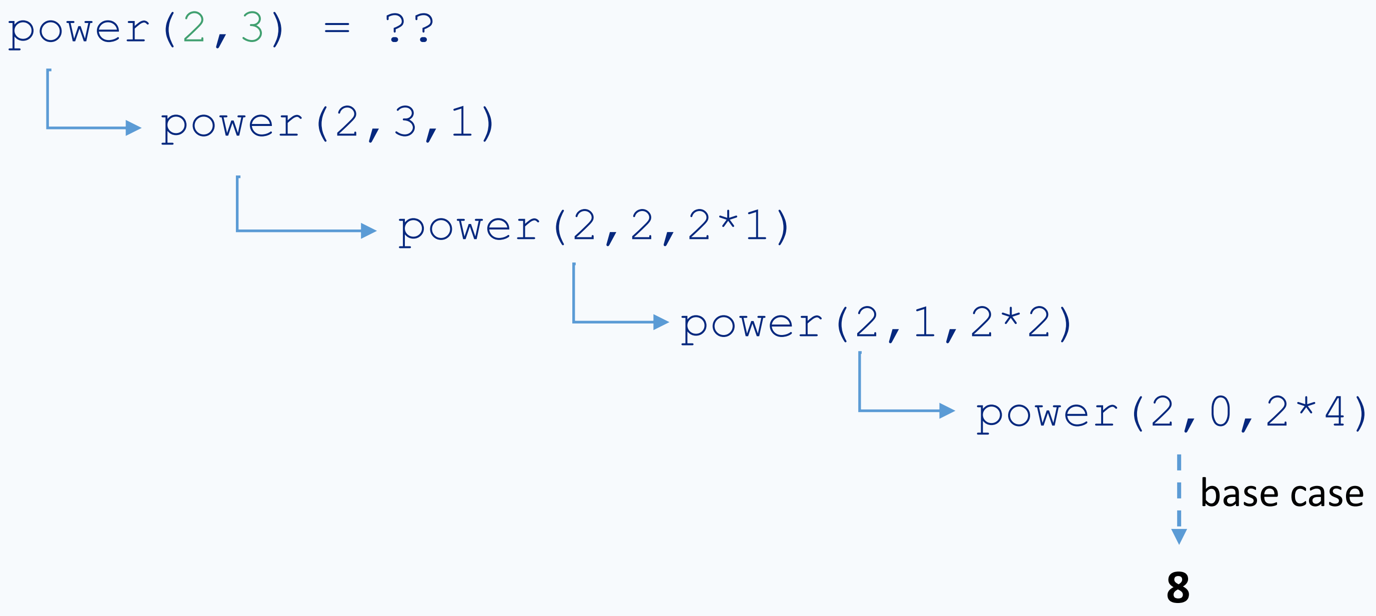
~~power(2,2)~~ 4

~~power(2,3)~~ 8

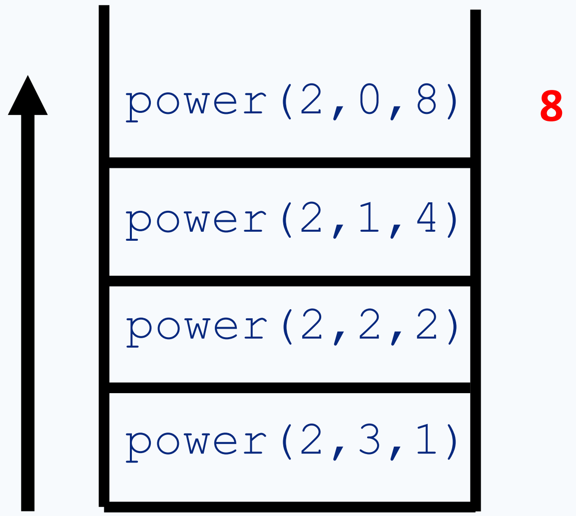
General Recursion vs Tail Recursion

Tail recursion:

```
% tail recursive:
power(X, N) ->  power(X, N, 1).
power(_, 0, Accumulator) ->  Accumulator;
power(X, N, Accumulator) ->  power(X, N-1, X*Accumulator).
```



Stack



Impure and higher-order functions

Where are all the statements, assignments, loops?

Statements, assignments, and loops are not available as such in Erlang

Everything is an expression that gets evaluated:

- (side-effect free) expressions are used instead of statements
- (pure) functions return modified copies of their arguments instead of modifying the arguments themselves
- one-time bindings are used instead of assignments that change values to variables
- recursion is used instead of loops

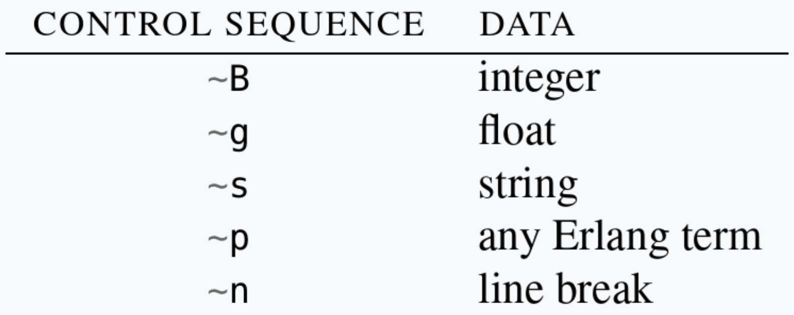
The sparse presence of side effects helps make functional programs higher level than imperative ones

Printing to screen

The expressions we have used so far have no side effects, that is they do not change the state but simply evaluate to a value

- Not all expressions are side-effect free in Erlang
  - Input/output is an obvious exception: to print something to screen, we evaluate an expression call, whose side effect is printing

```
io:format(Format, Data) % print the string Format, interpreting control sequences on Data
```



CONTROL SEQUENCE	DATA
~B	integer
~g	float
~s	string
~p	any Erlang term
~n	line break

You can use `fwrite` instead of `format`

```
1> io:format("~s ~B. ~p~n~s ~B~n", ["line", 1, true, "line", 2]).
line 1. true
line 2
```

Exception handling

Erlang has an exception handling mechanism that is similar to a functional version of Java's `try/catch/finally` blocks:

```
try Expr of
       Success1 -> Expr1;
              …
catch
       Error1:Fail1 -> Recov1;
              …
after After end
```

- The `try` blocks behaves like a `case` block
- If evaluating `Expr` raises an exception, it gets pattern matched against the clauses in `catch` (`Errork`'s are error types, `Failk`'s are patterns, and `Recovk`'s are expressions)
- Expression `After` in the `after` clause always gets evaluated in the end (but does not return any value: used to close resources)

Exception handling: Example

Function `safe_plus` tries to evaluate the sum of its arguments; if evaluation succeeds, it returns the result; if evaluation raises a `badarith` exception, it returns `false`:

```
safe_plus(X, Y) ->
  try X + Y of
    N -> N
  catch
    error:badarith -> false
  end.
```

Example of using it:

```
1> safe_plus(2, 3).
5
2> safe_plus(2, []).
false
```

Functions are values too

Functions are first-class objects in Erlang: they can be passed around like any other values, and they can be arguments of functions

- A function `f/k` defined in module `m` is passed as argument `fun m:f/k`

This makes it easy to define functions that apply other functions to values following a pattern

```
% apply function F to all elements in list L
map(F, [])     -> [];
map(F, [H|T]) -> [F(H)|map(F,T)].
```

```
1> map(fun m:age/1, [12, 1, 30, 56]).  % we defined age before
[adolescent,baby,adult,adult]
```

A function that takes another function as argument is called higher-order

High-Order Functions

```
% apply function F to all elements in list L
map(F, [])     -> [];
map(F, [H|T]) -> [F(H)|map(F,T)].
```

Let's define a function:

```
doub(X)-> 2*X;
```

What is the result of calling `map (doub/1, [12,1,30,56])` ?

```
map (doub/1, [12,1,30,56]) = ??
 └─> [doub(12)|map(doub,[1,30,56])]
       └─> [doub(12)|[doub(1)|map(doub,[30,56])]]
             └─> [doub(12)|[doub(1)|[doub(30)|map(doub,[56])]]]
                   └─> [doub(12)|[doub(1)|[doub(30)|[doub(56)|map(doub,[])]]]]
                                                               []
                                                         [112]
                                                   [60,112]
                                            [2,60,112]
                                    [24,2,60,112]
```

Inline functions

Sometimes it is necessary to define a function directly in an expression where it is used

For this we can use anonymous functions – also called lambdas, ***closures***, or funs (the last is Erlang jargon):

```
fun
        (A1) -> E1;
             ⋮
        (An) -> En
end
```

where each `Ak` is a sequence of patterns, and each `Ek` is a body

```
      % double every number in the list
1> map(fun (X)->2*X end, [12, 1, 30, 56]).
[24,2,50,112]
```

Working on lists

Module `lists` includes many useful predefined functions to work on lists

These are some you should know about – but check out the full module documentation at http://erlang.org/doc/man/lists.html:

```
all(Pred, List)      % do all elements E of List satisfy Pred(E)?
any(Pred, List)      % does any element E of List satisfy Pred(E)?
filter(Pred, List)   % all elements E of List that satisfy Pred(E)
last(List)           % last element of List
map(Fun, List)       % apply Fun to all elements of List
member(Elem, List)   % is Elem an element of List?
reverse(List)        % List in reverse order
seq(From, To)        % list [From, From+1, ..., To]
seq(From, To, I)     % list [From, From+I, ...,
```

Folds

Several functions compute their result by recursively accumulating values from a list:

```
sum([])     -> 0;
sum([H|T]) -> H + sum(T).
```

```
len([])     -> 0;
len([H|T]) -> 1 + len(T).
```

We can generalize this pattern into a single higher-order function `fold(F, R, L)`: starting from an initial value `R`, combine all elements of list `L` using function `F` and accumulate the result:

```
fold(_, Result, [])     -> Result;
fold(F, Result, [H|T]) -> F(H, fold(F, Result, T)).
```

Using `fold`, we can define `sum` and `len`:

```
sum(L) ->
 fold(fun (X,Y)->X+Y end, 0, L).
```

```
len(L) ->
 fold(fun (X,Y)->1+Y end, 0, L).
```

Erlang module `lists` offers functions `foldr/3` (which behaves like our `fold`) and `foldl/3` (a tail-recursive version of `fold`, with the same arguments)

Folds: Example

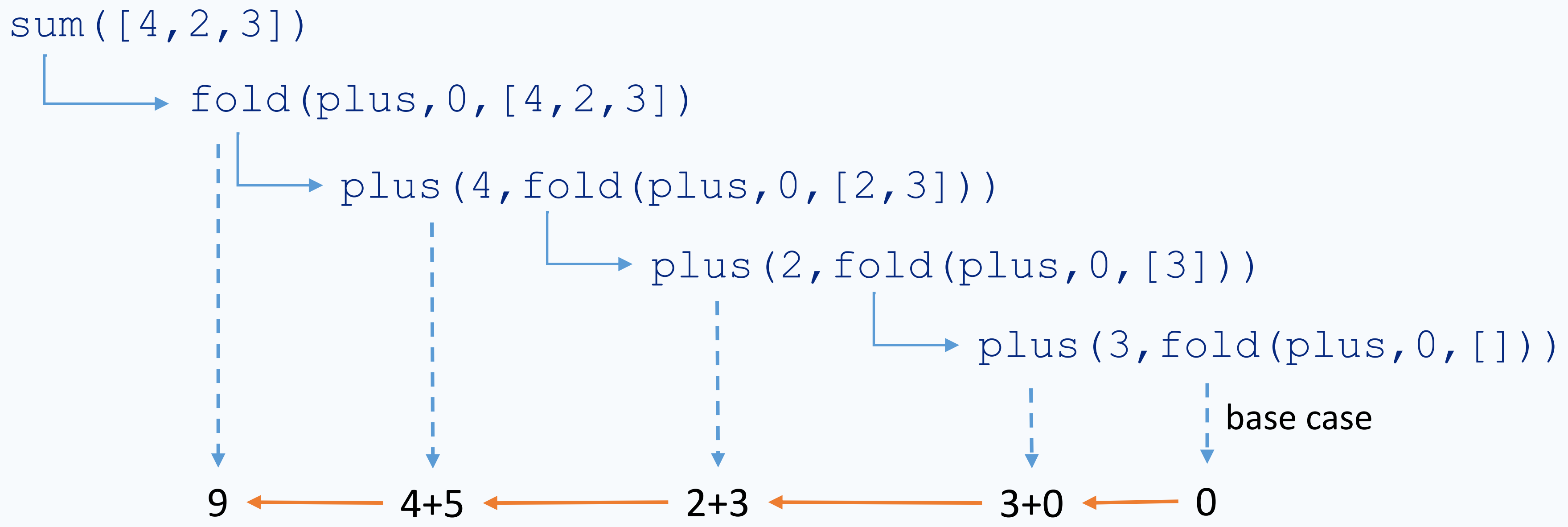
```
fold(_, Result, [])     -> Result;
fold(F, Result, [H|T]) -> F(H, fold(F, Result, T)).
```

Let's call this function `plus`

Let's define a `sum` of a list using `fold`

```
sum(L) -> fold( fun(X,Y)-> X+Y end, 0, L )
```

Let's try it!



```
sum([4,2,3])
fold(plus,0,[4,2,3])
plus(4,fold(plus,0,[2,3]))
plus(2,fold(plus,0,[3]))
plus(3,fold(plus,0,[]))
```

base case

9 ← 4+5 ← 2+3 ← 3+0 ← 0