Where is the math?

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Datavetenskap
Chalmers

lunch talk 2015-2018

Where is the math?

Matematisk modellering kan sägas vara länken mellan matematiken och verkligheten, där man ofta behöver arbeta kreativt för att hitta användbara förenklingar. I detta lunchföredrag vill jag ge lite personliga exempel, bland annat från mitt eget arbete med optimering för flygbolag. Jag vill också illustrera hur matematiskt tänkande, inklusive den viktiga förmågan att se saker på ett matematiskt sätt, hänger ihop med mycket av vad vi gör inom datavetenskapen. Jag vill även förutom den kurs i matematisk modellering och problemlösning som jag själv ger i tvåan, kort beskriva inriktningar inom utbildningen där datavetenskap och matematik går hand i hand, och hur detta i högsta grad ligger i tiden.

If you want to do things with computers…

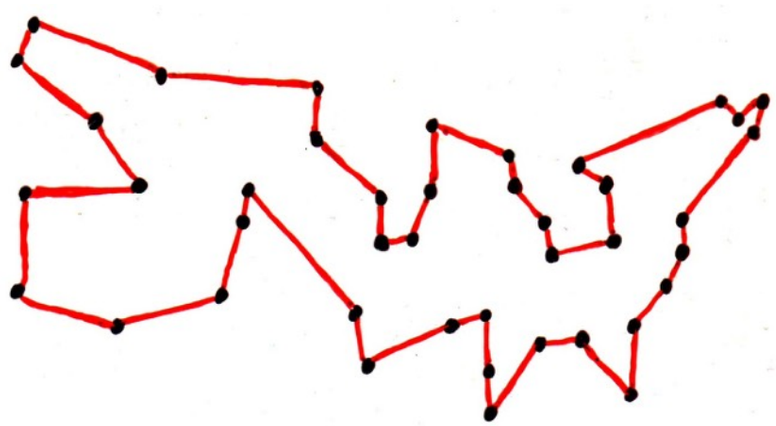
You need programming!

```
import java.awt.*;

public abstract class Animation
  extends java.applet.Applet
    implements java.lang.Runnable {

    protected Dimension d;  // bitmap size
    protected Image im;  // extra image for drawing
    protected Graphics offscreen; // the offscreen bitmap to draw in
    protected int delay = 100; // in milliseconds
    protected Thread animationThread;

    final public void init() {
   d = getSize();
   im = createImage(d.width, d.height);
   offscreen = im.getGraphics();
   initAnimator();
    }

    //final public void paint(Graphics g) {update(g);}

    final public void update(Graphics g) {
   paintAnimator(offscreen);  // first draw offscreen to reduce flicker
   g.drawImage(im, 0, 0, this);  // then put on screen
    }

    // To be implemented in subclass that does the actual drawing
    protected void initAnimator() {} // init for drawing routines
    abstract protected void paintAnimator(Graphics g); // the actual
drawing will be here

    public void setDelay(int d) {delay=d;}

    public void start() {
   animationThread = new Thread(this);
   animationThread.start();
    }

    public void stop() {
```

But if you want to do something more complicated…

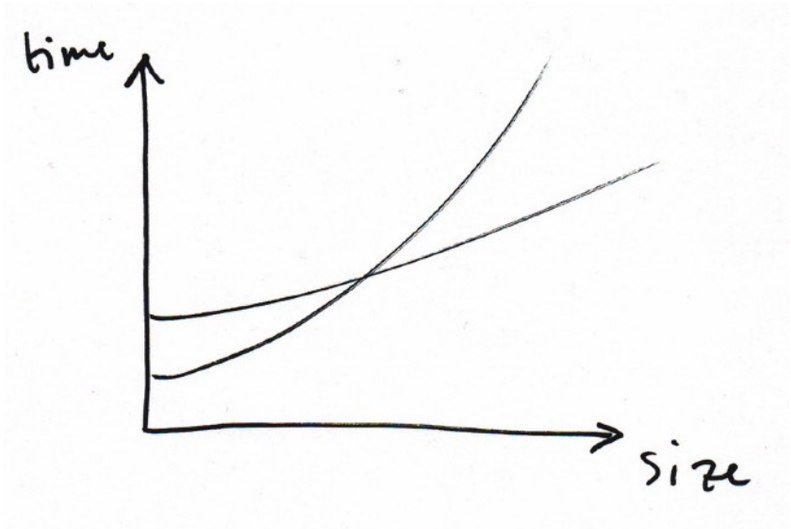
You need algorithms!

Nilsson, Sven
Lundberg, Lovisa
Larssson, Erik
Sjöberg, Anna
Carlén, Johanna
Persson, Mikael



But to use, understand and create algorithms…

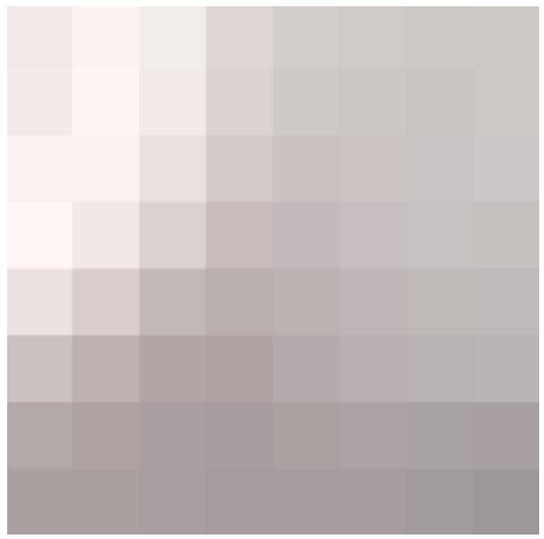
You often need math!



minimize $\sum_{ij \in A} w_{ij} x_{ij}$ subject to $x \geq 0$

$$\sum_j x_{ij} - \sum_j x_{ji} = \begin{cases} 1, & \text{if } i = s; \\ -1, & \text{if } i = t; \\ 0, & \text{otherwise.} \end{cases}$$

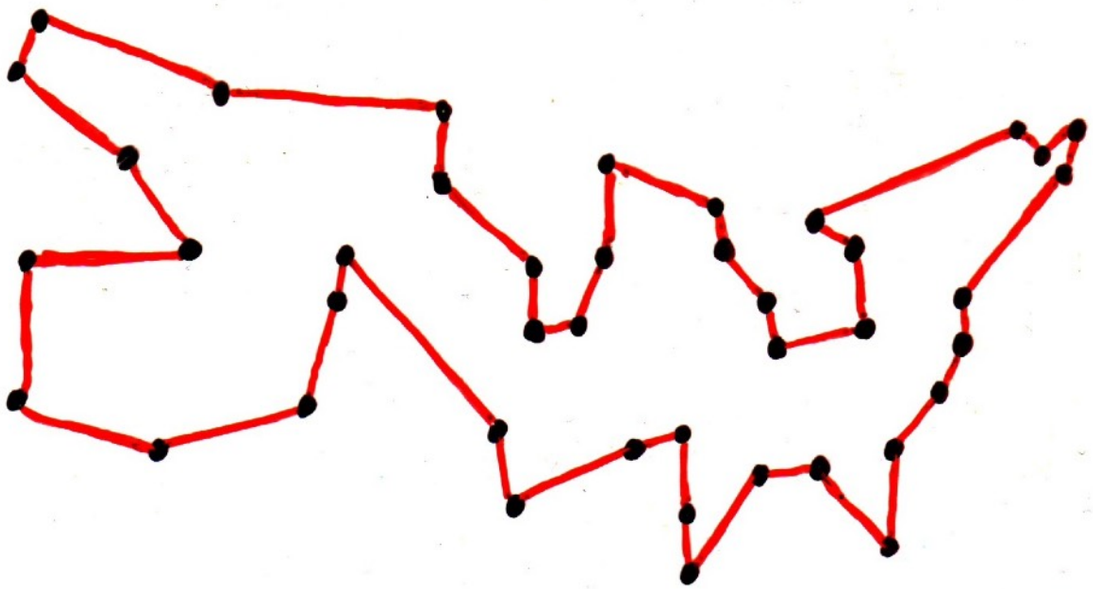
jpeg compression



$$
\begin{aligned}
Y' &= 16 + (65.481 \cdot R' + 128.553 \cdot G' + 24.966 \cdot B') \\
C_B &= 128 + (-37.797 \cdot R' - 74.203 \cdot G' + 112.0 \cdot B') \\
C_R &= 128 + (112.0 \cdot R' - 93.786 \cdot G' - 18.214 \cdot B')
\end{aligned}
$$

$$
G_{u,v} = \sum_{x=0}^{7} \sum_{y=0}^{7} \alpha(u)\alpha(v) g_{x,y} \cos\left[\frac{\pi}{8}\left(x + \frac{1}{2}\right)u\right] \cos\left[\frac{\pi}{8}\left(y + \frac{1}{2}\right)v\right]
$$

The Travelling Salesperson Problem



n	c 2^n
10	0,001 s
20	1 s
30	18 min
40	13 days
50	36 years
60	36600 years

TSP

Sweden Tour

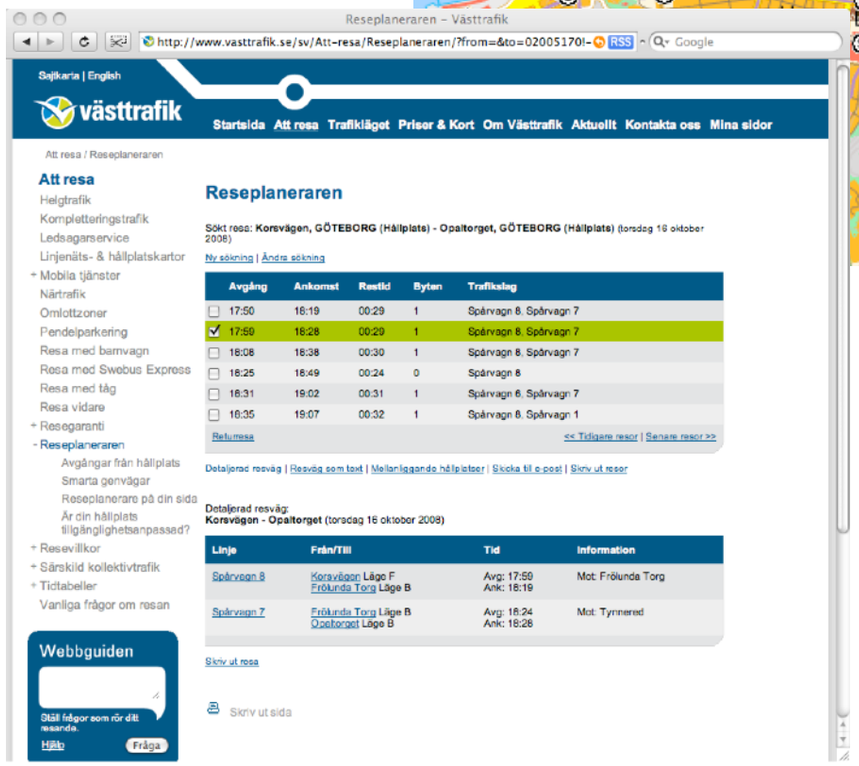
24,978 Cities

0 50 100 Kilometers
0 50 100 Miles

Lambert Conformal Conic Projection, SP 47N/62N

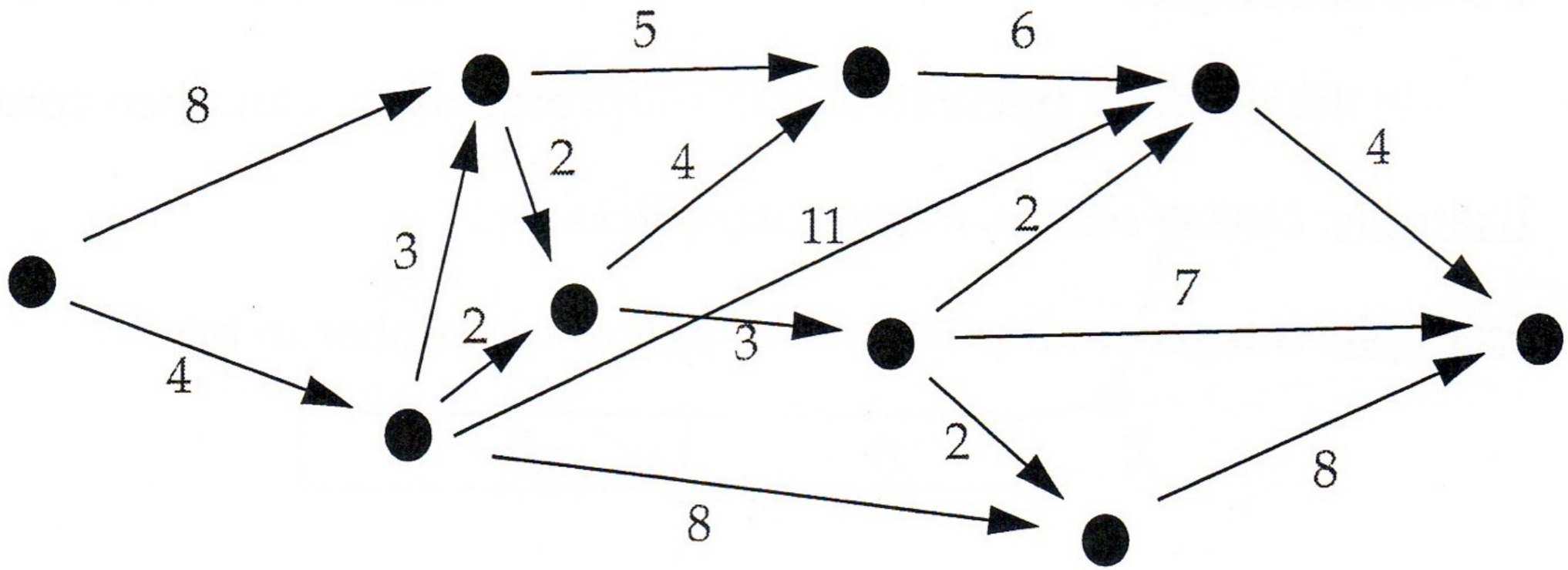
VESTERÅLEN
LOFOTEN
Harstad
Narvik
Bodø
Mo
Sandnessjøen
Mosjøen
Norwegian Sea
Namsos
Steinkjer
Trondheim
Molde
Ålesund
Andalsnes
Florø
NORWAY
Lillehammer
Hamar
Bergen
Drammen
Oslo
Haugesund
Stavanger
Porsgrunn
Arendal
Kristiansand
Skagerrak
Fredrikshavn
Ålborg
North Sea
Kattegat
Holstebro
Viborg
Århus
DENMARK
Helsingborg
Copenhagen
Vejle
Esbjerg
Roskilde
Odense
Korsør
Vänern
Gotland
Öland
Baltic Sea
Gulf of Bothnia
Vaasa
Seinäjoki
Pori
Tampere
Rauma
Hämeenlinna
Lahti
Kouvola
Kotka
Turku
Helsinki
ALAND ISLANDS
Gulf of Finland
Tallinn
ESTONIA
Hiiumaa
Saaremaa
Pärnu
Ventspils
Riga
LATVIA
Jelgava
Liepāja
Šiauliai
Klaipėda
LITHUANIA
NORWAY
RUSSIA
Ivalo
Kolari
Kemijärvi
Kelloselkä
Kuolo-yarvi
Rovaniemi
Kemi
Oulu
Kajaani
Iisalmi
Kokkola
FINLAND
Kuopio
Jämsä
Mikkeli
Aanekoski
Jyväskylä
Varkaus

The Shortest Path Problem



How solve?

Solving the directed shortest path problem with dynamic programming



Traverse nodes from left to right and mark with distance from origin.

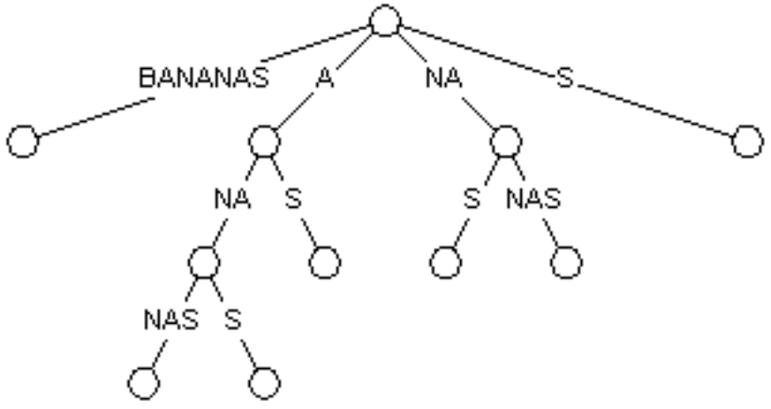
Circumvents the combinatorial explosion!
(not possible for all kinds of problems)

Text search is mathematical

statistical models and algorithms

search algorithms

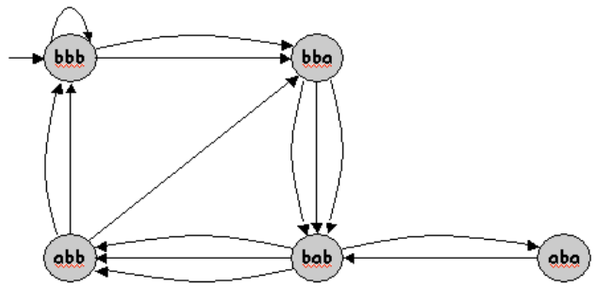
... my urgent need for a foreign partner after going through your profile that made me contact you for business partner Transaction. I am a banker by profession...



$$p(C|F_1,\ldots,F_n) = \frac{1}{Z}p(C)\prod_{i=1}^{n}p(F_i|C)$$

machine learning, pattern recognition

```
FTFTALILL-AVAV
--FTAL-LLAAV--
```



The result of the whole of a conception can be affirmed or denied of the whole of this critique, and obliges us to fall back on the old mode of metaphysical procedure.

Graphic Pairing Construction

Data | Plan | Rule | APC | Report | Options | Help

15:25

Window: 01/11 | 02/11 | 03/11 | 04/11 | 05/11 | 06/11 | 07/11 | 08/11 | 09/11 | 10/11 | 11/11 | 12/11 | 13/11 | 14/11

	01/11	02/11	03/11	04/11
1 | 12 0/0/0/0/0/1/2	FRA 3 3 3 LIN	LIN 36 325 5—0 3251 5—0 FRA		
1 | 1 0/0/0/0/0/1/2	MUC 43 43 40 0-0 40 00 MUC			
1 | 12345 0/0/0/0/0/1/2	FRA 40 40 40 40 HAM	HAM 40 40 41 41 HAM	HAM 0 3726 5—5 ATH	ATH 373 0—5 380 5—0 IST
1 | 123456 0/0/0/0/0/1/2	FRA 4 4 4 4 4 DUS	DUS 4 4 4 4 DUS	DUS 4 4 4 4 DUS	DUS 4 4 4 4 DUS
1 | 123456 0/0/0/0/0/1/2	FRA 0 HAJ 40 40 HAJ	HAJ 40 40 41 41 HAJ	HAJ 40 40 41 41 HAJ	HAJ 40 40 41 41 HAJ
1 | 12 0/0/0/0/0/1/2	FRA 31 31 3210 00-05 SVO	SVO 3213 20-35 481 0—0 470 0—0 FRA		
1 | 12356 0/0/0/0/0/1/2	FRA 3210 00-05 3211 05-20 4 GVA	GVA 45 4 BRU	BRU 4 36 3 NAP	
1 | 12 0/0/0/0/0/1/2	MUC 40 0-0 40 00 3 BUD	BUD 33 05 33 3 MUC		
1 | 12345 0/0/0/0/0/1/2	FRA 3 34 3816 15-15 IST	IST 380 5—0 373 0—5 ATH	ATH 3723 5050 3846 5—5 IST	IST 380 5—0 373 0—5 ATH
1 | 12 0/0/0/0/0/1/2	FRA 4 4 4 GVA	GVA 45 006S 0830—1530 FRA		
1 | 12 0/0/0/0/0/1/2	FRA 4 4 4 CDG	CDG 4 007S 0830—1530 FRA		
1 | 123 0/0/0/0/0/1/2	FRA 3 35 4 BRU	BRU 4 3 3 4 BRU	BRU 4 4916 10—45 4901 45—10 FRA	
1 | 123456 0/0/0/0/0/1/2	FRA 3846 5—5 3847 5055 0 HAM	HAM 40 40 HAM	HAM 40 40 HAM	HAM 40 40 HAM
1 | 1 0/0/0/0/0/1/2	FRA 4 46 005 0—0 FRA			
1 | 123 0/0/0/0/0/1/2	FRA 3736 5—0 ATH	ATH 3723 5060 3 LIN	LIN 36 325 5—0 3251 5—0 FRA	
1 | 1 0/0/0/0/0/1/2	FRA 4916 10—45 4901 45—10 FRA			
1 | 123 0/0/0/0/0/1/2	FRA 3 3 0 HAJ	HAJ 0 28 55 28 55 4 GVA	GVA 45 4 4 FRA	
1 | 1234 0/0/0/0/0/1/2	FRA 2 3230 10-20 3221 20-40 DUS	DUS 4 4 4 4 DUS	DUS 3230 10-20 SVO	SVO 3213 20-35 481 0—0 470 0—0 FRA
1 | 123456 0/0/0/0/0/1/2	FRA 481 0—0 470 0—0 3 STR	STR 40 00 40 STR	STR 40 00 40 STR	STR 40 00 40 STR
1 | 123456 0/0/0/0/0/1/2	FRA 33 3 8 DUS	DUS 4 4 DUS	DUS 4 4 DUS	DUS 4 4 DUS
1 | 1 0/0/0/0/0/1/2	FRA 480 0-5 475 5—0 3 3 FRA			
1 | 1 0/0/0/0/0/1/2	FRA 325 5—0 3251 5—0 FRA			
1 | 1 0/0/0/0/0/1/2	FRA 341 0—5 341 5—0 4 45 FRA			
1 | 1 0/0/0/0/0/1/2	FRA 47 5-5 471 5—0 43 43 55 FRA			
1 | 123 0/0/0/0/0/1/2	MUC 35 0-5 NAP	NAP 3 35 4 GVA	GVA 45 3 3 1 MUC	

Window: 01/11 | 02/11 | 03/11 | 04/11 | 05/11 | 06/11 | 07/11 | 08/11 | 09/11 | 10/11 | 11/11 | 12/11 | 13/11 | 14/11

01/11 | 02/11 | 03/11 | 04/11 | 05/11 | 06/11 | 07/11 | 08/11 | 09/11 | 10/11 | 11/11 | 12/11 | 13/11 | 14/11

Assign value: 0/0/0/0/0//1/2, Crew filter: On
SVO - FRA LH 3211 -1 J 123.56. A320 LH3306 0 F000 C144 M000 1/0/1/0/0//1/2
Gmt : 1605 - 1920 GDOR 1 Date(GDOP): 931101 SSIM 320 area :EU LH LH
Local : 1905 - 2020 Crew corp: booked:0/0/0/0/0//1/2

931101 - 931114 : READY
A320Nov01No cab14_scrat MTV
352 rows. Dated CRRs 931101 - 931104
0 . - 931101 - 931114

The resulting optimization problem

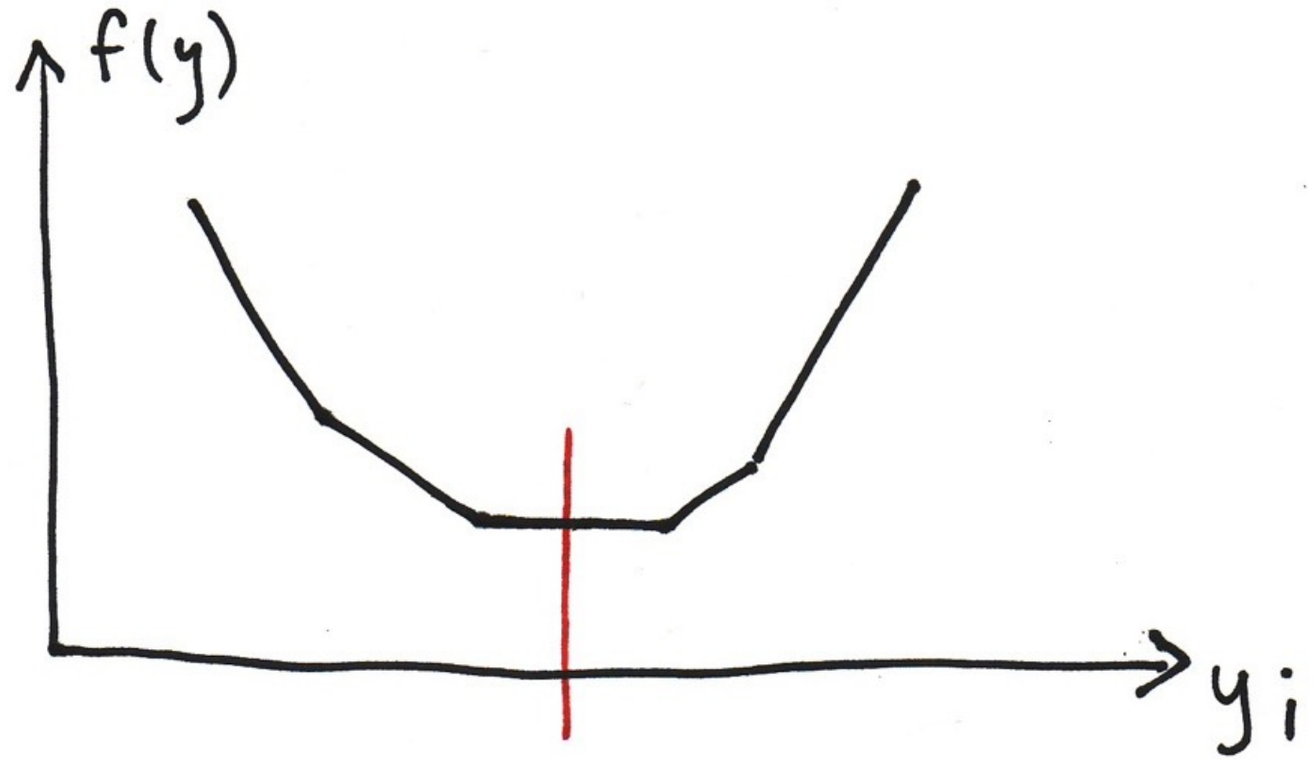
$$\begin{aligned}
\text{minimize} \quad & 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 + x_6 + x_7 + x_8 + 2x_9 + 2x_{10} \\
& +2x_{11} + 2x_{12} + 2x_{13} + 2x_{14} + 2x_{15} + 2x_{16} + 2x_{17} \\
\text{subject to} \quad & \\
& x_1 + x_2 + x_3 + x_4 + x_9 + x_{10} = 1 \\
& x_1 + x_2 + x_3 + x_4 + x_9 + x_{10} = 1 \\
& x_2 + x_5 + x_6 + x_8 + x_{11} + x_{12} + x_{13} + x_{14} + x_{15} = 1 \\
& x_2 + x_3 + x_5 + x_8 + x_{11} + x_{13} + 2x_{14} + 2x_{15} + x_{16} + x_{17} = 1 \\
& x_3 + x_4 + x_7 + x_8 + x_{10} + x_{13} + x_{14} + 2x_{15} + x_{16} + 2x_{17} = 1 \\
& x_4 + x_6 + x_7 + x_8 + x_{10} + x_{12} + x_{13} + x_{15} + x_{17} = 1
\end{aligned}$$

$$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17} \in \{0, 1\}$$

PAROS

A "dual" algorithm

Minimize piecewise linear convex function with coordinate descent



$$\min_y f(y) = yb + \max_{0 \le x \le 1} \bar{c}x$$

Aerolíneas Argentinas

AEROMEXICO.

AEROMEXICO. connect

Aer Lingus

airBaltic

AIR CANADA

AIRFRANCE

Air Wisconsin

Alaska Airlines

ASTAR AIR CARGO

BRITISH AIRWAYS

COMAIR

Continental Airlines

DB

DELTA

DSB

FINNAIR

green cargo

IBERIA

KLM

KLM Cityhopper

LAN

Lufthansa

MEXICANA

SL

SINGAPORE AIRLINES CARGO

SWISS Swiss International Air Lines

miami air INTERNATIONAL

Philippine Airlines

QANTAS

QATAR AIRWAYS القطرية

الخطوط الجوية العربية السعودية SAUDI ARABIAN AIRLINES

SAS Scandinavian Airlines

SINGAPORE AIRLINES

Spanair

Thomson Airways

الخطوط التونسية TUNISAIR

TURKISH AIRLINES

UNITED

U·S AIRWAYS

widerøe

Virgin atlantic

1990: "*datorn kan inte planera!*"

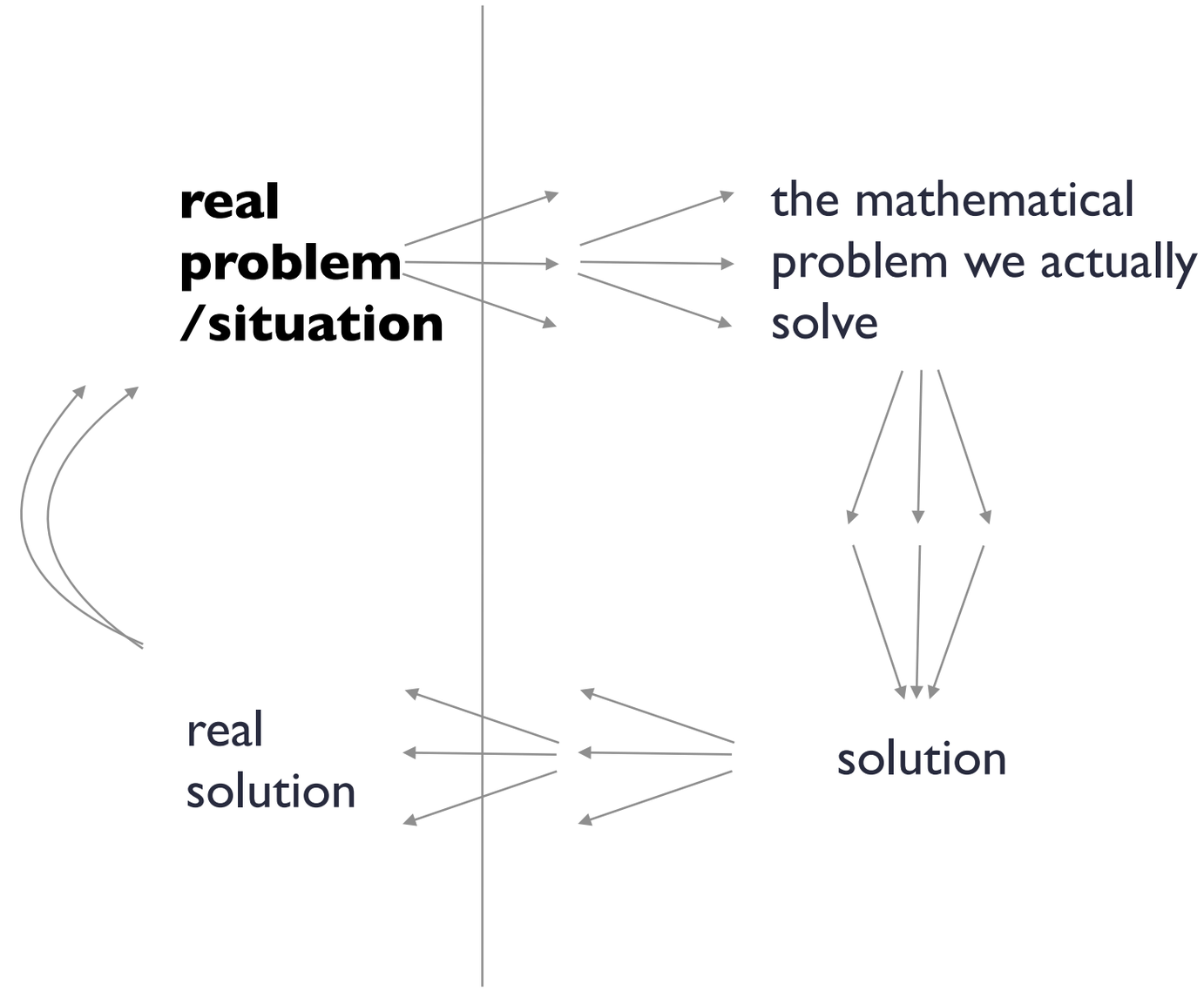
2000: "*optimization matters*"

A student project quite long ago:

Scheduling system for schools

not so easy…

Solving real problems mathematically - **modelling**!

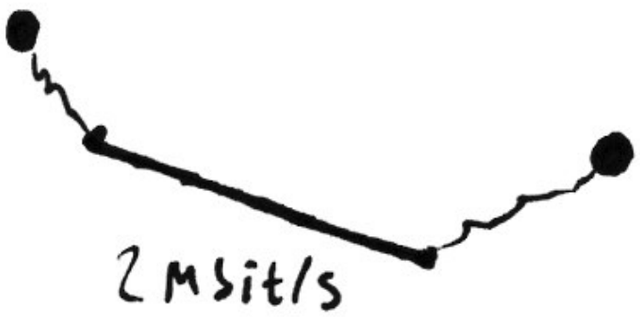


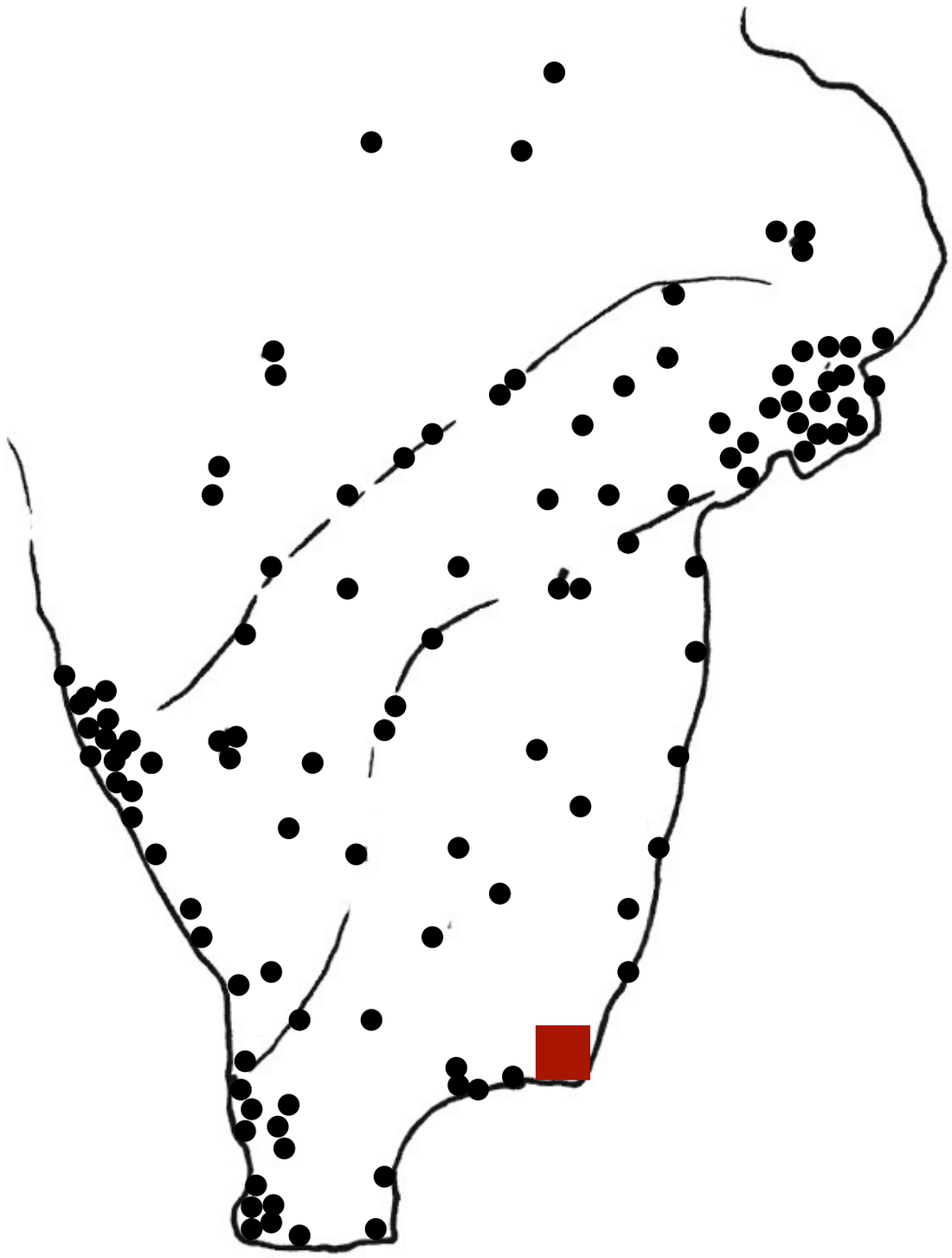
An iterative and creative design process!

Telephone operator problem (real applied problem)

A Swedish mobile phone operator needs to connect all base station to its main switch.

How can we best rent communication lines from the national fixed network?





A simulation based arrival forecast algorithm for public transportation

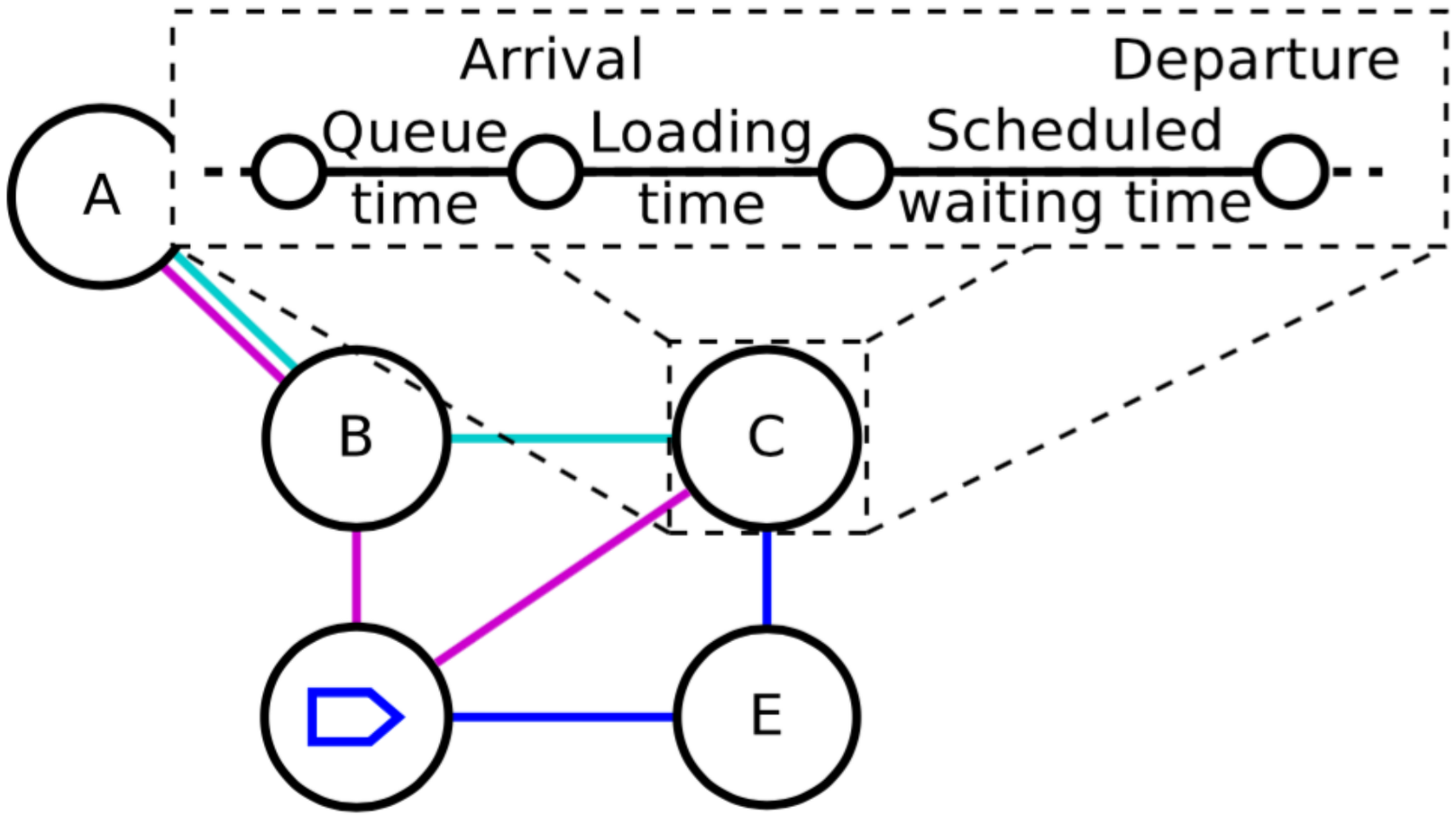


Figure 4.7: Dwell time is estimated by summing up the queue time, loading time and the scheduled waiting time. All these times are estimated separately.

Some recent master's theses

Driving context classification using pattern recognition

Fördelning av platsreservationer på tåg

Innovation in mathematics education - a synthesis of the debate

In the past

main task to build basic IT-infrastructure

significant hardware limitations

not always a very mathematical approach,
especially not in AI-applications

Today

now we want to do more things
with all the data we have!

no hardware limitations

considerable advances due to a more mathematical approach e.g. in machine learning

Some courses

mathematical modelling and problem solving

discrete optimization

algorithms

machine learning

applied optimization

data science

AI

signal and image processing…

cryptography

simulation….

computer graphics

also more computer and computation-oriented branches with other kinds of math…

END