Finite Automata Theory and Formal Languages

TMV026/TMV027/DIT321 – Responsible: Ana Bove

Wednesday 21 of August 2013
Total: 60 points

TMV027/DIT321 registration VT13 Exam valid 6hp	TMV026/DIT321 registration before VT13 Exam valid 7.5 hp
CTH: $\geqslant 27$: 3, $\geqslant 40$: 4, $\geqslant 50$: 5 GU: $\geqslant 27$: G, $\geqslant 45$: VG	CTH: $\geqslant 33$: 3, $\geqslant 43$: 4, $\geqslant 53$: 5 GU: $\geqslant 33$: G, $\geqslant 50$: VG

No help material but dictionaries to/from English or Swedish.

Write in English or Swedish, and as readable as possible (think that what we cannot read we cannot correct).

OBS: All answers should be well motivated. Points will be deduced when you give an unnecessarily complicated solution or when you do not properly justify your answer.

Good luck!

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1. (5pts) Prove by using induction that $\sum_{i\geqslant 0}^{n} i(i+1) = n(n+1)(n+2)/3$. Do not forget to clearly state which kind of induction you are using, the property you will prove, the base case(s) and the inductive hypothesis(es)!

2. Consider the language $\{a^n b \mid n \neq 4m, m \geqslant 0\}$.
   (a) (3pts) Construct a DFA that accepts this language.
   (b) (3pts) Use your intuition and give a (simple) regular expression that generates this language.

3. Consider the following NFA

	0	1
$\rightarrow q_0$	$\{q_1, q_2\}$	$\{q_4\}$
q_1	$\emptyset$	$\{q_0\}$
q_2	$\emptyset$	$\{q_3\}$
q_3	$\{q_0\}$	$\emptyset$
$^*q_4$	$\emptyset$	$\emptyset$

   (a) (2pts) User your intuition and give a (simple) regular expression that generates the language accepted by this NFA.
   (b) (4pts) Convert this NFA to an equivalent DFA.

4. (4.5pts) Minimise the following automaton. Show the intermediate table and justify the construction of the new automaton.

	0	1
$\rightarrow q_0$	q_1	q_2
q_1	q_2	q_3
q_2	q_2	q_4
$^*q_3$	q_3	q_4
$^*q_4$	q_3	q_4
$^*q_5$	q_5	q_4

5. (4pts) Compute, using any of the methods given in class, a regular expression generating the language accepted by the DFA below. Show enough intermediate steps to follow what you are doing!

	0	1
$\rightarrow q_0$	q_1	q_2
q_1	q_1	q_3
${}^*q_2$	q_1	q_2
q_3	q_2	q_3

6. (a) (2pts) Under which operations are regular languages closed? Name all those you can remember.
   (b) (2ts) Show that if $\mathcal{L}_1$ and $\mathcal{L}_2$ are regular, then $\{w \mid w \notin \mathcal{L}_1 \text{ and } w \notin \mathcal{L}_2\}$ is also regular.
   (c) (3pts) Is it true that if $\mathcal{L}_1$ is regular, $\mathcal{L}_2$ is not regular and $\mathcal{L}_1 \cap \mathcal{L}_2$ is regular, then $\mathcal{L}_1 \cup \mathcal{L}_2$ is not regular? Justify your answer as formal as you can.

7. (a) (2pts) What is the definition of a context-free grammar?
   (b) (2pts) What are the steps one needs to perform, and in which order, if one wants to simplify a grammar?

8. (a) (5pts) Give a context free grammar that generates the language $\{a^n b^m c^k \mid k = |n-m|\}$. Explain your grammar!
   (b) (1.5pts) Is your grammar ambiguous? Justify.
   (c) (2.5pts) Give the leftmost derivation and the parse tree of the word *aabbbc*.

9. (a) (2pts) Formulate the Pumping lemma for context-free languages.
   (b) (4.5pts) Use this lemma to show that $\{a^i b^j a^i b^j \mid i, j \geqslant 0\}$ is not context-free.

10. (4pts) Consider the following grammar with start symbol S:

$$S \rightarrow AB \qquad A \rightarrow BB \mid a \qquad B \rightarrow AB \mid b$$

Apply the CYK algorithm to determine if the string *aabab* is generated by this grammar. Show the resulting table and justify your answer.

11. (4pts) Assume your input tape has at most one word written in it. Define a Turing machine (or give its high-level description) over the alphabet $\{a, b\}$ which does nothing if the input tape has less than three non-empty symbols, and otherwise writes the 3rd symbol from the left at the end of the input. For example, if the input is *aabaaba* the output becomes *aabaabab*. Explain your machine.