# Measurement theory
## - for the interested student

Erland Jonsson

Department of
Computer Science and Engineering
Chalmers University of Technology

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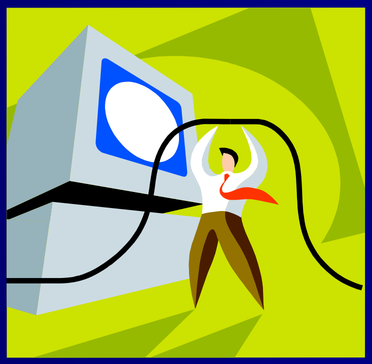
1. This presentation is partly based on Norman E Fenton: Software Metrics, 1st. ed.

# What is measurement ?



# Definition of measurement

- **Definition:**
  - Measurement is the process of empirical, objective encoding of some property of a selected class of entities in a formal system of symbols (A. Kaposi based on Finkelstein)
  - Cp Metrology is the field of knowledge con-cerned with measurement. Metrology can be split up into theoretical, methodology, technology and legal aspects.

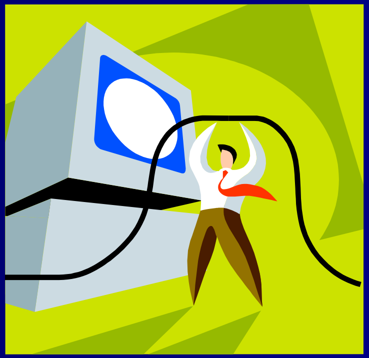
# General requirements on measurement operations

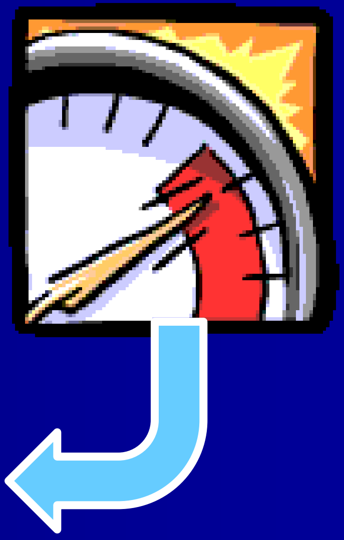
- Operations of measurement involve collecting and recording data from observation
- It means identifying the class of entities to which the measurement relates
- Measurements must be independent of the views and preferences of the measurerer
- Measurements must not be corrupted by an incidental, unrecorded circumstance, which might influence the outcome

# Specific requirements on measurement operations

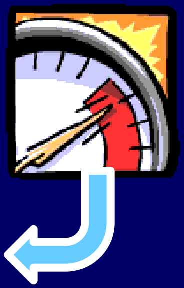
- Measurement must be able to characterize abstract entities as well as to describe properties of real-world objects
- The result of measurement may be captured in terms of any well-defined formal system, i.e. not necessarily involving numbers

# Relational systems
## - from measurement theory



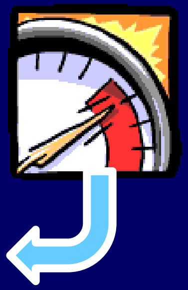


# Relational systems



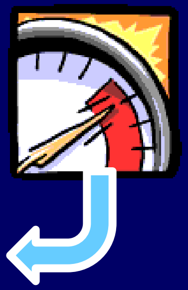
- There are two types of relational systems:
  - the empirical relational system
  - the formal relational system
- These two relational systems gives the theoretical basis for defining measurement scales

# Empirical relational system



- Let ***A*** = {*a,b,c,.....,z*} be the target set and $\kappa$ the chosen key property
- Ex. A is the set of schoolchildren in a class and $\kappa$ is the property of their height.
- Now let A = {a,b,c,.....,z} be the model of ***A*** which describes each child in terms of the property height
- The empirical relational system comprises this model set together with all the operations and relations defined over the set

# **Empirical relational system** (con't)

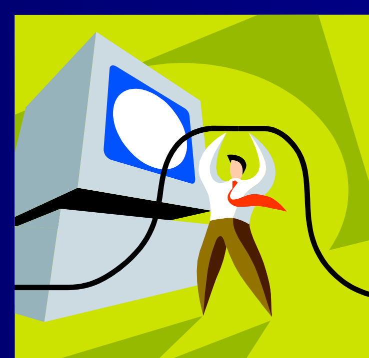


- We can now attempt to describe the empirical relational system as an ordered set
  E = (A,R,O) = (A, {r1,r2,r3},{o}), where
- R is a set of relations:
  r1 = taller than
  r2 = the same height as
  r3 = heads and shoulders above, and
- O is a set of binary operations:
  o is the operator "standing on the head of"

# Formal relational system

- We can now attempt to describe the formal relational system as a nested set F = (A',R',O')
- The formal relational system F must be capable of expressing all of the relations and operations of the empirical relational system E
- The mapping from E to F must actually represent all of the observations, preserving all the relations and operations of E
- If this is true we say that the mapping A -> A' is a homomorphism and F is homomorphic to E

# Scaling and scale types
## - from measurement theory



# Scale of measurement

- Assume that:
  1. The set A models the target set ***A***, wrt to $\kappa$
  2. We have the empirical system E = (A,R,O) and the formal system F = (A′,R′,O′)
  3. m is a homomorphic mapping from E to F
- Then S = (E, F, m) is called the scale of measurement for the key property $\kappa$

# Scale of measurement

- Now if F is defined over some subset of real numbers, then:
  - measurement maps the key property of each object of the target set into a number
  - further, if the mapping is homomorphic then:
    1. the measured data will be representative of the key property of the corresponding object, and
    2. empirical relations and operations on the properties will have correct representations on the corresponding numbers

# Scale of measurement

- The homomorphism assures that these formal conclusions, drawn in the number domain will have corresponding conclusions in the empirical domain and thus that the purpose of the measurement is fulfilled
- Or in more general terms:
  Our theoretical conclusions will be valid to the real world and let us draw corresponding conclusions for it
- A homomorphism is seldom unique, e.g cost can be expressed in EUROs or in SEK

# Measurement scales

- Mesurement theory distinguishes five types of scale:

  - nominal scale
  - ordinal scale
  - interval scale
  - ratio scale
  - absolute scale

- Here they are given in an ascending order of ”strength”, in the sense that each is permitting less freedom of choice and imposing stricter conditions than the previous one

# Nominal scale

- The nominal scale can be used to denote membership of a class for purposes such as labelling or colour matching
- There are no operations between E and F
- The only relation is equivalence
- One-to-one mapping

# Ordinal scale

- The ordinal scale is used when measurement expresses comparitive judgement
- The scale is preserved under any montonic, transformation:

$$x>=y \Leftrightarrow \phi(x)>=\phi(y),$$

  where $\phi$ is an admissible transformation
- Used for grading goods or rating candidates

# Interval scale

- The interval scale is used when measuring ”distance” between pairs of items of a class according to the chosen attribute
- The scale is preserved under positive linear transformation:

$$\phi(x) = \alpha m + \beta, \text{ where } \alpha > 0$$

- Used for measuring e.g. temperature in centigrade or Fahrenheit (but not Kelvin) or calendar time

# Ratio scale

- The ratio scale denotes the degree in relation to a standard. It must preserve the origin.
- It is the most frequently used scale
- The scale is preserved under the transformation:

  $$\phi(x) = \alpha m \text{ , where } \alpha > 0$$

- Used for measuring e.g. mass, length, elapsed time and temperature in Kelvin

# Absolute scale

- The absolute scale is a ratio scale which includes a ”standard” unit.
- The scale is only preserved under the identity transformation:

  $$\phi(x) = x,$$

  which means that it is not transformable
- Used for counting items of a class

# Meaningfulness

- Meaningfulness means that the scale measurement should be appropriate to the type of property measured, such that
  once measurement has been performed – and data expressed on some scale - sensible conclusions can be drawn from it
- Example 1: Point A is twice as far as point B (meaningless, since distance is a ratio scale, but position is not)
- Example 2: Point A is twice as far from point X as point B (is meaningful)

# Summary

- We have given a brief and heuristic overview of a few concepts from measurement theory
- We have described a number of scales
- We have defined ”scale of measurement” as a homomorphic mapping from an empirical relational system to a formal relational system