Homotopy limits in cubical sets

Homotopy limits

We consider a semisimplicial diagram of cubical sets $s_k^{n+1} : A_n \to A_{n+1}$ and we want to take the homotopy limit of this diagram.

The point of this note is to describe a possible coding of this limit L.

An object u of L consists of a sequence of elements $u(i_0, \ldots, i_n)$ in A_n for $i_0 = 1 \vee \cdots \vee i_n = 1$ satisfying the compatibility conditions

$$u(i_0, \ldots, i_{k-1}, 0, i_k, \ldots, i_n) = s_k^n u(i_0, \ldots, i_{k-1}, i_{k+1}, \ldots, i_n)$$

So we have one element $u(1)$ in A_0 then two lines $u(1, i)$ and $u(i, 1)$ in A_1 with $u(1, 0) = s_0^1 u(1)$ and $u(0, 1) = s_1^1 u(1)$ and so on.

We have a map $L \to A_n$ defined by $u \mapsto u(1, \ldots, 1)$.

Special case

We consider a (strict) pointed endofunctor E which commutes strictly with such limit.

Each such functor defines a semisimplicial diagram for any object A by taking $A_n = E^{n+1}A$

The homotopy limit of this diagram DA defines then a new (strict) pointed endofunctor with a map $\eta_A : A \to DA$ and which satisfies that there is a path between $D(\eta_A)$ and η_{DA}.

Acknowledgement

References