Finite Automata: Homework 2

1. Let Σ be $\{0, 1\}$. Give a NFA with four states equivalent to the regular expression $(01 + 011 + 0111)^*$, and give then a DFA equivalent to it by the subset construction.

2. Let Σ be $\{0, 1\}$. Give a regular expression which represents the complement of the language represented by $(0 + 1)^*01(0 + 1)^*$.

3. (Representation of Gilbreath's theorem[1].) Let $\Sigma = \{0, 1\}$. We use words in Σ^* to represent abstract card decks, with 0 for black and 1 for red. We say that a word in Σ^* is *alternating* iff it is in the language defined by $0(10)^*(\epsilon + 1)$ or by $1(01)^*(\epsilon + 0)$. What does this mean in term of card decks? The goal is to prove using automata theory the following result: *if one shuffles two alternating words, one starting by* 0 *and the other by* 1*, then one obtains a word in* $(01 + 10)^+(\epsilon + 0 + 1)$.

   For this, build a NFA on the following set of states in order to represent abstractly the shuffle operation:

   - p_0: two alternating words, one starting with 0 and one with 1
   - p_1: two alternating words, each starting with 0
   - p_2: two alternating words, each starting with 1
   - p_3: one alternating word starting with 0 and the empty word
   - p_4: one alternating word starting with 1 and the empty word
   - p_5: two empty words

   One can go from one state to another by choosing one non empty word and by taking away the first letter of this word, the observable event being then this first letter. For instance one has $\delta(p_0, 0) = \{p_2, p_4\}$: this corresponds in choosing in the state p_0 the word starting with 0; one goes to the state p_4 if this word was reduced to 0 (and becomes ϵ) and to the state p_2 if this word was not 0. Similarly $\delta(p_3, 0) = \{p_4, p_5\}$, etc...

   Take p_0 to be the starting state and p_5 to be the only accepting state to define the NFA A. Compute the language $L(A)$ as a regular expression and show that it is equal to $(01 + 10)^+(\epsilon + 0 + 1)$.

[1] This mathematical result is a model of the following card trick. "The two words represent abstract card decks, with 0 for black and 1 for red. Take an even deck, that you have previously arranged alternatively red, black, red, black, etc. Ask a spectator to cut the deck, into sub-decks u and v. Now shuffle u and v into a new deck. Before shuffling note carefully whether u and v starts with opposite colors (state p_0) or not (state p_1 or p_2). If they do, the resulting deck is composed of pairs red-black or black-red; otherwise, you get the property by first rotating the deck by one card. When showing the pairing property, show the card by pairs and say "red back red black..." in order to confuse in the spectator's mind the weak paired property with the strong alternate one."