Erratum in
"Verifying a Semantic $\beta\eta$-Conversion Test for Martin-Löf Type Theory"
and
"Irrelevance in Type Theory with a Heterogeneous Equality Judgement"

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The Kripke model for completeness presented in Section 5 of the article [ACD08] has problems. The relation Ⓢ cannot be shown transitive, at least not directly, thus Lemma 1 breaks.

The definition is:

$$\Delta \vdash f : \mathsf{Fun}\, X F \;\text{Ⓢ}\; \Delta' \vdash f' : \mathsf{Fun}\, X' F'$$
$$\iff \quad \text{for all } \hat{\Delta} \le \Delta, \hat{\Delta}' \le \Delta', d, d', \;\; \hat{\Delta} \vdash d : X \;\text{Ⓢ}\; \hat{\Delta}' \vdash d' : X'$$
$$\text{implies } \hat{\Delta} \vdash f \cdot d : F \cdot d \;\text{Ⓢ}\; \hat{\Delta}' \vdash f' \cdot d' : F' \cdot d'$$

To show transitivity, assume $\Delta_1 \vdash f_1 : \mathsf{Fun}\, X_1\, F_1$ Ⓢ $\Delta_2 \vdash f_2 : \mathsf{Fun}\, X_2\, F_2$ and $\Delta_2 \vdash f_2 : \mathsf{Fun}\, X_2\, F_2$ Ⓢ $\Delta_3 \vdash f_3 : \mathsf{Fun}\, X_3\, F_3$ and show $\Delta_1 \vdash f_1 : \mathsf{Fun}\, X_1\, F_1$ Ⓢ $\Delta_3 \vdash f_3 : \mathsf{Fun}\, X_3\, F_3$. That is, assume arbitrary $\hat{\Delta}_1 \le \Delta_1$ and $\hat{\Delta}_3 \le \Delta_3$ and $\hat{\Delta}_1 \vdash d_1 : X_1$ Ⓢ $\hat{\Delta}_3 \vdash d_3 : X_3$ and show $\hat{\Delta}_1 \vdash f_1 \cdot d_1 : F_1 \cdot d_1$ Ⓢ $\hat{\Delta}_3 \vdash f_3 \cdot d_3 : F_3 \cdot d_3$. We would like to proceed by induction hypothesis, i.e., transitivity, on $\hat{\Delta}_1 \vdash f_1 \cdot d_1 : F_1 \cdot d_1$ Ⓢ $\hat{\Delta}_3 \vdash f_2 \cdot d_3 : F_2 \cdot d_3$ and $\hat{\Delta}_3 \vdash f_2 \cdot d_3 : F_2 \cdot d_3$ Ⓢ $\hat{\Delta}_3 \vdash f_3 \cdot d_3 : F_3 \cdot d_3$. However, this requires us to show $\hat{\Delta}_3 \le \Delta_2$, which does not hold.

While a *homogeneous* Kripke logical relation $\Delta \vdash d\,\text{Ⓢ}\,d' : X$ over a transitive base relation $\Delta \vdash d\,\text{Ⓡ}\,d' : X$ is always transitive, this breaks down for a *heterogeneous* Kripke logical relation $\Delta \vdash d : X$ Ⓢ $\Delta' \vdash d' : X'$, even if the base relation is transitive. This is what I overlooked.

The article [Abe11] has inherited this problem; the relation presented in Section 4.2 is not transitive.

A newer article [AS11] shows how to get the meta-theory right.

References

[Abe11] Andreas Abel. Irrelevance in type theory with a heterogeneous equality judgement. In Martin Hofmann, editor, *Foundations of Software Science and Computational Structures, 14th International Conference, FOSSACS 2011, Held as Part of the Joint European Conferences on Theory and Practice of Software, ETAPS 2011, Saarbrücken, Germany, March 26 - April 3, 2011. Proceedings*, volume 6604 of *Lecture Notes in Computer Science*, pages 57–71. Springer-Verlag, 2011.

[ACD08] Andreas Abel, Thierry Coquand, and Peter Dybjer. Verifying a semantic $\beta\eta$-conversion test for Martin-Löf type theory. In Philippe Audebaud and Christine Paulin-Mohring, editors, *Mathematics of Program Construction, 9th International Conference, MPC 2008, Marseille, France, July 15-18, 2008. Proceedings*, volume 5133 of *Lecture Notes in Computer Science*, pages 29–56. Springer-Verlag, 2008.

[AS11] Andreas Abel and Gabriel Scherer. On irrelevance and algorithmic equality in predicative type theory. *TYPES'10 post proceedings*, 2011. Submitted.