Matematisk Modellering

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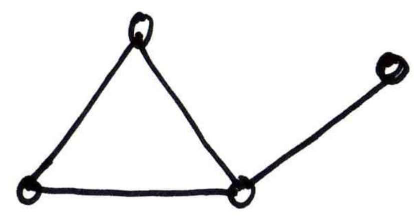
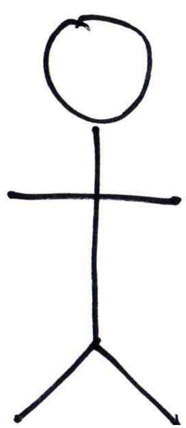
Different kinds of models

$$F = G \frac{m_1 m_2}{r^2}$$

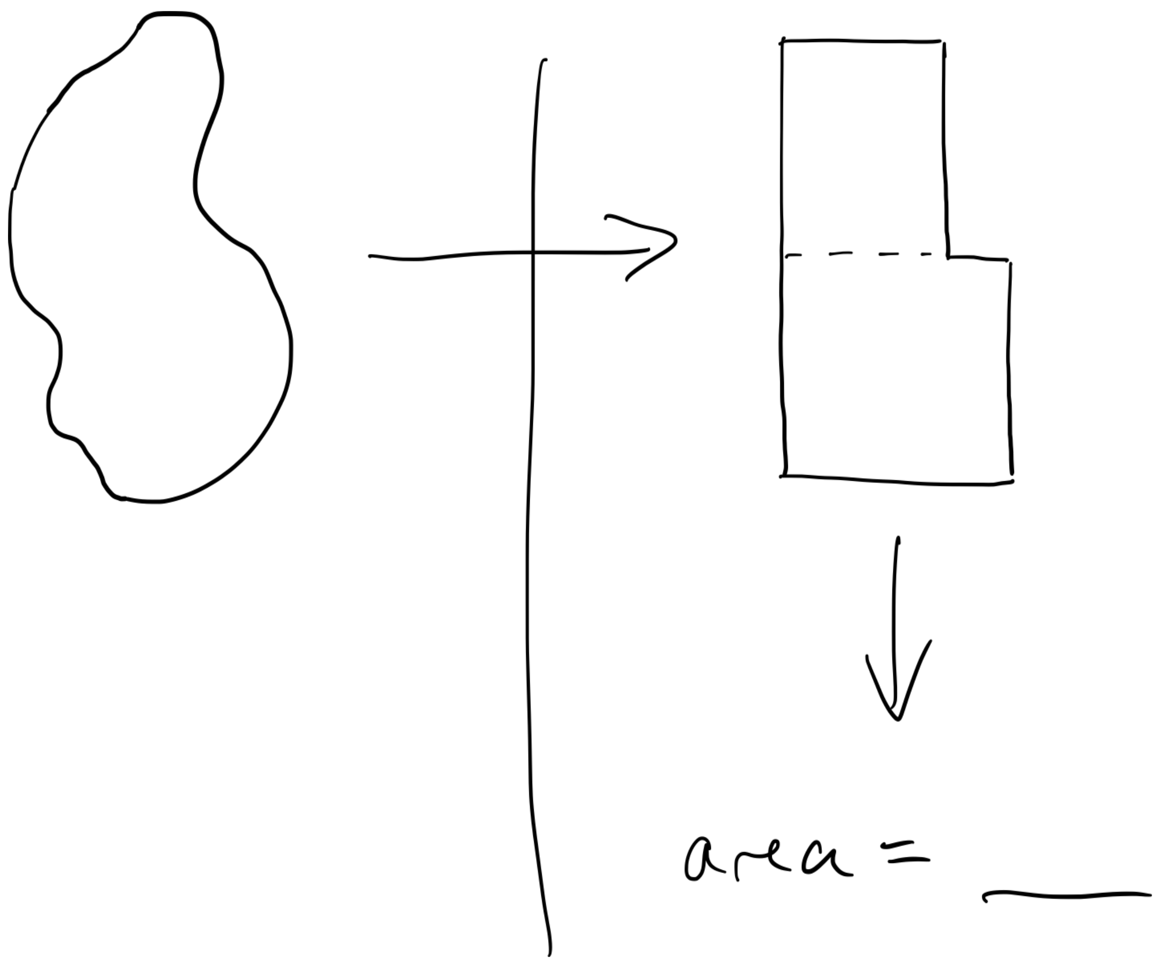




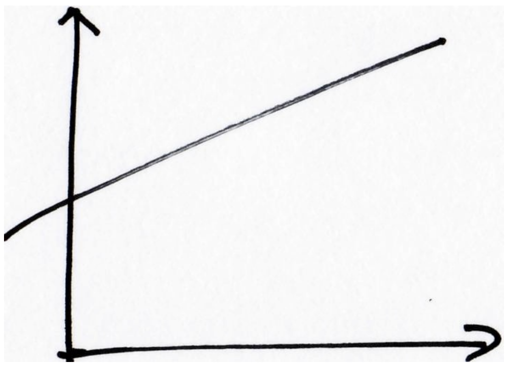
man(Socrates)

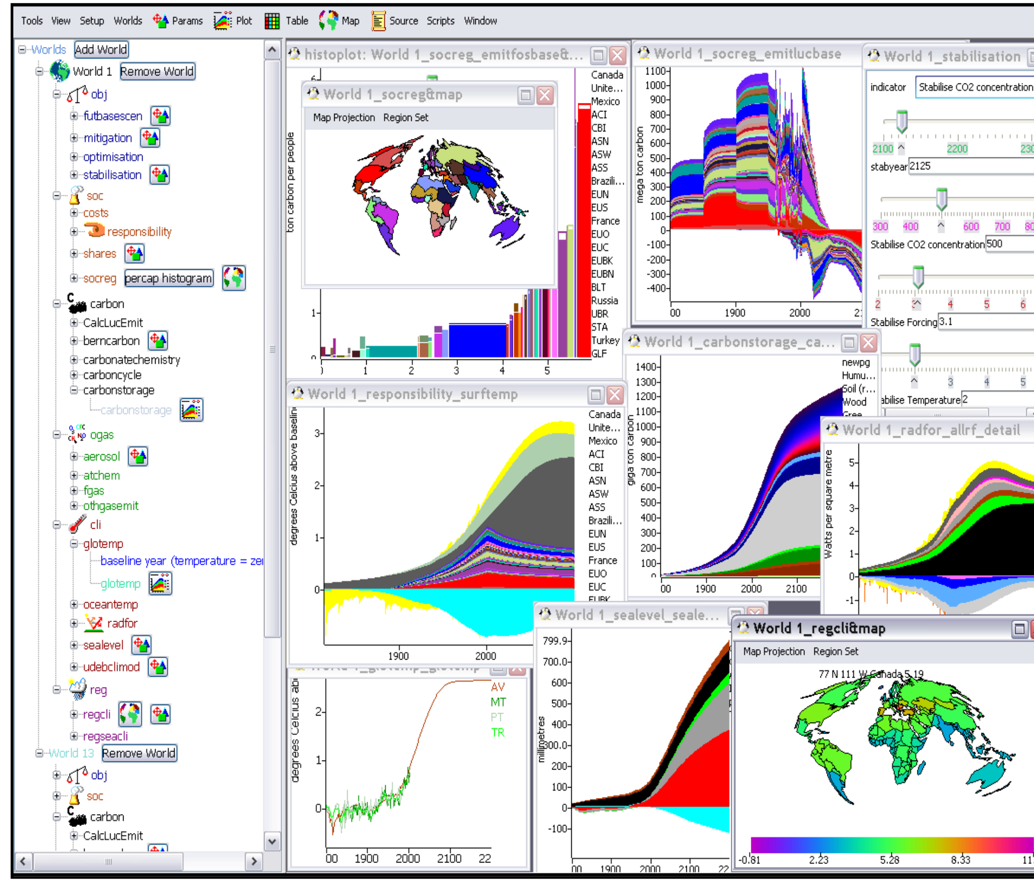
man(X) ⇒ mortal(X)

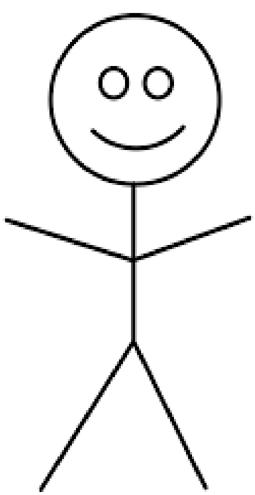
Why models?



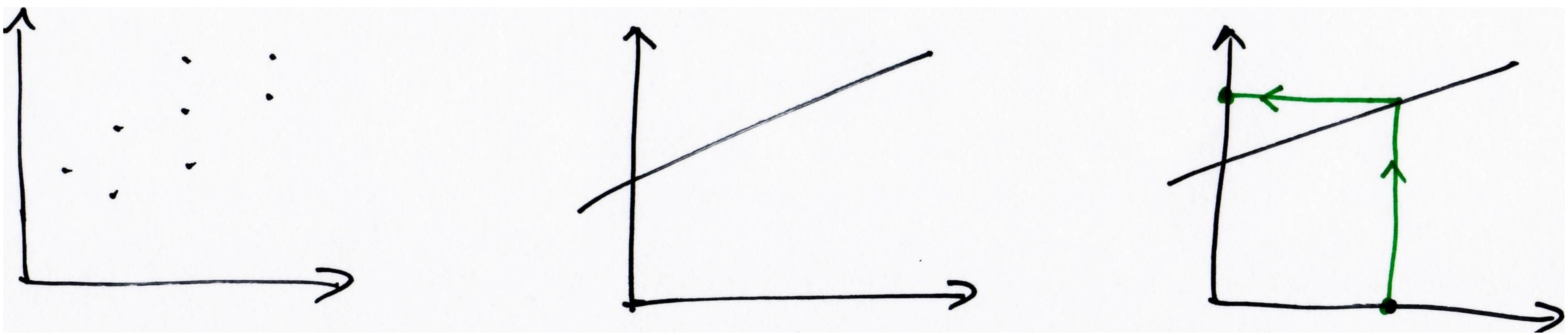
Simple and complicated models







Mathematical modelling = descriptive mathematics!

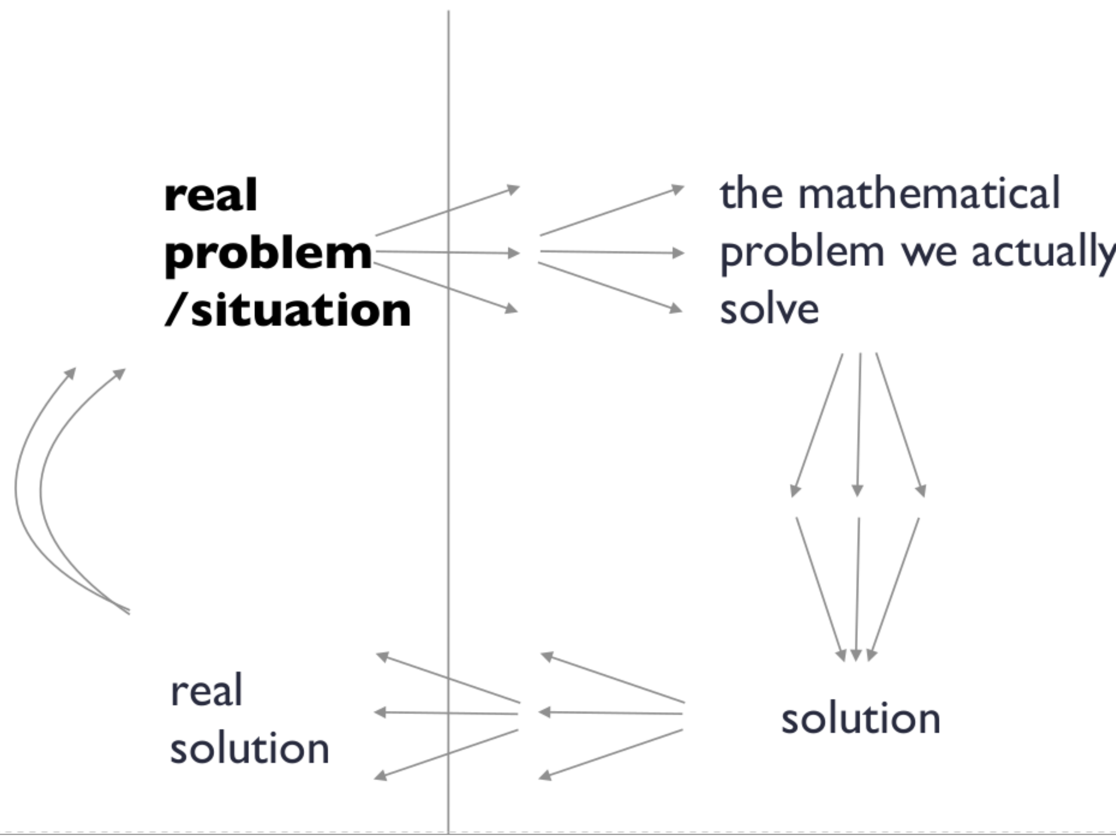


Some aspect of reality → Mathematical Model → Draw conclusions from model

not exact
creative

often exact
more analytical

Solving real problems mathematically - **modelling**!



Weekly exercise modules

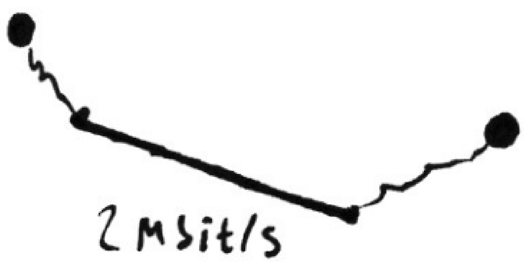
1. Introduction lecture
2. Do exercises and get supervision during the week
3. Follow-up lecture next week
4. Reflection

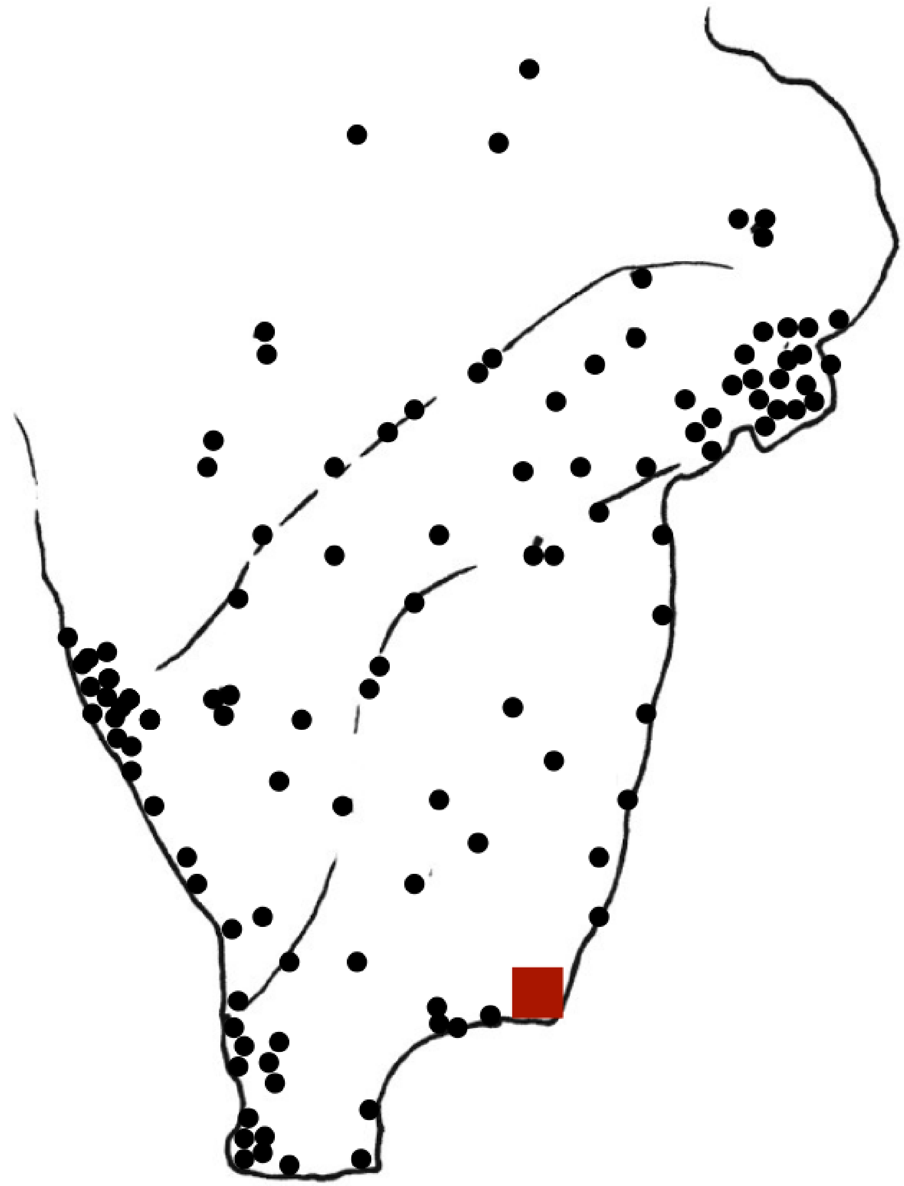
The different modules focus on different model types.

Telephone operator problem (real applied problem)

A Swedish mobile phone operator needs to connect all base station to its main switch.

How can we best rent communication lines from the national fixed network?






Graphic Pairing Construction
Data
Plan
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APC
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Options
Help
15:25
Window
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01/11 02/11 03/11 04/11
Assign value: 0/0/0/0/0//1/2, Crew filter: On
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931101 - 931114 : READY
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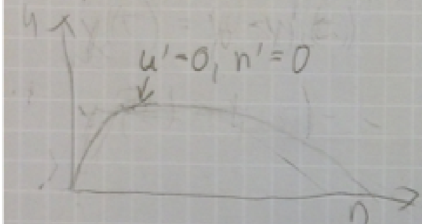
med lågt valbestånd. (När krill tar slut måste valarna gå av innan krillbeståndet återhämtar sig. Tills krillbeståndet växer fortare än valarna)

$y(0)=0$

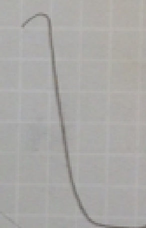
$y'(t) = max - y(t)$

upptagning $u(t)$

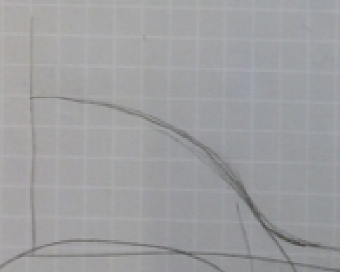
nedbrytning $n(t)$

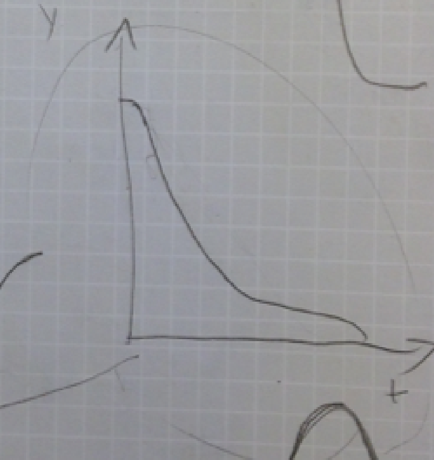


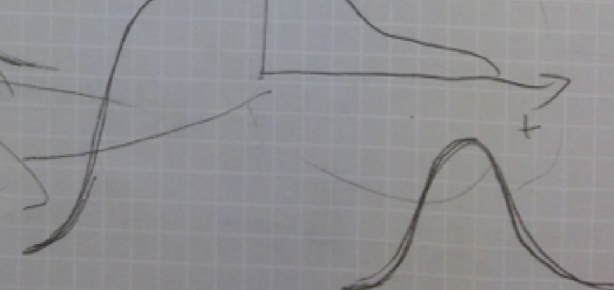
$u'(t) =$



$y(t) = max - y'(t)$



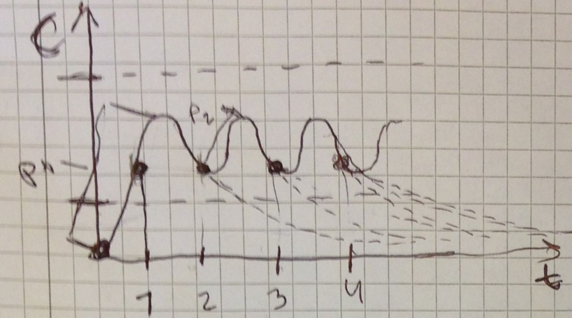




$y'(t) = - \frac{y(t)}{a}$

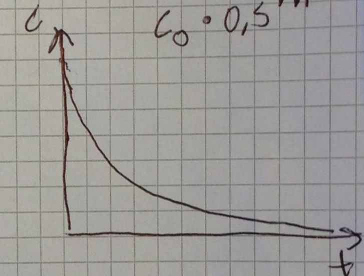
$u'(t) = \frac{y(t)}{b}$

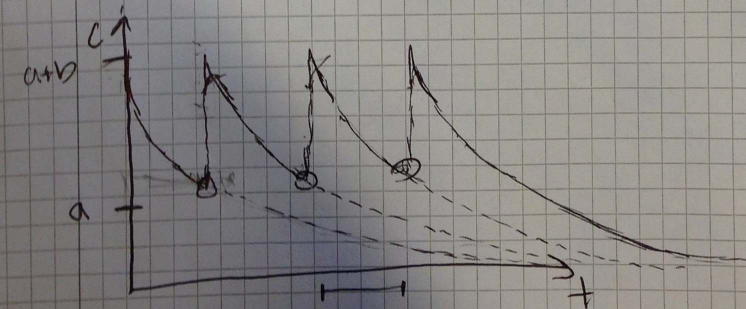
$n'(t) = n(t)(a-b)$



Interested in dose & time interval.
Depends on accepted low & high,
half-life of the substance

$c_0 \cdot 0{,}5^{t/h}$





$$\left(\left(\left(\left(D\left(\tfrac{1}{2}\right)^{t/n}+D\right)\tfrac{1}{2}^{t/n}\right)+D\right)\left(\tfrac{1}{2}\right)^{t/n}\right)\text{-----}\Big)\left(\tfrac{1}{2}\right)^{t/n}$$

$$D\cdot \tfrac{1}{2}^{nt/n}+D\cdot\tfrac{1}{2}^{(n-1)t/n}+D\cdot\tfrac{1}{2}^{(n-2)t/n}+D\cdot\tfrac{1}{2}^{(n-3)t/n}+$$
$$+\text{---}+D\cdot\tfrac{1}{2}^{t/n}=D$$

$$\tfrac{1}{2}^{nt/n}+\tfrac{1}{2}^{(n-1)t/n}+\text{--}+\tfrac{1}{2}^{t/n}=1$$

$$\frac{1}{2^{nt/n}}+\frac{1}{2^{(n-1)t/n}}+\dots+\frac{1}{2^{1\cdot t/n}}=1$$

$$\Big(D\cdot\big(\underbrace{\tfrac{1}{2}^{n}+\tfrac{1}{2}^{n-1}+\tfrac{1}{2}^{n-2}+\tfrac{1}{2}^{n-3}+\dots+\tfrac{1}{2}^{2}+\tfrac{1}{2}^{1}}_{M}\big)\Big)^{t/n}=D$$

$$D^{t/n}\cdot M^{t/n}=D$$

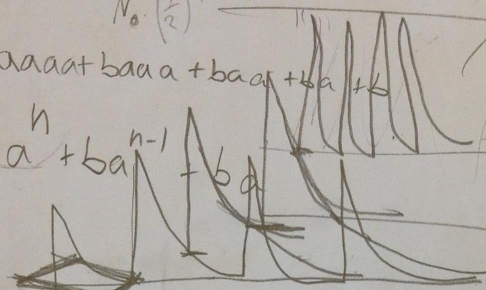
$((a+b)a+b)a+b$

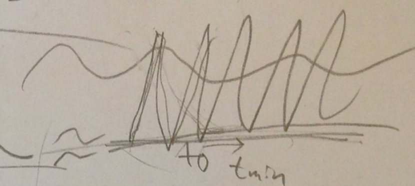
$N_0 \cdot \left(\frac{1}{2}\right)^{\frac{t}{T}}$

$k' = \frac{-\ln(2)}{h} \cdot k$

aaaa+baaa+baa+ba+b

$a^n + ba^{n-1} + ba$



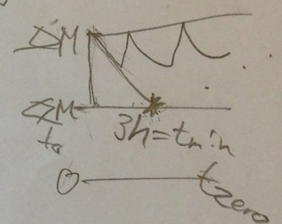


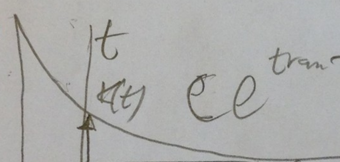
$K(t) = C = (Max - min)$

t

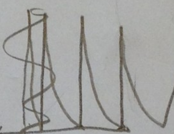
$a^n + ba^{n-1} + ba^{n-2} + \dots + ba^1$ varde

$k[0]$





$e e^{tran}$



$K(t) \approx (max - min)(1/2)^{t/h} = (max - min)$

$K(t) = D \cdot (1/2)^{t/h}$

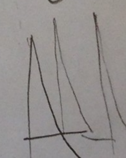
$K(t_{min}) \approx (max - min)\ D\ (1/2)^{t_{min}/h}$, $n \cdot t_{min} \to \infty$

$n \cdot t_{min}$

$F = \frac{1}{s}e^{sx}$

$f(x) = e^{sx}$

$f'(x) = se^{sx}$



$a^n + b^{n-1} + b^{n-2} + b^{n-3} + b^{n-4} + b^1$

$\lim_{t \to \infty} \max(A(t)) = \max$

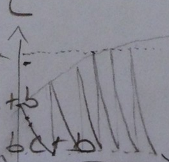
-1 ?

$((a+b)c+b)c$

$ac + bc + b$ max

$acc + bcc + bc + b$

$accc + bcc + bc + b$



$\left(\left(D(1/2)^{t/h} + D\right)(1/2)^{t/h} + D\right)(1/2)^{t/h}$

$f(0) = 0, \quad f(n) = (f(n-1) + D)(1/2)^{t/h}$

$D(1/2)$

Vi använder sprutor för att administrera dosen så upptagningen blir direkt.

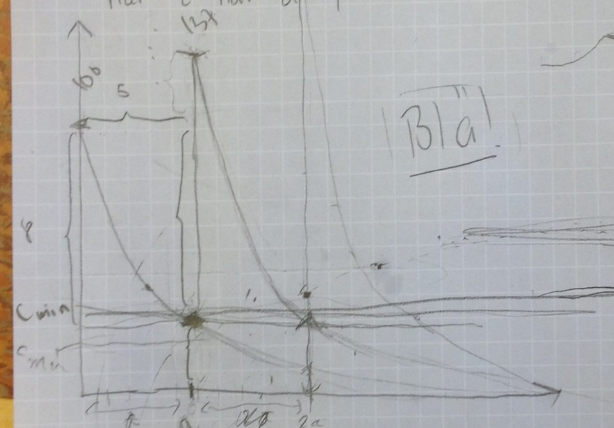
nedbrytning i kroppen $\Rightarrow$ $y(t)$

$$y'(t) = \frac{-y(t)}{a} \Rightarrow y'(t) = -y(t)\cdot\frac{1}{a}$$

→ a värde

$$\Rightarrow y(t) = c_1 e^{-\frac{t}{a}} \Rightarrow 0{,}37 \cdot c$$

när t når a finns ≈37% kvar av dosen



Blå

$e^{-1} \approx 0{,}37$

$c_1 e^{-ta}$

$$1:\ c_1 \frac{1}{e^{t/a}}$$

$$2:\ c_1 e^{-\frac{t+a}{a}} + c_1 e^{-t/a} = y(t+a) + y(t)$$

$$3:\ c_1 e^{-\frac{t+2a}{a}} + c_1 e^{-\frac{t+a}{a}} + e^{-t/a} c_1$$

$$\frac{1}{e^{ta}}$$

$$\frac{1}{e^{\frac{t}{a}}}$$

$$n:\ \sum_{i=0}^{n-1} y(t+i\cdot a) \Rightarrow c_{min}$$

$$\Rightarrow \frac{c\cdot e^{1-\frac{t}{a}}}{e-1} \Rightarrow c_{min}$$

Think and
struggle!

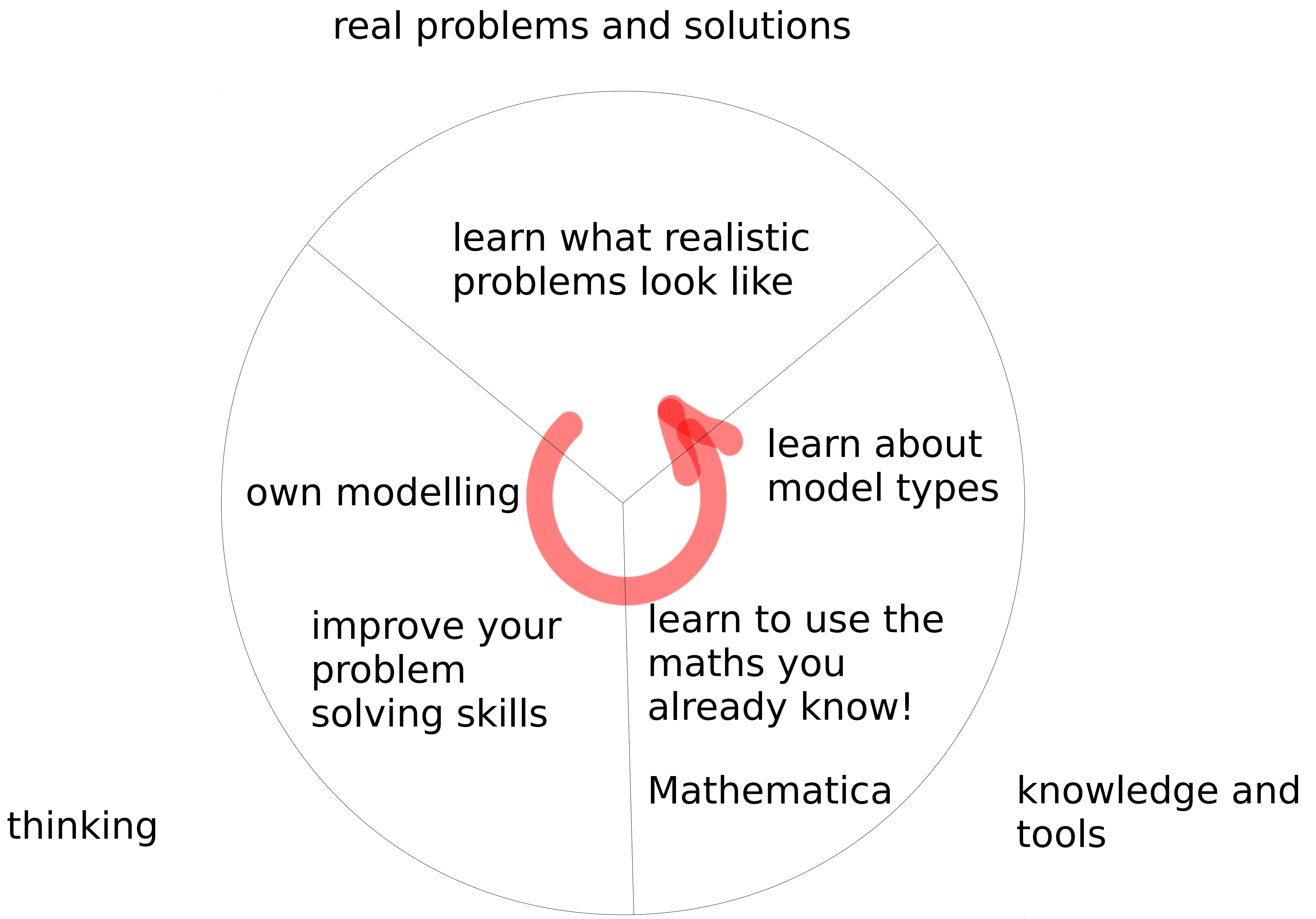
What is needed to solve a problem?

very different balance
for different problems

knowledge needed for solving a problem = knowledge created by own thinking + knowledge from others

because of the variation
you often have to add
something here!

What will
you learn?


real problems and solutions
learn what realistic problems look like
own modelling
learn about model types
improve your problem solving skills
learn to use the maths you already know!
Mathematica
thinking
knowledge and tools

Vienna Symphonic Library

Bösendorfer mic'd for sampling

Example of changing underlying models: electronic pianos

1960's: simple waveform and decay synthesis

1980's: sampling synthesis

2000- : physical modelling

Bösendorfer mic'd for sampling